REVIEW OF LECTURES 01-04
### Chapter 1 Relationships

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohm’s law</td>
<td>( i = \frac{v}{R} )</td>
</tr>
<tr>
<td>Current</td>
<td>( i = \frac{dq}{dt} )</td>
</tr>
<tr>
<td>Direction of ( i )</td>
<td>direction of flow of (+) charge</td>
</tr>
<tr>
<td>Charge transfer</td>
<td>( q(t) = \int_{-\infty}^{t} i , dt )</td>
</tr>
<tr>
<td>( \Delta Q )</td>
<td>( q(t_2) - q(t_1) = \int_{t_1}^{t_2} i , dt )</td>
</tr>
<tr>
<td>Voltage</td>
<td>potential energy difference</td>
</tr>
<tr>
<td>Passive sign convention</td>
<td>Direction of ( i ) is into +( v ) terminal of device</td>
</tr>
<tr>
<td>Power</td>
<td>( p = vi )</td>
</tr>
</tbody>
</table>

- If \( p > 0 \): device absorbs power
- If \( p < 0 \): device delivers power
Units, Multiples, Notation

Table 1-1: Fundamental SI units.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Electric Current</td>
<td>ampere</td>
<td>A</td>
</tr>
<tr>
<td>Temperature</td>
<td>kelvin</td>
<td>K</td>
</tr>
<tr>
<td>Amount of substance</td>
<td>mole</td>
<td>mol</td>
</tr>
</tbody>
</table>

Table 1-2: Multiple and submultiple prefixes.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>exa</td>
<td>E</td>
<td>$10^{18}$</td>
</tr>
<tr>
<td>peta</td>
<td>P</td>
<td>$10^{15}$</td>
</tr>
<tr>
<td>tera</td>
<td>T</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>$10^{9}$</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^{6}$</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>$10^{3}$</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>micro</td>
<td>$\mu$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>femto</td>
<td>f</td>
<td>$10^{-15}$</td>
</tr>
<tr>
<td>atto</td>
<td>a</td>
<td>$10^{-18}$</td>
</tr>
</tbody>
</table>

As a general rule, we shall use:

- A lowercase letter, such as $i$ for current, to represent the general case:
  
  \[ i \] may or may not be time varying

- A lowercase letter followed with $(t)$ to emphasize time:
  
  \[ i(t) \] is a time-varying quantity

- An uppercase letter if the quantity is not time varying; thus:
  
  \[ I \] is of constant value (dc quantity)

- A letter printed in boldface to denote that:
  
  \[ \textbf{I} \] has a specific meaning, such as a vector, a matrix, the phasor counterpart of $i(t)$, or the Laplace or Fourier transform of $i(t)$
Charge & Current

- **Unit of charge** = coulomb \( e = 1.6 \times 10^{-19} \) \( \text{C} \)

1. Charge can be either positive or negative.

2. The fundamental (smallest) quantity of charge is that of a single electron or proton. Its magnitude usually is denoted by the letter \( e \).

3. According to the law of conservation of charge, the (net) charge in a closed region can neither be created nor destroyed.

4. Two like charges repel one another, whereas two charges of opposite polarity attract.

\[
i = \frac{dq}{dt} \quad \text{(A)},
\]
Voltage & Power

The voltage between location \( a \) and location \( b \) is the ratio of \( dw \) to \( dq \), where \( dw \) is the energy in joules (J) required to move (positive) charge \( dq \) from \( b \) to \( a \) (or negative charge from \( a \) to \( b \)).
Power

Rate of expending or absorbing energy

\[ P = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = vi \]

\[ \sum P = 0 \]

Energy conservation

Units: watts

One watt = power rate of one joule of work per second. 1 W = 1 A x 1 V
Passive Sign Convention

\[ p = vi \]

- \( p > 0 \) power delivered to device
- \( p < 0 \) power supplied by device

*Note that \( i \) direction is defined as entering (\(+\)) side of \( v \).
Summary of Chapter 2

Chapter 2 Relationships

Linear resistor
\[ R = \frac{\rho \ell}{A} \]
\[ p = i^2 R \]

