ECE 541/ME 541
Microelectronic Fabrication Techniques

Review of Semiconductor Fundamentals

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Electronic properties: Silicon in general.

\[ E_G = 1.12 \text{ eV} \]

Boltzmann constant: \( k = 8.62 \times 10^{-5} \text{ eV/K} \)

Fundamental materials property:

\[ n = N_c \times e^{-(E_c-E_F)/kT} \]

Where \( n \) = concentration of negative (electron) carriers (typically in \( \text{cm}^{-3} \))

- \( E_c \) is the energy level of the conduction band
- \( E_F \) is the Fermi level.
- \( N_c \) is the intrinsic density of states in the conduction band (\( \text{cm}^{-3} \)).

Similarly,

\[ p = N_v \times e^{-(E_F-E_v)/kT} \]

Where \( p \) = concentration of positive (hole) carriers (typically in \( \text{cm}^{-3} \))

- \( E_v \) is the energy level of the valence band
- \( N_v \) is the intrinsic density of states in the valence band (\( \text{cm}^{-3} \)).

\[
\begin{align*}
\frac{n}{p} &= n^2 \\
\Rightarrow \quad \frac{ni}{n_i} &= \frac{E_F - E_i}{k_B T} \\
\Rightarrow \quad n_i &= \frac{k_B T}{E_F - E_i} \frac{1}{n_i} \\
\Rightarrow \quad n_i &= 10^{10} \text{ cm}^{-3}
\end{align*}
\]
**Electronic properties: intrinsic (undoped) silicon.**

Density of states in conduction band, $N_c$ (cm$^{-3}$) 3.22E+19
Density of states in valence band, $N_v$ (cm$^{-3}$) 1.83E19

Note: without doping, $n = p = n_i$ where $n_i$ is the intrinsic carrier concentration.
For pure silicon, then

$$n_i^2 = N_c N_v \exp \left( -E_G / kT \right)$$

Thus $n_i = 1 \times 10^{10}$ cm$^{-3}$

Similarly the Fermi level for the intrinsic silicon is,

$$E_i = E_v + \left( E_C - E_v \right) / 2 + \left( 1 / 2 \right) kT \ln \left( N_v / N_c \right)$$

Where we have used $E_i$ to indicate intrinsic Fermi level for Si.

Consider doping with n-type (or electron donating) dopant (such as Arsenic).

Then $n \approx N_D$ where $N_D$ is the arsenic doping concentration.

The injection of negative (electron) carriers dramatically alters the Fermi level of the system since there are now a
The injection of negative (electron) carriers dramatically alters the Fermi level of the system since there are now a significant sea of negative carriers available.

We can determine the new Fermi level as well as the resulting change in positive carriers.

\[ n_i^2 = p_n = N_c N_v \exp\left(-\frac{E_G}{kT}\right) \]

Thus \( p = n_i^2/N_D \).

And \( E_F = E_i + kT \ln\left(N_D/n_i\right) \).
Correspondingly, for p-type (acceptor) dopants such as Boron:

Thus \( n = n_i^2 / N_A \). 

And \( E_F = E_i - kT \ln(N_A/n_i) \)

\[
\rho = \frac{1}{q(n\mu_n + p\mu_p)}
\]

Where \( q \) is electron charge and \( \mu \) are mobilities.

\[
p = n_i \times e^{(E_i - E_F) / kT}
\]

\[
n = n_i \times e^{(E_F - E_i) / kT}
\]
Equations to remember

\[ n = n_i e^{(E_f - E_i)/kT} \]
\[ p = n_i e^{(E_i - E_f)/kT} \]
\[ np = n_i^2 \]
\[ p - n + N_D - N_A = 0 \]

Note: Our interest was in determining \( n \) and \( p \). Free carriers strongly influence the properties of semiconductors.
Example 1

(a) Consider Si doped with \(10^{14} \text{ cm}^{-3}\) boron atoms. Calculate the carrier concentration \((n\text{ and } p)\) at 300 K.

\[ p = N_A = 10^{14} \text{ cm}^{-3} \]

\[ n = \frac{n_i^2}{p} = \frac{(10^{10})^2}{10^{14}} = 10^6 \text{ cm}^{-3} \]

\[ n = n_i \exp \left( \frac{E_f - E_i}{k_B T} \right) \]

\[ \Rightarrow E_f - E_i = k_B T \ln \frac{n}{n_i} = \frac{26 \text{ meV} \cdot 10^6}{10^{10}} = 2.6 \times 10^{-4} \text{ meV} \]

\[ \frac{E_f}{E_i} = 5.6 \times 10^3 \text{ meV} \]

\[ E_f = 33.9 \text{ meV} \]

\[ E_i = 3.2 \text{ meV} \]

\[ E_{2h} = 3.12 \text{ meV} \]

\[ E_{2.3} = 2.3 \text{ meV} \]

\[ E_{2.08} = 2.08 \text{ meV} \]

\[ E_{23.92} = 23.92 \text{ meV} \]
Example 2

Consider a Si sample doped with $5 \times 10^{16}$ cm$^{-3}$ of phosphorous (P) atoms and $10^{16}$ cm$^{-3}$ of boron (B) atoms.

