ECE 541/ME 541
Microelectronic Fabrication Techniques

Lecture 02 Review of PN-Junctions

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Diode Current-Voltage Characteristics

“Rectification”

![Ideal diode model]

\[ I_D = I_S (e^{V_D/qT} - 1) \]

(a) On a linear scale.

(b) On a logarithmic scale (forward bias).

\( V_D \) (V)

\( I_D \) (mA)

3.3 \( \text{mA} \text{decade current} \)

\[ \text{pn-junctions, heterojunction } \rightarrow \text{home junction} \]

Different materials

\( p/i/n \)

\( p/n \) - same material

Equilibrium energy band diagram for the pn junction

\[ n = n_i \exp \left( \frac{E_i - E_F}{kT} \right) \]

\[ p = n_i \exp \left( \frac{E_F - E_i}{kT} \right) \]

\[ N_D - N_A \]

\[ E_F \]

\[ E_C \]

\[ E_V \]

\[ E_i \]

\[ E_F \]

\[ E_C \]

\[ E_V \]

\[ E_i \]

\[ E_F \]

\[ V_{i} = k_B T \frac{p}{n_i} + k_B T \frac{n}{n_i} = k_B T \frac{n_i + p_i}{n_i} \]

\[ E_F = \text{same everywhere under equilibrium} \]

Join the two sides of the band by a smooth curve.
Electrostatic variables for the equilibrium pn junction

Potential, \( V = -\left(\frac{1}{q}\right) (E_C - E_{\text{ref}}) \). So, potential difference between the two sides (also called built-in voltage, \( V_{bi} \)) is equal to \(-\left(\frac{1}{q}\right)(\Delta E_C)\).

\[
V = -\frac{1}{q} (E_C - E_{\text{ref}})
\]

\[
\mathcal{E} = \frac{1}{q} \frac{dE_C}{dx} = \frac{1}{q} \frac{dE_i}{dx}
\]

\[
\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon}
\]

\( \rho = \text{charge density} \)

\( \varepsilon = K_s \varepsilon_o \)

\[
\nabla \cdot \mathbf{D} = \rho
\]

\[
\mathbf{D} = \varepsilon \mathbf{E}
\]

The built-in potential, \( V_{bi} \)

When the junction is formed, electrons from the n-side and holes from the p-side will diffuse leaving behind charged dopant atoms. Remember that the dopant atoms cannot move! Electrons will leave behind positively charged donor atoms and holes will leave behind negatively charged acceptor atoms.

The net result is the build up of an electric field from the positively charged atoms to the negatively charged atoms, i.e., from the n-side to p-side. When steady state condition is reached after the formation of junction the net electric field (or the built-in potential) will prevent further diffusion of electrons and holes. In other words, there will be drift and diffusion currents such that net electron and hole currents will be zero.
Conceptual pn-junction formation

Holes and electrons will diffuse towards opposite directions, uncovering ionized dopant atoms. This will build up an electric field which will prevent further movement of carriers.

Equilibrium conditions

Under equilibrium conditions, the net electron current and hole current will be zero.

\[
N_A = 10^{17} \text{ cm}^{-3} \quad \text{and} \quad N_D = 10^{16} \text{ cm}^{-3}
\]

- Hole diffusion current
- Hole drift current
- Electron diffusion current (opposite to electron flux)
- Electron drift current (opposite to electron flux)

\[\mathcal{E}-\text{field} \quad \rightarrow \quad \nabla \cdot \mathbf{j} = 0\]

\[\mathcal{E} = \frac{k_B T}{q} \ln \left( \frac{n_i^2}{n} \right)\]

\[V_i = \frac{k_B T}{q} \ln \left( \frac{n_i^2}{n} \right)\]

\[V_i = 26 \times \ln 10^{13} \times 10^{15} \text{ mV} \]

\[V_i = 26 \times 13 \times 2.3 \text{ mV} \]

\[V_i = 0.78 \text{ V}\]
Majority and minority carrier concentrations

<table>
<thead>
<tr>
<th>p-side</th>
<th>$N_A$</th>
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<th>n-side</th>
<th>$N_D$</th>
<th>$n_n$</th>
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The built-in potential, $V_{bi}$

$E_C$  
$E_i$  
$E_V$

$E_i - E_F = kT \ln \left( \frac{p}{n} \right)$

$q V_{bi} = (E_i - E_F)_{p-side} + (E_F - E_i)_{n-side}$

$E_F - E_i = kT \ln \left( \frac{n}{n_i} \right)$
The built-in potential, $V_{bi}$

The built-in potential, $V_{bi}$, is numerically equal to the “shift” in the bands expressed in eV.

