Problem 5.59  The input-voltage waveform shown in Fig. P5.59(a) is applied to the circuit in Fig. P5.59(b). Determine and plot the corresponding $v_{out}(t)$.

(a) Waveform of $v_i(t)$

(b) Op-amp circuit

Figure P5.59

Solution: The circuit in Fig. P5.59(a) is a differentiator circuit with

$$RC = 50 \times 10^3 \times 2 \times 10^{-6} = 0.1.$$  

For the time segment between $t = 0$ and $t = 2$ s, the slope of the input signal is $(12/2) = 6$ V/s. The output voltage is given by

$$v_{out} = -RC \frac{dv_i}{dt}$$

$$= -0.1 \times 6 = -0.6$$ V.

Hence, $v_{out}$ is a square wave with an amplitude of 0.6 V, as shown in the figure.
Problem 5.60  Relate $v_{\text{out}}$ to $v_i$ in the circuit of Fig. P5.60. Assume $v_i(0) = 0$.

**Solution:**

\[ v_n = v_p = v_i \]
\[ \frac{v_n}{R} + C \frac{d}{dt}(v_n - v_{\text{out}}) = 0 \]

Hence,
\[ \frac{d}{dt}v_{\text{out}} = \frac{1}{RC} v_i + \frac{d}{dt}v_i. \]

Integrating all terms from $t = 0$ to $t$,
\[ \int_0^t \left( \frac{d}{dt}v_{\text{out}} \right) dt = \frac{1}{RC} \int_0^t v_i dt + \int_0^t \left( \frac{d}{dt}v_i \right) dt, \]

which simplifies to
\[ v_{\text{out}}(t)|_0^t = \frac{1}{RC} \int_0^t v_i dt + v_i(t)|_0^t \]

or
\[ v_{\text{out}}(t) = v_{\text{out}}(0) + v_i(t) + \frac{1}{RC} \int_0^t v_i dt. \]
Problem 5.62  Relate $v_{out}$ to $v_i$ in the circuit of Fig. P5.62. Assume $v_C = 0$ at $t = 0$.

Figure P5.62: Circuit for Problem 5.62.

Solution:

$$i_1 = \frac{v_n - v_i}{R_1}$$

$$v_n - v_{out} = i_2 R_2 + \frac{1}{C} \int_0^t i_2 \, dt$$

But $v_n = v_p = 0$, and

$$i_2 = -i_1 = \frac{v_i}{R_1},$$

which leads to

$$v_{out} = -\left( \frac{R_2}{R_1} v_i + \frac{1}{R_1 C} \int_0^t v_i \, dt \right).$$
Problem 5.68  The two-stage op-amp circuit in Fig. P5.68 is driven by an input step voltage given by $v_{i}(t) = 10u(t)$ mV. If $V_{cc} = 10$ V for both op amps and the two capacitors had no charge prior to $t = 0$, determine and plot:

(a) $v_{out_1}(t)$ for $t \geq 0$
(b) $v_{out_2}(t)$ for $t \geq 0$

Solution:
(a)

$$v_{out_1}(t) = -\frac{1}{R_1C_1} \int_{0}^{t} v_{i} \, dt$$

$$R_1C_1 = 5 \times 10^3 \times 4 \times 10^{-6} = 0.02.$$ 

Hence,

$$v_{out_1}(t) = -50 \int_{0}^{t} 10 \times 10^{-3} \, dt = -0.5t \quad (V), \quad \text{for} \quad t \geq 0.$$ 

(b) For the second stage:

$$v_{out_2}(t) = -\frac{1}{R_2C_2} \int_{0}^{t} v_{out_1}(t) \, dt$$

$$R_2C_2 = 1 \times 10^6 \times 5 \times 10^{-6} = 5$$

$$v_{out_2}(t) = -\frac{1}{5} \int_{0}^{t} (-0.5t) \, dt = 0.1 \frac{t^2}{2} = 0.05t^2 \quad (V), \quad \text{for} \quad t \geq 0.$$ 

Plots of $v_{i}(t)$, $v_{out_1}(t)$, and $v_{out_2}(t)$ are shown below. We note that $v_{out_1}(t)$ reaches saturation at $-V_{cc} = -10$ V after 20 s, and $v_{out_2}(t)$ reaches saturation at $V_{cc} = +10$ V at $t = 14.14$ s.
Figure P5.68