Notes on Experiment #8

Theorems of Linear Networks

Prepare for this experiment!

If you prepare, you can finish in 90 minutes. If you do not prepare, you will not finish even half of this experiment. So, do your preliminary work. Set up data tables and graphs before you come to lab.

Bring cm × cm graph paper

Measure the Resistors First!

The resistors must be accurate in this experiment. Discard any with an error greater than 5%. Ask your lab instructor for a replacement.

The resistor values should be:

- Part 1:
  \[ R_S = 3.3K \text{ (DC case)}; \ R_S \text{ will be determined experimentally (AC case)} \]

- Parts 2 and 3:
  \[ R_1 = 3.3K; \ R_2 = 6.8K; \ R_3 = 4.7K; \ R_4 = 10K \]

Procedure

We will do the experiment almost "as is" in the experiment. The discussion below gives a bit more detail about the procedures of this experiment.
Part 1: Maximum Power Transfer Theorem

We will do this part twice. The first time through we will use a pure DC source. See Figure 1. The second time through we will use a pure AC source. See Figure 2.

For each case above we will **measure and record** $V_L$ for ten different test values of $R_L$ in the range $0.1R_S$ to $10R_S$. This, of course, will require you to know the value of $R_S$. It is very important to include $R_L = R_S$ as the center test value of set of $R_L$. So use this set of $R_L$:

$$R_L = \{0.1R_S, 0.3R_S, 0.5R_S, 0.7R_S, R_S, 2R_S, 5R_S, 8R_S, \text{and } 10R_S\}$$

You will then calculate the power absorbed by $R_L$:

$$P_{ABS,RL} = \frac{(V_{RL})^2}{R_L}$$

for each value of $R_L$. Use your data to plot $P_{ABS,RL}$ as a function of $R_L$.

To begin each case you will measure $V_{OC}$, the "open-circuit" voltage. See Figure 3. This is the case when $R_L = \infty$, i.e. there is no $R_L$ connected. Note that $V_{OC} = V_S$. Then connect a variable resistor as $R_L$ and adjust $R_L$ until the voltage $V_L$ becomes exactly $0.5V_{OC}$. When $V_L = 0.5V_{OC}$ then we know that $R_L$ is exactly equal to $R_S$. (See circuit analysis below.) So, we have just experimentally found $R_S$! Use this value of $R_S$ to determine the test values required as explained above and measure the voltages $V_L$ as explained above.

**Part 1A: DC Case**

Build the circuit using these discreet values:

- $V_S = 8$ volts DC. (Use one side on the dual DC supply)
- $R_S = 3.3K$ (So we know $R_S$ in advance. However use the above technique to verify that $R_L = R_S$ when $V_L = 0.5V_{OC}$)

Now get the data for the various $R_L$ and plot the power curve.

**Part 1B: AC Case**

The circuit is the Function Generator! $R_S$ and $V_S$ are inside the function generator. DO NOT INCLUDE AN EXTERNAL $R_S$!!!

Set $V_S = 5$ Volts RMS (Pure AC. The DC = 0.) To set this just use the DMM to measure the AC voltage at the terminals of the function generator and adjust the amplitude control until the AC (RMS) meter reads 5.00 Volts. Now connect the resistor decade box as $R_L$ and follow the above procedures to determine the value of the internal $R_S$ of the function generator. Now get the data for the various $R_L$ and plot the power curve.

Answer these questions:
1. Does $R_L = R_S$ when $V_L = 0.5V_{OC}$?
2. Does $R_L = R_S$ when the maximum power is being delivered to $R_L$?

**Part 2: Linearity**

**Part 2A: DC Point by Point Plot (The hard way)**

1. Set up the circuit in Figure 4. Use a DC supply for $V_S$.
2. Measure $V_O$ for these values of $V_S$:
   $$V_S = \{-4, -2, -1, 0, 1, 2, \text{ and } 4\} \text{ Volts.}$$
3. Plot $V_O$ as a function of $V_S$. Connect the points to get a continuous relation. Is the relation linear?
4. Verify that the slope $V_O/V_S$ is the same value as calculated in your circuit analysis.

