\[ \text{KCL (ch2)} \]
\[ \sum i_n = 0 \]

\[ \text{KVL (ch2)} \]
\[ \sum V_n = 0 \]

(oh3) Node Voltage Analysis
(Nodal Analysis)

Mesh Current Analysis (oh3)
(Mesh Analysis)

By - Inspection Method
(direct approach to build up the equations)
Chapter 3

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Objectives

Learn to:

- Apply the node-voltage and mesh-current methods to analyze an electric circuit of any configuration, so long as it is linear and planar.
- Apply the by-inspection methods to circuits that satisfy certain conditions.
- Use the source-superposition method to evaluate the sensitivity of a circuit to the various sources in the circuit.

The basic laws of Chapter 2 are used in the present chapter to develop standard solution methods that can be applied to analyze any linear circuit, no matter how complex.

- Determine the Thévenin and Norton equivalent circuits of any input circuit and use them to evaluate the response of an external load (or an output circuit) to the input circuit.
- Establish the conditions for maximum transfer of current, voltage, and power from an input circuit to an external load.
- Learn the basic properties of the bipolar junction transistor.

\[ \text{kCL} \]
Node-Voltage Method

\[ \begin{align*}
(\frac{1}{R_1} + \frac{1}{R_2 + R_3} + \frac{1}{R_4}) V_1 - \left(\frac{1}{R_4}\right) V_2 &= \frac{V_0}{R_2 + R_3}, \\
-\left(\frac{1}{R_4}\right) V_1 + \left(\frac{1}{R_4 + \frac{1}{R_6}}\right) V_2 - \frac{V_3}{R_6} &= I_0,
\end{align*} \tag{3.8a} \]

and

\[ \left(-\frac{1}{R_6}\right) V_2 + \left(\frac{1}{R_5 + \frac{1}{R_6}}\right) V_3 = -I_0. \tag{3.8c} \]

Three equations in 3 unknowns:
Solve using Cramer’s rule, matrix inversion, or MATLAB

by inspection

\[
\begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3
\end{pmatrix}
= 
\begin{pmatrix}
b_1 \\
b_2 \\
b_3
\end{pmatrix}
\]

Nodal Analysis by Inspection

□ Requirement: All sources are independent current sources

\[
G_{kk} = \text{sum of all conductances connected to node } k
\]

\[ G_{kl} = G_{lk} = \text{negative of conductance(s) connecting nodes} \]

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\( G_{kk} \) = sum of all conductances connected to node \( k \)

\( G_{k\ell} = G_{\ell k} = \text{negative of conductance(s) connecting nodes } k \text{ and } \ell, \text{ with } k \neq \ell \)

\( V_k \) = voltage at node \( k \) \text{ unknowns}

\( I_{Ik} \) = total of current sources entering node \( k \) (a negative sign applies to a current source leaving the node).
Example 3-7: Nodal by Inspection

\[
\begin{align*}
G_{11} &= \frac{1}{1} + \frac{1}{5} + \frac{1}{10} = 1.3, \\
G_{22} &= \frac{1}{5} + \frac{1}{2} + \frac{1}{10} = 0.8, \\
G_{33} &= \frac{1}{10} + \frac{1}{20} = 0.15, \\
G_{44} &= \frac{1}{10} + \frac{1}{20} = 0.15.
\end{align*}
\]

Off-diagonal elements
\[
\begin{align*}
G_{12} &= G_{21} = -\frac{1}{5} = -0.2, \\
G_{13} &= G_{31} = -\frac{1}{10} = -0.1, \\
G_{14} &= G_{41} = 0, \\
G_{23} &= G_{32} = 0, \\
G_{24} &= G_{42} = -\frac{1}{10} = -0.1, \\
G_{34} &= G_{43} = -\frac{1}{20} = -0.05.
\end{align*}
\]

Currents into nodes
\[
\begin{align*}
I_{t1} &= 2 \text{ A}, \\
I_{t2} &= 3 \text{ A}, \\
I_{t3} &= 4 \text{ A}, \\
I_{t4} &= -4 \text{ A}.
\end{align*}
\]

\[
\begin{bmatrix}
1.3 & -0.2 & -0.1 & 0 \\
-0.2 & 0.8 & 0 & -0.1 \\
-0.1 & 0 & 0.15 & -0.05 \\
0 & -0.1 & -0.05 & 0.15
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix}
= \begin{bmatrix}
2 \\
3 \\
4 \\
-4
\end{bmatrix}
\]

\[
\begin{align*}
V_1 &= 3.73 \text{ V}, \\
V_3 &= 23.43 \text{ V}, \\
V_2 &= 2.54 \text{ V}, \\
V_4 &= -17.16 \text{ V}.
\end{align*}
\]
**Problem 3.52** Apply the by-inspection method to develop a node-voltage matrix equation for the circuit in Fig. P3.52 and then use MATLAB or MathScript software to solve for $V_1$ and $V_2$.

![Circuit Diagram](image)

**Figure P3.52:** Circuit for Problem 3.52.

**Solution:**

Node 1:

$G_{11} = \left( \frac{1}{6} + \frac{1}{12} \right) = 0.25$

$G_{12} = G_{21} = -\frac{1}{12} = -0.083$

$G_{22} = \left( \frac{1}{6} + \frac{1}{12} \right) = 0.25$

$I_n = 2 + 4 = 6 \text{ A}$

$I_n = 3 - 4 = -1 \text{ A}$

Application of Eq. (3.26) gives:

\[
\begin{bmatrix}
0.25 & -0.083 \\
-0.083 & 0.25
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
6 \\
-1
\end{bmatrix}
\]

Matrix inversion gives

$V_1 = 25.5 \text{ V}, \quad V_2 = 4.5 \text{ V}.$

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Problem 3.53 Use the by-inspection method to establish a node-voltage matrix equation for the circuit in Fig. P3.53. Solve the matrix equation by MATLAB® or MathScript software to find $V_1$ to $V_4$.

![Circuit diagram](image)

Figure P3.53: Circuit for Problem 3.53.

Solution:

$$G_{11} = \frac{1}{2+1} + \frac{1}{3+4} = 0.476$$

$G_{12} = G_{21} = -\frac{1}{2+1} = -0.333$

$G_{13} = G_{31} = 0$

$G_{14} = G_{41} = -\frac{1}{3+4} = -0.143$

$G_{22} = \frac{1}{1+2} + \frac{1}{7} + \frac{1}{6} = 0.643$

$G_{23} = G_{32} = -\frac{1}{6} = -0.167$

$G_{24} = G_{42} = 0$

$G_{33} = \frac{1}{5} + \frac{1}{6} + \frac{1}{9} = 0.478$

$G_{34} = G_{43} = -\frac{1}{5} = -0.2$

$G_{44} = \frac{1}{3+4} + \frac{1}{5} = 0.343$

Application of Eq. (3.26) gives:

$$\begin{bmatrix}
0.476 & -0.333 & 0 & -0.143 \\
-0.333 & 0.643 & -0.167 & 0 \\
0 & -0.167 & 0.478 & -0.2 \\
-0.143 & 0 & -0.2 & 0.343 \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
\end{bmatrix} =
\begin{bmatrix}
2 \\
0 \\
-2 \\
-3 \\
\end{bmatrix}$$

Matrix inversion gives:

$$V_1 = -8.1689 \text{ V}, \quad V_2 = -8.4235 \text{ V}, \quad V_3 = -16.155 \text{ V}, \quad V_4 = -21.5748 \text{ V}.$$