Kirchhoff current law (KCL)
\[ \sum_{n=1}^{N} i_n = 0 \]
\[ i_n = \text{current entering node } n \]

Kirchhoff voltage law (KVL)
\[ \sum_{n=1}^{N} v_n = 0 \]
\[ v_n = \text{voltage across branch } n \]

Resistor combinations
In series
\[ R_{eq} = \sum_{i=1}^{N} R_i \]

In parallel
\[ \frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i} \]
\[ or \quad G_{eq} = \sum_{i=1}^{N} G_i \]

Voltage division
\[ v_1 = \frac{R_1}{R_1 + R_2} v_s \]
\[ v_2 = \frac{R_2}{R_1 + R_2} v_s \]

Current division
\[ i_1 = \frac{R_2}{R_1 + R_2} i_s \]
\[ i_2 = \frac{R_1}{R_1 + R_2} i_s \]

Source transformation
\[ v_s \]
\[ i_s = \frac{v_s}{R_s} \]

Y–Δ transformation
Table 2-6
Ohm’s Law

Voltage across resistor is proportional to current

\[ \nu = iR \]

\[ R = \frac{\nu}{i} \]

Resistance: ability to resist flow of electric current

\[ R = \frac{\ell}{\sigma A} = \rho \frac{\ell}{A} \quad (\Omega), \]

\[ \rho = \text{resistivity} \]
Kirchhoff’s Current Law (KCL)

Sum of currents entering a node is zero
Also holds for closed boundary

\[
\sum_{n=1}^{N} i_n = 0 \quad \text{(KCL)},
\]

\[
i_1 - i_2 - i_3 + i_4 = 0
\]

\[
i_1 + i_4 = i_2 + i_3
\]
Kirchhoff’s Voltage Law (KVL)

Sum of voltages around a closed path is zero
Sum of voltage drops = sum of voltage rises

\[ \sum_{n=1}^{N} v_n = 0 \] (KVL),

Sign Convention

- Add up the voltages in a systematic clockwise movement around the loop.
- Assign a positive sign to the voltage across an element if the (+) side of that voltage is encountered first, and assign a negative sign if the (−) side is encountered first.

\[-4 + V_1 - V_2 - 6 + V_3 - V_4 = 0\]
Resistors in Series

Equivalent resistance (series) is sum of resistances

Voltage Divider

\[ R_{eq} = \sum_{i=1}^{N} R_i \quad \text{(resistors in series)}, \]

\[ v_i = \left( \frac{R_i}{R_{eq}} \right) v_s. \]

Voltage divided over resistors (voltage divider)
Resistors in Parallel

Combining In-Parallel Resistors

\[ \begin{align*}
\text{(a) Original circuit} \\
R_{eq} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} \\
\text{Current Division} \\
R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \\
i_1 = \left( \frac{R_2}{R_1 + R_2} \right) i_s \\
i_2 = \left( \frac{R_1}{R_1 + R_2} \right) i_s \\
i_s = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3} \\
\text{(b) Equivalent circuit} \\
\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i} \quad \text{(resistors in parallel).}
\end{align*} \]
Hence, for the two circuits to be equivalent:

\[-v_s + iR_1 + v_{12} = 0\]

\[i = \frac{v_s}{R_1} - \frac{v_{12}}{R_1}\]

\[i = i_s - iR_2\]

\[= i_s - \frac{v_{12}}{R_2}\]

\[R_1 = R_2\]

\[i_s = \frac{v_s}{R_1}\]
Wye–Delta (Y–Δ) Transformation

Circuit with no two resistors sharing the same current or same voltage
Wye–Delta (Y–Δ) Transformation

Hence,

\[ R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}. \]  \hspace{1cm} (2.57a)

When applied to the other two combinations of nodes, the foregoing procedure leads to:

\[ R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \]  \hspace{1cm} (2.57b)

and

\[ R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}. \]  \hspace{1cm} (2.57c)
Wye–Delta (Y–Δ) Transformation

Simultaneous solution leads to:

Δ → Y Transformation

\[
R_1 = \frac{R_b R_c}{R_a + R_b + R_c}
\]

\[
R_2 = \frac{R_a R_c}{R_a + R_b + R_c}
\]

\[
R_3 = \frac{R_a R_b}{R_a + R_b + R_c}
\]

Y → Δ Transformation

\[
R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}
\]

\[
R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}
\]

\[
R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}
\]
Table 2-5: Equivalent circuits.