(a) Is the semiconductor n-type or p-type?

(b) Determine the free carrier concentration (hole and electron concentrations, or $p$ and $n$) at 300K.

(c) Determine the position of the Fermi level and draw the band diagram.

\[ n = N_0 - N_A = 2 \times 10^{16} - 10^{16} = 10^{16} \text{ cm}^{-3} \]

\[ p = \frac{n_i^e}{n} = \frac{(10^{10})^2}{10^{16}} = 10^4 \text{ cm}^{-3} \]

\[ \Rightarrow E_f - E_i = k_B T \ln \frac{n}{n_i} = 26 \times \ln \frac{10^{16}}{10^{13}} \]

\[ \frac{E_f - E_i}{E_i} \approx 2.3 \text{ meV} \approx 26 \times 6 \times 2.3 = 359 \text{ meV} \]
Introduction to Hall Effects

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\[ \vec{F} = g \vec{E} + q \vec{v} \times \vec{B} \]
Discovery of Hall Effect

Edwin Herbert Hall
(1855~1938)

Hall effect was discovered by Edwin H. Hall in 1878, when he was a graduate student in Johns Hopkins University.
Basic Picture of Hall Effect

Lorentz Force
\[ \vec{F} = e \left( \vec{E} + \vec{v} \times \vec{B} \right) \]

\[ J_x = \frac{I_x}{dW} \]

\[ \frac{V_H}{W} \]

\[ E_H \propto B_y, J_x \]

\[ R_H = \frac{E_H}{B_y J_x} = \frac{I_x}{dW} B_y \]

\[ = \frac{V_H}{I_x} \cdot \frac{d}{B_y} \]
Physics of Hall Effect

\[ E_{H} \propto J_{x} \cdot B_{z} \]

**Define a coefficient**

\[ R_{H} = \frac{E_{H}}{J_{x} \cdot B_{z}} \]

- \( E_{H} \) — Electric Field Intensity;
- \( J_{x} \) — Current Density;
- \( B_{z} \) — Magnetic Field Intensity.

\[ qE_{H} = qv_{x}B_{z} \Rightarrow E_{H} = B_{z}v_{x} \]

\[ J_{x} = nqv_{x} \]

\[ R_{H} = \frac{1}{nq} \]

\[ n = \frac{1}{R_{H}I} \]

**\( \rho \)-type materials**

\[ R_{H} = \frac{1}{pe} \]

**\( n \)-type materials**

\[ R_{H} = -\frac{1}{ne} \]

**If both holes and electrons are present**

\[ R_{H} = \frac{(p - b^{2}n) + (\mu_{n}B)^{2}(p - n)}{e \cdot [(p + bn)^{2} + (\mu_{n}B)^{2}(p - n)^{2}]} \]

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Mobility and Hall Effect

Definition of Mobility

\[ \mu = \left| \frac{\nabla}{E_x} \right| \]

\[ J_x = nqv_x \]
\[ J_x = \sigma E_x \]

\[ \mu = \frac{v_x}{E_x} = \frac{\sigma}{nq} = \sigma \cdot R_H \]

Hall Mobility

\[ \mu_H = \sigma \cdot R_H \]

- \( p \)-type materials: \[ \mu_p = \frac{\sigma}{pe} \]
- \( n \)-type materials: \[ \mu_n = \frac{\sigma}{ne} \]
- If both holes and electrons are present:
  \[ \sigma = e \cdot (\mu_p p + \mu_n n) \]
Uses of Hall Measurements

➢ To check the type of a semiconductor (n- or p-type)

  - *electrons* as carriers $\rightarrow R_{Hi} < 0 \rightarrow n$-type semiconductor
  - *holes* as carriers $\rightarrow R_{Hi} > 0 \rightarrow p$-type semiconductor

➢ To estimate the carrier concentration of a semiconductor

  \[ n = \frac{|R_{Hi}|}{e} \]

➢ To measure Hall mobility

  \[ \mu_{Hi} = \sigma \cdot R_{Hi} \]