\[
V_{bi} = \left( \frac{1}{q} \right) \left\{ \left( E_i - E_F \right)_{p \text{-side}} + \left( E_F - E_i \right)_{n \text{-side}} \right\} \\
= kT \ln \left( \frac{p}{n_i} \right) + kT \ln \left( \frac{n}{n_i} \right) \\
= kT \ln \left( \frac{p_p n_n}{n_i^2} \right)
\]

where $p_p =$ hole concentration on p - side
and $n_n =$ electron concentration on n - side

An interesting fact: $\frac{p_p}{p_n} = \frac{n_n}{n_p} = \exp \left( \frac{q V_{bi}}{kT} \right)$

### Example 1

A p-n junction is formed in Si with the following parameters.

Calculate the built-in voltage, $V_{bi}$.

\[
N_D = 10^{16} \text{ cm}^{-3}, \quad N_A = 10^{17} \text{ cm}^{-3}
\]

Calculate majority carrier concentration in n-side and p-side.
Assume $n_n = N_D = 10^{16} \text{ cm}^{-3}$ and $p_p = N_A = 10^{17} \text{ cm}^{-3}$.

\[
V_{bi} = \frac{kT}{q} \ln \left( \frac{p_p n_n}{n_i^2} \right) = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) = 0.7 \text{ V}
\]

Plug in the numerical values to calculate $V_{bi}$. 

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Example 2

A pn junction is formed in Si with the following parameters. Calculate the built-in voltage, $V_{bi}$.

\[ N_n = 10^{16} \text{ cm}^{-3} \quad N_D = 2 \times 10^{16} \text{ cm}^{-3} \]

\[ N_A = 10^{16} \text{ cm}^{-3} \quad N_D = 2 \times 10^{17} \text{ cm}^{-3} \]

\[ P = N_A - N_D = 10^7 \text{ cm}^{-3} \]

Calculate majority carrier concentration in n-side and p-side.

\[ n_n = \text{“effective } N_D = 10^{16} \text{ cm}^{-3}, \quad p_p = \text{“effective } N_A = 10^{17} \text{ cm}^{-3}. \]

\[
V_{bi} = \frac{kT}{q} \ln \left( \frac{p_p n_n}{n_i^2} \right) = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)
\]

Here $N_A$ and $N_D$ are “effective” or net values.

Plug in the numerical values to calculate $V_{bi}$.

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Built-in potential as a function of doping concentration for an abrupt p$^+$n or n$^+$p junction

![Graph showing built-in potential as a function of doping concentration](image.png)
Depletion approximation

Poisson equation

\[
\frac{dE}{dx} = \frac{\rho}{K_s \varepsilon_0} = \begin{cases} 
q \frac{(N_D - N_A)}{K_s \varepsilon_0} & \text{for } -x_p \leq x \leq x_n \\
0 & \text{everywhere else}
\end{cases}
\]

**Depletion Approximation:**
We assume that the free carrier concentration inside the depletion region is zero.

**1D Approximation:** In the discussion here and following, we make the analysis in one dimension, although actual diode may not be a perfect one-dimensional system. It makes the analysis simple.

Quantitative analysis: Electric field \( E \)

\[
\frac{dE}{dx} = \frac{\rho}{\varepsilon} \quad \text{where} \quad \varepsilon = K_s \varepsilon_0
\]

\[
= -q \frac{N_A}{\varepsilon} \quad -x_p < x < 0
\]

\[
= q \frac{N_D}{\varepsilon} \quad 0 < x < x_n
\]

\[
= 0 \quad x > x_n \quad x < -x_p
\]

\[
E(x) = -q \frac{N_A}{\varepsilon}(x_p + x) \quad -x_p \leq x \leq 0
\]

\[
= -q \frac{N_D}{\varepsilon}(x_n - x) \quad 0 \leq x \leq x_n
\]

\[
= 0 \quad x < -x_p; \quad x > x_n
\]
Relationship between $x_n$ and $x_p$

\[
\mathcal{E}_{\text{max}} = -q \frac{N_A x_p}{\varepsilon} = -q \frac{N_D x_n}{\varepsilon}
\]

\[
N_A x_p = N_D x_n
\]

Net charge on p-side = Net charge on n-side

Depletion layer width: \( W = x_n + x_p \)

\[
x_n = \frac{W \frac{N_A}{N_A + N_D}}{x_p} = \frac{W \frac{N_D}{N_A + N_D}}{x_p}
\]

If \( N_A \gg N_D \), then \( W \approx x_n \) and if \( N_A \ll N_D \), then \( W \approx x_p \).

