Problem 5.37  Prior to \( t = 0 \), capacitor \( C_1 \) in the circuit of Fig. P5.37 was uncharged. For \( I_0 = 5 \) mA, \( R_1 = 2 \) k\( \Omega \), \( R_2 = 50 \) k\( \Omega \), \( C_1 = 3 \) \( \mu \)F, and \( C_2 = 6 \) \( \mu \)F, determine:

(a) The equivalent circuit involving the capacitors for \( t \geq 0 \). Specify \( v_1(0) \) and \( v_2(0) \).

(b) \( i(t) \) for \( t \geq 0 \).

(c) \( v_1(t) \) and \( v_2(t) \) for \( t \geq 0 \).

Solution:

Figure P5.37

(a) At \( t = 0^- \),

\[
\begin{align*}
\nu_2(0^-) &= I_0 R_1 = 5 \times 10^{-3} \times 2 \times 10^3 = 10 \text{ V}.
\nu_1(0^-) &= 0 \quad \text{(Given)}.
\end{align*}
\]

At \( t = 0 \), circuit is as shown in Fig. P5.37(b), with:

\[
C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} \mu \text{F} = 2 \mu \text{F},
\]
and
\[ \nu_{\text{eq}}(0) = \nu_2(0) - \nu_1(0) = 10 - 0 = 10 \text{ V}, \]
\[ \tau = R_2 C_{\text{eq}} = 5 \times 10^4 \times 2 \times 10^{-6} = 0.1 \text{ s}. \]

(b) For \( t \geq 0 \),
\[ \nu_{\text{eq}}(t) = \nu_{\text{eq}}(\infty) + [\nu_{\text{eq}}(0) - \nu_{\text{eq}}(\infty)]e^{-t/\tau} = 0 + (10 - 0)e^{-10t} = 10e^{-10t}. \]
\[ i(t) = C_{\text{eq}} \frac{d}{dt} \nu_{\text{eq}} = 2 \times 10^{-6} \frac{d}{dt}(10e^{-10t}) = -0.2e^{-10t} \quad (\text{mA}). \]

(c)
\[ \nu_1(t) = \nu_1(0) + \frac{1}{C_1} \int_{0}^{t} -i(t) \, dt \]
\[ = \frac{1}{3 \times 10^{-6}} \int_{0}^{t} 0.2e^{-10t} \times 10^{-3} \, dt = 6.7(1 - e^{-10t}) \quad (\text{V}), \quad \text{for } t \geq 0, \]
\[ \nu_2(t) = \nu_2(0) + \frac{1}{C_2} \int_{0}^{t} i(t) \, dt \]
\[ = 10 + \frac{1}{6 \times 10^{-6}} \int_{0}^{t} -0.2e^{-10t} \times 10^{-3} \, dt = [6.7 + 3.3e^{-10t}] \quad (\text{V}), \quad \text{for } t \geq 0. \]
Problem 5.38  The switch in the circuit of Fig. P5.38 had been closed for a long time before it was opened at \( t = 0 \). Given that \( V_s = 10 \text{ V} \), \( R_1 = 20 \text{ k}\Omega \), \( R_2 = 100 \text{ k}\Omega \), \( C_1 = 6 \mu\text{F} \), and \( C_2 = 12 \mu\text{F} \), determine \( i(t) \) for \( t \geq 0 \).

Solution:

At \( t = 0^- \), the circuit is as depicted in Fig. P5.38(b).

\[
i(0^-) = 0
\]

\[
V_1(0^-) = V_2(0^-) = V_s = 10 \text{ V}.
\]

At \( t > 0 \), the circuit becomes as shown in Fig. P5.38(c). Hence,

\[
\tau = R_2 \left( \frac{C_1 C_2}{C_1 + C_2} \right) = 10^5 \times \left( \frac{6 \times 12}{6 + 12} \right) \times 10^{-6} = 0.4 \text{ s},
\]

\[
C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 4 \mu\text{F},
\]

but

\[
V_{eq}(0^-) = V_2(0^-) - V_1(0^-) = 10 - 10 = 0.
\]

Consequently, because the two capacitors have equal initial voltages at \( t = 0 \), the net voltage is zero, resulting in no current flow through \( R_2 \). That is,

\[
V_1(t) = V_2(t) = 10 \text{ V} \quad \text{for} \ t \geq 0,
\]

and

\[
i(t) = 0.
\]
Problem 5.44  Given that in Fig. P5.44, $I_1 = 4 \text{ mA}$, $I_2 = 6 \text{ mA}$, $R_1 = 3 \text{ k}\Omega$, $R_2 = 6 \text{ k}\Omega$, and $C = 0.2 \text{ mF}$, determine $v(t)$. Assume the switch was connected to terminal 1 for a long time before it was moved to terminal 2.

