Ohm’s Law

Voltage across resistor is proportional to current

\[ V = IR \]
\[ R = \frac{V}{I} \]

Resistance: ability to resist flow of electric current

\[ R = \frac{\rho}{A} \]

\[ \rho = \text{resistivity} \]
Example 2-1: dc Motor

What fraction of power supplied by the battery is dissipated in the motor?

\[ P = IV \]

Solution:

\[ R = \frac{f}{I} \]

\[ R = \frac{12}{2.34} \]

\[ R = 5.18 \Omega \]

\[ R = R_a + R_w \]

\[ R = 0.04 + 2 \]

\[ = 2.04 \Omega \]

\[ P_a = \frac{I^2 R_a}{2} \]

\[ P_a = \frac{(5.88)^2 \times 2}{2} \]

\[ P_a = 69.15 \text{ W} \]

\[ = 98\% \text{ of } P \]
Tech Brief 2: Superconductivity

Critical temperature $T_c$ is the temperature at which a material becomes superconducting (zero resistance & no power dissipation)

Kirchhoff’s Current Law

Sum of currents entering a node is zero
Also holds for closed boundary

$$\sum_{k=1}^{N} i_k = 0 \quad \text{(KCL)}$$

$$i_1 - i_2 - i_3 + i_4 = 0$$
$$i_1 + i_4 = i_2 + i_3$$
Kirchhoff’s Voltage Law (KVL)

Sum of voltages around a closed path is zero
Sum of voltage drops = sum of voltage rises

Example: KCL/KVL

Solution:
Loop 1

Loop 2

Next, we apply KCL at node 1 which gives:

Three equations w/three unknowns:

\[ I_1 = 4 \text{ A}, \]
\[ I_2 = 3 \text{ A}, \]
\[ V_2 = 51 \text{ V}. \]
Equivalent Circuits

- If the current and voltage characteristics at nodes are identical, the circuits are considered "equivalent".
- Identifying equivalent circuits simplifies analysis.

### Resistors in Series

- Equivalent resistance (series) is sum of resistances.
  \[
  R_{eq} = \sum_{i=1}^{n} R_i \quad \text{(resistors in series)}
  \]
- Voltage divided over resistors (voltage divider):
  \[
  v_i = \left( \frac{R_i}{R_{eq}} \right) v_s
  \]
**Adding Sources In Series**

Unrealizable Circuit

Combining voltage sources

**Resistors in Parallel**

Combining in Parallel

Current Division

Hence,

\[
\frac{1}{R_{eq}} = \sum \frac{1}{R_i} \quad \text{(resistors in parallel)}
\]

**Source Transformation**

Hence,

\[
\frac{1}{R_1} = \frac{1}{R_{eq}} = \frac{1}{R_2}
\]

For the two circuits to be equivalent:

\[
\frac{1}{R_1} = \frac{1}{R_2}
\]
Example 2-10: Source Transformation

\[ R_1, R_2, R_3 = \Delta \]
\[ R_3, R_4, R_5 = \Delta \]
\[ R_1, R_3, R_4 = \gamma \]

Circuit with no two resistors sharing the same current or same voltage

Wye–Delta (Y–Δ) Transformation

Hence,

\[ R_1 = R_2 = R_3 \] (Y circuit)
\[ R_{12} = \frac{R_1 R_2}{R_1 + R_2 + R_3} \] (Delta circuit)

Between nodes 1 and 2:

\[ R_{12} = \frac{R_1 R_2}{R_1 + R_2 + R_3} \]

\[ R_{13} = \frac{R_1 R_3}{R_1 + R_2 + R_3} \]

\[ R_{23} = \frac{R_2 R_3}{R_1 + R_2 + R_3} \]

\[ R_{1} = R_{2} = R_{3} \] (Assuming no connection at node 3)
Wye–Delta (Y–Δ) Transformation

Simultaneous solution leads to:

\[ R_y = \frac{R_1 \cdot R_2 \cdot R_3}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \]

\[ R_\Delta = R_1 + R_2 + R_3 \]

Example 2-12: Y–Δ Circuit

Simplify the circuit as Fig. 2-3(b) for applying the Y–Δ transformation as a 3-olunteer for (closed 1).