**Part 2B: Automatic Plotting (The easy way)**

1. Set up the circuit in Figure 5. Use the function generator for $V_S$.
2. Connect the scope as indicated in Figure 5.
3. **Scope Setup**
   a. Put the scope in "X-Y" mode.
   b. Set both channels to GND and position the "dot" to center screen.
   c. Now set both channels to 1 Volt/DIV
4. **Function Generator Setup:**
   a. Turn DC to Off
   b. Use a sinusoidal waveform
   c. Set AC amplitude to maximum
   d. Set frequency to a "low" value ~60 to 120 Hz (whatever frequency gives the best or "cleanest" image)
5. You should now see a continuous plot of $V_O$ as a function of $V_S$. Sketch it. Is the relation linear?
6. Verify that the slope $V_O/V_S$ is the same value as calculated in your circuit analysis.

Are the plots from the above two methods the same? Which method was easier?

**Part 3: Superposition**

1. Set up the circuit in Figure 6.
2. Use the DMM to accurately set:
   a. $V_{S1} = 5.00$ Volts.
   b. $V_{S2} = 4.00$ Volts.
3. Now verify that superposition holds for $V_1$ and $I_2$. This requires that you show that:
   a. $V_1|_{(V_{S1} = 5, V_{S2} = 0)} + V_1|_{(V_{S1} = 0, V_{S2} = 4)} = V_1|_{(V_{S1} = 5, V_{S2} = 4)}$
and

\[ I_2(\text{VS1} = 5, \text{VS2} = 0) + I_2(\text{VS1} = 0, \text{VS2} = 4) = I_2(\text{VS1} = 5, \text{VS2} = 4) \]

4. **HINT:** After setting the sources, the best way to go back to Zero Volts (as is needed during data taking) is to remove the cables from a voltage source terminals and connect the cables together. You will have the Zero Volts required. Then, when you need the non-zero value again, just plug the cables back into the source. That way you do not waste time re-setting the source voltages.

5. So, fill in a data table like the one below and verify that the addition of rows one and two is equivalent to row three for each column.

<table>
<thead>
<tr>
<th>Row</th>
<th>( V_{S1} )</th>
<th>( V_{S2} )</th>
<th>( V )</th>
<th>( I_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Superposition Data Table

Set up appropriate data tables and plots for all the expected data for each part.

You will then compare this data to the calculated values from your circuit analysis and do error analysis for each part.

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**Circuit Analysis**

**Note:** An arrow through a resistor is the circuit symbol for a variable resister. Your Lab instructor will show you how to use the POWER RESISTOR DECADE BOX as a variable resistor.

**Part 1A: DC Case**

- \( R_S = 3.3K \), and
- \( V_S = 8 \text{ Volts DC} \)
Part 1B: AC Case

- $R_S = 50$ Ohms, and
- $V_S = 5$ Volts AC (RMS)

For each circuit above the "open circuit voltage" $V_{OC}$ is the value of $V_L$ when $R_L$ is infinite. Note that in that case

$V_{OC} = V_S$. See Figure 3.

Note that in Figures 1 and 2 if $R_L = R_S$ then

$V_L = 0.5V_S = 0.5V_{OC}$.

Which can be found easily by voltage division.

Also, when we have the above conditions, $R_L$ is absorbing the maximum power that the circuit is able to deliver. See pages 143-145 in your text for a proof.

Part 2: DC Point-by-Point Plot

For the circuit in Figure 4. find the ratio of $V_O/V_S$. You can do this using by successive voltage division of $V_S$. Note that this ratio is a constant now matter what the value of $V_S$. Show all of your work.
Part 2 Elements:

\[ R_1 = 3.3\, \text{K} \]
\[ R_2 = 6.8\, \text{K} \]
\[ R_3 = 4.7\, \text{K} \]
\[ R_4 = 10\, \text{K} \]

\( V_S = \{ -4, -2, -1, 0, 1, 2, \text{ and } 4 \, \text{volts} \} \)

Part 2: AC Continuous Plot

The circuit in Figure 5 shows how to connect the oscilloscope to easily verify linearity.