Solution:

![Figure P5.44](image)

At $t = 0^-$ (Fig. P5.44(b)),

$$v(0^-) = I_1R_1 = 4 \times 10^{-3} \times 3 \times 10^3 = 12 \text{ V}.$$  

At $t \geq 0$,

$$\tau = R_2C = 6 \times 10^3 \times 0.2 \times 10^{-3} = 1.2 \text{ s},$$

$$v(0) = v(0^-) = 12 \text{ V},$$

$$v(\infty) = I_2R_2 = 6 \times 10^{-3} \times 6 \times 10^3 = 36 \text{ V}.$$  

Hence, for $t \geq 0$:

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$= [36 + 12 - 36]e^{-t/1.2}$$

$$= [36 - 24e^{-t/1.2}] \text{ (V)}.$$  

**Problem 5.48** Determine $i(t)$ for $t \geq 0$ given that the circuit in Fig. P5.48 had been in steady state for a long time prior to $t = 0$. Also, $I_0 = 5\, \text{A}$, $R_1 = 2\, \Omega$, $R_2 = 10\, \Omega$, $R_3 = 3\, \Omega$, $R_4 = 7\, \Omega$, and $L = 0.15\, \text{H}$.

**Solution:**

At $t = 0^-$, current division in the circuit of Fig. P5.48(b) gives

$$i(0^-) = \frac{I_0 R_1}{R_1 + R_3} = \frac{5 \times 2}{2 + 3} = 2\, \text{A}.$$

Hence,

$$i(0) = i(0^-) = 2\, \text{A}.$$

At $t = \infty$,

$$i(\infty) = 0 \quad \text{(no active sources)}.$$

At $t \geq 0$,

$$\tau = \frac{L}{R_{\text{eq}}}$$

$$R_{\text{eq}} = \frac{R_2(R_4 + R_3)}{R_2 + R_3 + R_4} = \frac{10 \times (3 + 7)}{10 + 3 + 7} = 5.$$
\[ \tau = \frac{0.15}{5} = 0.03 \text{ s.} \]

\[ i_L(t) = [i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}] \]

\[ = 2e^{-100t/3} \quad \text{(A)}. \]
Problem 5.49  For the circuit in Fig. P5.49, determine $i_L(t)$ and plot it as a function of $t$ for $t \geq 0$. The element values are $I_0 = 4$ A, $R_1 = 6$ Ω, $R_2 = 12$ Ω, and $L = 2$ H. Assume that $i_L = 0$ before $t = 0$.

Solution:

1. Time segment $0 \leq t \leq 0.5$ s

Before closing the switches, $L$ was not part of any closed circuit. Hence

$$i_L(0^-) = 0.$$  

Continuity requires that

$$i_{L1}(0) = i_L(0^-) = 0. $$

After closing the first switch at $t = 0$, the circuit looks as in Fig. P5.48(b). The response of the circuit can be calculated as if no change will happen at $t = 0.5$ s. Hence,

$$i_{L1}(\infty) = I_0.$$  

$$\tau_1 = \frac{L}{R_1} = \frac{2}{6} = \frac{1}{3} \text{ s}.$$  

$$i_{L1}(t) = [i_{L1}(\infty) + [i_{L1}(0) - i_{L1}(\infty)]e^{-t/\tau}]$$  
$$= I_0\left[1 - e^{-R_1t/L}\right] = 4\left[1 - e^{-3t}\right] \text{ (A)}.$$  

2. Time segment $t > 0.5$ s

At $t = 0.5$ s,

$$i_{L1}(0.5) = 4\left[1 - e^{-3\times0.5}\right] = 3.11 \text{ A}.$$  

$$i_{L2}(0.5) = i_{L1}(0.5) = 3.11 \text{ A}.$$
\[ \tau_2 = \frac{L}{R_{eq}} = \frac{2}{\left( \frac{R_1 R_2}{R_1 + R_2} \right)} = \frac{2}{4} = 0.5 \text{ s} \]

\[ i_{L_2}(\infty) = I_0 = 4 \text{ A,} \]
\[ i_{L_2}(t) = \{i_{L_2}(\infty) + [i_{L_2}(0.5) - i_{L_2}(\infty)] e^{-(t-0.5)/\tau_2} \} u(t - 0.5) \]
\[ = [4 + (3.11 - 4)e^{-2(t-0.5)}] u(t - 0.5) \]
\[ = [4 - 0.89e^{-2(t-0.5)}] u(t - 0.5) \quad \text{(A).} \]
Problem 5.53  In the circuit of Fig. P5.53(a), $R_1 = R_2 = 20 \, \Omega$, $R_3 = 10 \, \Omega$, and $L = 2.5 \, \text{H}$. Determine $i(t)$ for $t \geq 0$ given that $v_s(t)$ is the step function described in Fig. P5.53(b).