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Equivalent</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Circuit 1" /></td>
<td><img src="image2" alt="Equivalent 1" /></td>
<td>$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$</td>
</tr>
<tr>
<td><img src="image3" alt="Circuit 2" /></td>
<td><img src="image4" alt="Equivalent 2" /></td>
<td>$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$</td>
</tr>
<tr>
<td><img src="image5" alt="Circuit 3" /></td>
<td><img src="image6" alt="Equivalent 3" /></td>
<td>$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$</td>
</tr>
<tr>
<td><img src="image7" alt="Circuit 4" /></td>
<td><img src="image8" alt="Equivalent 4" /></td>
<td>$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$</td>
</tr>
<tr>
<td><img src="image9" alt="Circuit 5" /></td>
<td><img src="image10" alt="Equivalent 5" /></td>
<td>$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$</td>
</tr>
<tr>
<td><img src="image11" alt="Circuit 6" /></td>
<td><img src="image12" alt="Equivalent 6" /></td>
<td>$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$</td>
</tr>
</tbody>
</table>

- For $R_a = R_b = R_c$  $\Rightarrow R_1 = R_2 = R_3 = R_a / 3$
- For $R_1 = R_2 = R_3$  $\Rightarrow R_a = R_b = R_c = 3R_1$
# Summary of Chapter 3 (Parts 1 & 2)

## Chapter 3 Relationships

<table>
<thead>
<tr>
<th>Method</th>
<th>Equation</th>
</tr>
</thead>
</table>
| Node-voltage method                   | $\sum$ of all current leaving a node = 0  
  [current entering a node is (−)]   |                   |
| Mesh-current method                   | $\sum$ of all voltages around a loop = 0  
  [passive sign convention applied to  
  mesh currents in clockwise direction] |                   |
| Nodal analysis by inspection          | $GV = I_t$        |
| Mesh analysis by inspection           | $RI = V_t$        |
Node-Voltage Method

Solution Procedure: Node Voltage

Step 1: Identify all extraordinary nodes, select one of them as a reference node (ground), and then assign node voltages to the remaining \( n_{ex} - 1 \) extraordinary nodes.

Step 2: At each of the \( n_{ex} - 1 \) extraordinary nodes, apply the form of KCL requiring the sum of all currents leaving a node to be zero.

Step 3: Solve the \( n_{ex} - 1 \) independent simultaneous equations to determine the unknown node voltages.

Node 1:

\[
I_1 + I_2 + I_3 = 0.
\]

\[
\frac{V_1}{R_1} + \frac{V_1 - V_0}{R_2 + R_3} + \frac{V_1 - V_2}{R_4} = 0 \quad \text{(node 1)}.
\]

Node 2

\[
\frac{V_2 - V_1}{R_4} - I_0 + \frac{V_2 - V_3}{R_6} = 0 \quad \text{(node 2)},
\]

Node 3

\[
\frac{V_3}{R_5} + \frac{V_3 - V_2}{R_6} + I_0 = 0 \quad \text{(node 3)}.
\]
Node-Voltage Method

Three equations in 3 unknowns:
Solve using Cramer’s rule, matrix inversion, or MATLAB

\[
\begin{aligned}
(\frac{1}{R_1} + \frac{1}{R_2 + R_3} + \frac{1}{R_4}) V_1 - (\frac{1}{R_4}) V_2 &= \frac{V_0}{R_2 + R_3}, \\
-(\frac{1}{R_4}) V_1 + (\frac{1}{R_4} + \frac{1}{R_6}) V_2 - \frac{V_3}{R_6} &= I_0, \\
-(\frac{1}{R_6}) V_2 + (\frac{1}{R_5} + \frac{1}{R_6}) V_3 &= -I_0.
\end{aligned}
\]
Mesh-Current Method

Solution Procedure: Mesh Current

Step 1: Identify all meshes and assign each of them an unknown mesh current. For convenience, define the mesh currents to be clockwise in direction.