![Figure 4](image4)

![Figure 5](image5)

Part 3: Superposition

Use the principle of superposition to find \( V_1 \) and \( I_2 \) for the circuit in Figure 6. Show all of your work.

Part 3 Elements:

\[ R_1 = 3.3\, \text{K} \]
\[ R_2 = 6.8\, \text{K} \]
\[ R_3 = 4.7\, \text{K} \]
\[ R_4 = 10\, \text{K} \]

![Figure 6](image6)
$V_{S1} = 5\text{ volts.}$
$V_{S2} = 4\text{ volts.}$

Have fun.
ECE 225 Experiment #8

Theorems of Linear Networks

Purpose: To illustrate linearity, superposition, and the maximum power transfer theorem.

Equipment: Agilent 54622A Oscilloscope, Agilent 34401A Digital Multimeter (DMM), Agilent E3631A Triple Output DC Power Supply, Universal Breadbox

I. Maximum Power Transfer Theorem

Set up the circuit in Figure 1. For the variable load resistor $R_L$ use a decade resistor box. Measure $V_L$ and calculate the power absorbed in $R_L$, for a variety of values of resistance from $R_S/10$ to $10R_S$. Plot the values of power absorbed vs. the load resistance $R_L$. Find the value of $R_L$ which corresponds to a maximum on the graph. This should be the same value as $R_S$. Is it? Comment. Comment also on the accuracy of this technique as a way of determining the value which maximizes the power transfer. Comment on the deviation from maximum which occurs when the load resistor deviates from the optimum value by 50 percent.

![Circuit Diagram](image)

Figure 1.

A much more accurate way to determine the value of $R_L$ which maximizes power transfer is to make use of the Thevenin equivalent of the network in question. If the network is represented by its Thevenin equivalent ($V_{OC}$ and $R_{TH}$ in series) then when $R_L = R_{TH}$, the voltage across the $R_L$ will be $V_{OC}/2$. Thus the Thevenin equivalent resistance of any linear network can be determined by (1) measuring $V_{OC}$, and (2) attaching an $R_L$ and changing it until the load voltage is $V_{OC}/2$. This value maximizes the power transfer. Use this technique on the circuit above.
This technique also works if the sources in the network are sinusoidal, the
difference being that RMS measurements are made rather than DC measurements.
Adjust the function generator for zero DC offset and a frequency of 1 KHz. Then
using the method of the previous paragraph, determine the $R_{TH}$ of the function
generator (which, although shown as an ideal source in the circuit, actually has a
nonzero internal resistance), and using the less accurate graphical method find the
value of $R_L$ which maximizes the power transfer from the generator to its load.

II. Linearity

Set up the circuit in Figure 2. Take enough readings of $V_S$ and $V_O$ to make an
accurate graph of $V_O$ (vertically) on the graph vs. $V_S$ (horizontally). A smart way
to do this is to use the scope in the "X-Y" mode, using $V_S$ as the X (CH1) input
and $V_O$ as the Y (CH2) input, with the signal generator, running as a triangle
generator, attached to the input terminals. Record the graph and comment on the
linearity of the input/output relationship.

![Figure 2.](image)

III. Superposition

Set up the linear circuit Figure 2, using the dual DC source. Set $V_{S1} = 5$ Volts
and $V_{S2} = 0$ Volts, and record $V_4$ and $I_1$. Then set $V_{S1} = 0$ Volts and
$V_{S2} = 4$ Volts, and record $V_4$ and $I_1$ again. Finally set $V_{S1} = 5$ and $V_{S2} = 4$
and record $V_4$ and $I_1$ once more. Comment on the relationship between the sets of
readings.
Figure 3.