Solution:

Because $v_s(t)$ was zero before $t = 0$,

$$i(0^-) = 0,$$

and continuity requires that

$$i(0) = i(0^-) = 0.$$

The circuit in Fig. P5.53(c) pertains to the conditions at $t = \infty$.

$$-v_s + R_1 i_1 + R_2 (i_1 - i_2) = 0$$

$$R_2 (i_2 - i_1) + R_3 i_2 = 0$$

Solution leads to:

$$i(\infty) = i_2(\infty) = 0.3 \, \text{A}.$$

Also,

$$R_{eq} = R_3 + \frac{R_1 R_2}{R_1 + R_2} = 10 + \frac{20 \times 20}{20 + 20} = 20 \, \Omega.$$

$$\tau = \frac{L}{R_{eq}} = \frac{2.5}{20} = 0.125 \, \text{s}$$

Hence, for $t \geq 0$:

$$i(t) = [i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}] = 0.3(1 - e^{-8t}) \quad \text{(A)}.$$
Problem 5.59  The input-voltage waveform shown in Fig. P5.59(a) is applied to the circuit in Fig. P5.59(b). Determine and plot the corresponding $v_{out}(t)$.

(b) Op-amp circuit

Figure P5.59

Solution: The circuit in Fig. P5.59(a) is a differentiator circuit with

$$RC = 50 \times 10^3 \times 2 \times 10^{-6} = 0.1.$$ 

For the time segment between $t = 0$ and $t = 2$ s, the slope of the input signal is $(12/2) = 6$ V/s. The output voltage is given by

$$v_{out} = -RC \frac{dv_i}{dt}$$

$$= -0.1 \times 6 = -0.6 \text{ V}.$$ 

Hence, $v_{out}$ is a square wave with an amplitude of 0.6 V, as shown in the figure.
Problem 5.60  Relate $v_{\text{out}}$ to $v_i$ in the circuit of Fig. P5.60. Assume $v_i(0) = 0$.

Solution:

$$v_n = v_p = v_i$$

$$\frac{v_n}{R} + C \frac{d}{dt}(v_n - v_{\text{out}}) = 0$$

Hence,

$$\frac{d}{dt}v_{\text{out}} = \frac{1}{RC}v_i + \frac{d}{dt}v_i$$

Integrating all terms from $t = 0$ to $t$,

$$\int_0^t \left( \frac{d}{dt}v_{\text{out}} \right) dt = \frac{1}{RC} \int_0^t v_i dt + \int_0^t \left( \frac{d}{dt}v_i \right) dt,$$

which simplifies to

$$v_{\text{out}}(t)\big|_0^t = \frac{1}{RC} \int_0^t v_i dt + v_i(0) \big|_0^t$$

or

$$v_{\text{out}}(t) = v_{\text{out}}(0) + v_i(t) + \frac{1}{RC} \int_0^t v_i dt.$$
Problem 5.62  Relate $v_{out}$ to $v_i$ in the circuit of Fig. P5.62. Assume $v_C = 0$ at $t = 0$.

\[ i_1 = \frac{v_n - v_i}{R_1} \]

\[ v_n - v_{out} = i_2 R_2 + \frac{1}{C} \int_0^t i_2 \, dt \]

But $v_n = v_p = 0$, and

\[ i_2 = -i_1 = \frac{v_i}{R_1}, \]

which leads to

\[ v_{out} = - \left( \frac{R_2}{R_1} v_i + \frac{1}{R_1 C} \int_0^t v_i \, dt \right). \]
Problem 5.68  The two-stage op-amp circuit in Fig. P5.68 is driven by an input step voltage given by \( v_i(t) = 10u(t) \) mV. If \( V_{cc} = 10 \) V for both op amps and the two capacitors had no charge prior to \( t = 0 \), determine and plot:

(a) \( v_{out1}(t) \) for \( t \geq 0 \)
(b) \( v_{out2}(t) \) for \( t \geq 0 \)

Solution:

(a) \[
v_{out1}(t) = -\frac{1}{R_1C_1} \int_0^t v_i \, dt
\]

\[
R_1C_1 = 5 \times 10^3 \times 4 \times 10^{-6} = 0.02.
\]

Hence,

\[
v_{out1}(t) = -50 \int_0^t 10 \times 10^{-3} \, dt = -0.5t \quad (V), \quad \text{for } t \geq 0.
\]

(b) For the second stage:

\[
v_{out2}(t) = -\frac{1}{R_2C_2} \int_0^t v_{out1} \, dt
\]

\[
R_2C_2 = 1 \times 10^6 \times 5 \times 10^{-6} = 5
\]

\[
v_{out2}(t) = -\frac{1}{5} \int_0^t (-0.5t) \, dt = 0.1 \frac{t^2}{2} = 0.05t^2 \quad (V), \quad \text{for } t \geq 0.
\]

Plots of \( v_i(t) \), \( v_{out1}(t) \), and \( v_{out2}(t) \) are shown below. We note that \( v_{out1}(t) \) reaches saturation at \( -V_{cc} = -10 \) V after 20 s, and \( v_{out2}(t) \) reaches saturation at \( V_{cc} = +10 \) V at \( t = 14.14 \) s.
Figure P5.68