Step 2: Apply kirchhoff’s voltage law (KVL) to each mesh.

Step 3: Solve the resultant simultaneous equations to determine the mesh currents.

Two equations in 2 unknowns:
Solve using Cramer’s rule, matrix inversion, or MATLAB

\[-V_0 + I_1 R_1 + (I_1 - I_2)R_3 = 0 \quad \text{(mesh 1)}\]

\[(I_2 - I_1) R_3 + I_2 R_2 = 0 \quad \text{(mesh 2)}\]
Mesh-Current Method

Solution Procedure: Mesh Current

Step 1: Identify all meshes and assign each of them an unknown mesh current. For convenience, define the mesh currents to be clockwise in direction.

Step 2: Apply Kirchhoff's voltage law (KVL) to each mesh.

Step 3: Solve the resultant simultaneous equations to determine the mesh currents.

\[ (1 + 2)I_1 - 2I_2 - I_3 = 10, \quad \text{Mesh 1} \]
\[ -2I_1 + (2 + 1 + 3)I_2 - I_3 = 0, \quad \text{Mesh 2} \]
\[ I_3 = I_x = 4V_1, \quad \text{Mesh 3} \]

But \[ V_1 = 2(I_1 - I_2). \]

Hence \[ -5I_1 + 6I_2 = 10, \]
\[ -10I_1 + 14I_2 = 0. \]

\[ I_1 = -14 \text{ A}, \quad I_2 = -10 \text{ A}. \]

\[ I_x = 8(I_1 - I_2) = 8(-14 + 10) = -32 \text{ A}. \]
Nodal Analysis by Inspection

- **Requirement:** All sources are independent current sources

\[
\begin{bmatrix}
G_{11} & G_{12} & \cdots & G_{1n} \\
G_{21} & G_{22} & \cdots & G_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
G_{n1} & G_{n2} & \cdots & G_{nn}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_n
\end{bmatrix}
= 
\begin{bmatrix}
i_{t1} \\
i_{t2} \\
\vdots \\
i_{tn}
\end{bmatrix}
\]

- \(G_{kk}\) = sum of all conductances connected to node \(k\)
- \(G_{k\ell} = G_{\ell k}\) = negative of conductance(s) connecting nodes \(k\) and \(\ell\), with \(k \neq \ell\)
- \(V_k\) = voltage at node \(k\)
- \(I_{tk}\) = total of current sources entering node \(k\) (a negative sign applies to a current source leaving the node).

All rights reserved. Do not reproduce or distribute. © 2013 National Technology and Science Press
Mesh by Inspection

Requirement: All sources are independent voltage sources

\[ \mathbf{RI} = \mathbf{V}_t, \]

\[
\begin{bmatrix}
R_{11} & R_{12} & \cdots & R_{1n} \\
R_{21} & R_{22} & \cdots & R_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
R_{n1} & R_{n2} & \cdots & R_{nn}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_n
\end{bmatrix}
= \begin{bmatrix}
v_{t1} \\
v_{t2} \\
\vdots \\
v_{tn}
\end{bmatrix}, \quad (3.29)
\]

where

- \( R_{kk} \) = sum of all resistances in mesh \( k \),
- \( R_{k\ell} = R_{\ell k} \) = negative of the sum of all resistances shared between meshes \( k \) and \( \ell \) (with \( k \neq \ell \))
- \( i_k \) = current of mesh \( k \)
- \( v_{tk} \) = total of all independent voltage sources in mesh \( k \), with positive assigned to a voltage rise when moving around the mesh in a clockwise direction.

\[
\begin{bmatrix}
(2 + 3 + 6) & -3 & -6 \\
-3 & (3 + 4 + 5) & -5 \\
-6 & -5 & (5 + 6 + 7)
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
= \begin{bmatrix}
6 - 4 \\
0 \\
4
\end{bmatrix}
\]