Built-in voltage: \( V_{bi} \)

\[
\mathcal{E} = -\frac{dV}{dx} \quad \text{or} \quad V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E}(x) \, dx
\]

\[
V_{bi} = -\left\{ \text{area under } \mathcal{E} \text{ versus } x \text{ curve} \right\}
\]

\[
= -(1/2) \left\{ W \left( -q \frac{N_D x_n}{\varepsilon} \right) \right\}
\]

\[
= \left( \frac{q}{2 \varepsilon} \right) N_D x_n W
\]

\[
= \frac{1}{2} \frac{q}{\varepsilon} \left( \frac{N_A N_D}{N_A + N_D} \right) W^2 \quad \text{since} \quad x_n = \frac{W \frac{N_A}{N_A + N_D}}{x_p}
\]

\[
W = \left[ \frac{2 \varepsilon \left( \frac{N_A + N_D}{N_A N_D} \right) V_{bi}}{q \left( \frac{N_A N_D}{N_A + N_D} \right)} \right]^{1/2}
\]
Quantitative analysis: Electrostatic potential

\[
\frac{dV}{dx} = \frac{qN_A}{\varepsilon} (x_p + x) \quad -x_p \leq x \leq 0
\]

\[
= \frac{qN_D}{\varepsilon} (x_n - x) \quad 0 \leq x \leq x_n
\]

with the reference potential at \( x = -x_p \) set to zero

\[
V(x) = \frac{qN_A}{2\varepsilon} (x_p + x)^2 \quad -x_p \leq x \leq 0
\]

\[
= V_{bi} - \frac{qN_D}{2\varepsilon} (x_n - x)^2 \quad 0 \leq x \leq x_n
\]

Summary of a \( p\!n \)-junction at the equilibrium

\[
V_{bi} = \frac{kT}{q} \ln \left( \frac{p_x n_n}{n_i^2} \right) = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)
\]

\[
V(x) = \frac{qN_A}{2\varepsilon} (x_p + x)^2 \quad -x_p \leq x \leq 0
\]

\[
= V_{bi} - \frac{qN_D}{2\varepsilon} (x_n - x)^2 \quad 0 \leq x \leq x_n
\]

\[
x(x) = \frac{qN_A}{\varepsilon} (x_p + x) \quad -x_p \leq x \leq 0
\]

\[
= \frac{qN_D}{\varepsilon} (x_n - x) \quad 0 \leq x \leq x_n
\]

\[
= 0 \quad x < -x_p, \; x > x_n
\]

\[
f_{max} = -q N_A x_p / \varepsilon = -q N_D x_n / \varepsilon
\]

\[
W = \left[ \frac{2e}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) V_{bi} \right]^{1/2}
\]
Step Junction

\[ I_0 = I_S \left( \exp\left(\frac{qV_p}{kT}\right) - 1 \right) \]

\[ Q_p' = -qN_a x_p \] [Coul/cm²]

\[ Q_n' = qN_d x_n \] [Coul/cm²]

Due to Charge neutrality

\[ Q_p' + Q_n' = 0, \Rightarrow N_a x_p = N_d x_n \]

Linear graded junction

Let the net donor concentration, \( N(x) = N_d(x) - N_a(x) = ax \), so \( \rho = qx \), \(-x_p < x < x_n = x_p = x_o\) (chg neu)

\[ \rho = qa \ x \]

\[ Q_n' = qax_o^2/2 \]

\[ Q_p' = -qax_o^2/2 \]
Linear graded junction (cont.)

Let $E_x(-x_0) = 0$, since this is the edge of the DR (also true at $+x_0$)

By Gauss' Law, \[
\int_{-x_0}^{x} dE_x = q \int_{-x_0}^{x} a x dx,
\]
so

\[
E_x(x) = -E_{\text{max}} \left[ 1 - \left( \frac{x}{x_0} \right)^2 \right], \text{ where}
\]
\[
E_{\text{max}} = \frac{qa}{2\varepsilon} x_0^2
\]
Linear graded junction (cont.)

\[ \delta V = V_{bi} - V_a = \frac{2qa}{3\varepsilon} x_0^3, \] and

\[ V_{bi} = 2V_t \ln \left( \frac{ax_0}{n_i} \right), \] so \[ x_0 = \left[ \frac{3\varepsilon(V_{bi} - V_a)}{2qa} \right]^\frac{1}{3}. \]

Letting \[ C'_j \equiv \left[ \frac{dQ'}{dV} \right] = C'_j(0) \left[ 1 - \frac{V_a}{V_{bi}} \right] \negthinspace \frac{1}{3}. \]