Solution: Notice the symmetry rules associated with the transformation, the 3-olunteer is used in order to reduce the Y circuit as shown in Fig. 2-3(b), with

\[ R_\Delta = R_1 + R_2 + R_3 \]

\[ R_y = \frac{R_1 \cdot R_2 \cdot R_3}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \]
Wheatstone Bridge

- Measurement instrument based on differential measurement
- Balanced Condition: $I_2 = 0$
- Determine unknown resistance based on “balanced” condition

\[ V_1 = V_2 \]

\[ \frac{R_3V_0}{R_1 + R_3} = \frac{R_4V_0}{R_2 + R_4} \]

For balanced condition

Wheatstone Sensor

- Determine Vout

Vout = $V_o = \frac{V_i}{2}$

Vout = $V_o = \frac{V_i}{2}$

$V_o = V_i \frac{R}{2}$

Determine Vout

\[ V_o = \frac{V_i}{2} \]

Piezoresistive Sensors

\[ R = R_0 + \Delta R = R_0 (1 + \alpha P) \]
Tech Brief 3: Resistors As Sensors

Thermistor Sensors

\[ \Delta R = k \Delta T \]

Figure 3.3: This microfabricated anemometer is a thermistor that measures fluid velocity. The resistor (red) serves as both a heater and a thermometer. During operation, a voltage across the resistor produces a current \( I = V/ R \), which heats the resistor and the fluid. The fluid flow, \( \dot{V} \), is measured by the thermistor (blue), which is connected to the heater (red). Since increasing the flow increases the cooling of the resistor, the temperature of the resistor can be inferred from the thermistor.

(Courtesy of Khalid Najafi, University of Michigan.)

Example 2-16: Piezoresistor Cantilever

\[ \Delta P = \text{Kl} \Delta P \]

\[ V_{\text{ref}} = \frac{L}{2} \left( \frac{\Delta P}{E} \right) = \frac{V}{2} \Delta P \]

\[ P = \frac{F \cdot L}{W \cdot H^2} \]

Example 2-16: Piezoresistor Cantilever

A piezoresistor cantilever with a length \( L = 10 \, \text{cm} \), width \( W = 1 \, \text{cm} \), and thickness \( H = 0.1 \, \text{cm} \) is submerged in a medium. The fluid flow is measured using the cantilever. The force \( F \) acting on the cantilever is given by:

\[ F = \frac{P \cdot L}{W} \]

Substituting the expressions for \( P \) and \( L \) into the equation for \( F \), we get:

\[ F = \frac{\frac{F \cdot L}{W \cdot H^2} \cdot L}{W} = \frac{F \cdot L^2}{W^2 \cdot H^2} \]

This expression shows the relationship between the force \( F \) and the other parameters. The dimensions and materials of the cantilever can be used to calculate the force acting on it.
**i–v Relationships**

Linear i–v relationships

- Current source $I_0$
- Resistor $R$
- Voltage source $V_0$

Current can flow only from + to – through a diode.

**The Diode**

- $V_1 = 5\, \text{V}$
- $V_2 = 0\, \text{V}$
- $R = 100\, \Omega$

If the diode is forward biased, $V_0 = 0.7\, \text{V}$, which leads to:

- $I = 2\, \text{mA}$

Current can flow only from + to – through a diode.

**Example 2.16:** Square-Wave Generator

The circuit in Fig. 2.16 contains two diodes, both with $V_{th} = 0.7\, \text{V}$.

- The waveforms of the voltage sources are a square wave of a square wave cycle of $1\, \text{V}$.

- The diode current $I_0$ in the diode is $0.1\, \text{mA}$.

The output voltage waveform during the second half of the cycle is $0.1\, \text{V}$ in the diode. For diode $A$, the output voltage waveform is $0.1\, \text{V}$.

- The output voltage waveform during the second half of the cycle is $0.1\, \text{V}$.
Summary

Chapter 2 Relationships

- Lumped System
- Kirchhoff’s current law
- Kirchhoff’s voltage law
- Ohm’s law
- Superposition

Voltage relations:

\[ V = \sum (V_i) \]

Current relations:

\[ I = \sum (I_i) \]

Ohm’s law:

\[ V = I \times R \]

Superposition theorem:

\[ V = \sum (V_i) \]