LOCALIZATION AND TRAJECTORY ESTIMATION OF MOBILE OBJECTS WITH A SINGLE SENSOR

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ABSTRACT
The localization problem in mobile sensors is aimed at identifying the spatial location of an object with reference to a known coordinate system. Existing solutions assume that multiple (generally three or more) sensors around the object know their position and the solution is obtained by using a triangulation scheme. Such solutions are not computationally feasible when the object or sensor are moving. In this paper, we present a novel method to solve the localization problem for an object whose position is unknown using a single moving sensor whose position is known. The proposed method relies on multiple time samples from the moving sensor to estimate the trajectory of the moving object. We derive the Cramer-Rao bound for the localization parameters and use an unscented Kalman filter to estimate the parameters from noisy measurements.

Index Terms— Sensors, Estimation, Kalman filtering.

1. INTRODUCTION
Sensor localization is a key enabling technology in wireless sensor networks. Existing solutions to the localization problem, for example [3,4], are based on estimating the Received Signal Strength (RSS) from a set of sensors whose positions and transmission power are known. Subsequently assuming some power fading model and employing triangulation methods, the localization problem is solved. In some of the existing approaches it is assumed that fixed known-position sensors are available [6]. In other approaches [5] anchor-free localization is performed, where no absolute positions are known. In most of the existing works, fixed or relative position of at least two sensors is required to solve the localization problem [6]. For the case of mobile sensors, localization may need to be solved frequently thus making it imperative to have power- and communication-efficient solutions. In existing sensor localization systems, fixed or relative position of at least two sensors are required to solve the localization problem. In this paper, we propose a method for sensor localization and trajectory estimation from a single moving sensor. We refer to the node with unknown position as an “object” and the node whose position is known as a “sensor”. Both the object and sensor are assumed to be moving. Cramer-Rao bounds on the estimation error of sensor localization from stationary sensors have been presented in [1]. We compute the Cramer-Rao bounds (CRB) for sensor localization in mobile networks. Finally, we use an unscented Kalman filter for non-linear estimation and prediction of the localization parameters from noisy measurements.

2. GEOMETRICAL ANALYSIS AND SENSOR LOCALIZATION
The premise of this paper is based on the observation that a motion trajectory can be represented as a sequence of piecewise continuous line segments. The accuracy of this representation increases with the decrease in the length of the line segments. Furthermore, as the length of a line segment further decreases, it is expected that adjacent line segments will locally form a straight line. In the context of sensor networks, this observation implies that, in a very short duration, the object’s direction of motion and velocity remain constant provided the measurement sampling rate is sufficiently high. In
the remainder of this paper, we assume that the direction and velocity of the object in four consecutive samples remains constant (i.e. the sampling rate is very high). Notice, however, that both the direction and velocity of the object can change when using the four consecutive samples based on the next measurement, i.e. the assumption of constant direction and velocity of the object is valid over a moving window of four samples. We further assume that the absolute position of the sensor at each sample is known.

Assuming that the initial location of the sensor is known, say (0, 0), e.g., node O in Fig. 2, the aim is to compute the object location (x, y), i.e. node A in Fig. 2, with reference to (0, 0). The direction of the velocity of the object is decomposed into two orthogonal directions v_x, v_y along x and y axis. By using RSS measurements, the distances between the moving object and the sensor can be expressed as r_0, r_1, r_2, r_3. The time spent from the first sample to the second, the third, and the fourth sample are t_1, t_2, t_3 respectively. The four sampling locations for the object are (0, 0), (x_1, y_1), (x_2, y_2), (x_3, y_3), denoted in Fig. 1 as O, B, C, D respectively. Under our linear assumption, the locations of the moving object at the four samples are (x, y), (x + v_x t_1, y + v_y t_1), (x + v_x t_2, y + v_y t_2), (x + v_x t_3, y + v_y t_3) respectively, denoted in Fig. 1 as A, E, F, G. Applying the geometry knowledge we can derive the following equations:

\[ r_0^2 = x^2 + y^2 \]  
\[ r_1^2 = (x + v_x t_1 - x_1)^2 + (y + v_y t_1 - y_1)^2 \]  
\[ r_2^2 = (x + v_x t_2 - x_2)^2 + (y + v_y t_2 - y_2)^2 \]  
\[ r_3^2 = (x + v_x t_3 - x_3)^2 + (y + v_y t_3 - y_3)^2 \]  

Let us derive the sufficient conditions which assure our models have solutions for (x, y). In fact, our problem is equivalent to the problem of finding sufficient conditions for four circles to have common intersections. For the four circles with the centers (0, 0), (x_1 - v_x t_1, y_1 - v_y t_1), (x_2 - v_x t_2, y_2 - v_y t_2), (x_3 - v_x t_3, y_3 - v_y t_3), and the radius r_0, r_1, r_2, r_3, we know that if two circles have intersections, the distance between the centers of two circles d_{i,j} is greater than the absolute value of the difference of the two radii and less than the sum of the two radii. So we have the following six inequalities: \( \forall i, j = 0, 1, 2, 3 \) and \( i \neq j \)

\[ (r_i - r_j)^2 \leq d_{i,j}^2 \leq (r_i + r_j)^2 . \]  

In order to write the results in matrix form, we subtract (2), (3), (4) from (1), respectively, as follows:

\[ \begin{bmatrix} 2(v_x t_1 - x_1) \\ 2(v_x t_2 - x_2) \\ 2(v_x t_3 - x_3) \end{bmatrix}, \begin{bmatrix} 2(v_y t_1 - y_1) \\ 2(v_y t_2 - y_2) \\ 2(v_y t_3 - y_3) \end{bmatrix} \] 

\[ \begin{bmatrix} r_1^2 - r_0^2 - (v_x t_1 - x_1)^2 - (v_y t_1 - y_1)^2 \\ r_2^2 - r_0^2 - (v_x t_2 - x_2)^2 - (v_y t_2 - y_2)^2 \\ r_3^2 - r_0^2 - (v_x t_3 - x_3)^2 - (v_y t_3 - y_3)^2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} . \]  

We write (6) as \( AX = B \), where X represents the location vector \([x, y] \). Therefore, we obtain

\[ X = A^T B , \]  

where "i" is used to denote the pseudo-inverse. Substituting (7) into (1) to cancel the unknown parameters \((x, y)\), we get an equation containing the relationship between the velocities and the distance. Combining this new equation with the six inequalities in (5), we arrive at the sufficient conditions required to ensure that the nonlinear system of equations (1)-(4) has at least one solution.

3. THE STATISTICAL MODEL AND CRAMER-RAO BOUND FOR LOCATION ESTIMATION

Through the equations (1)-(4), let us consider the following scenario: five stationary sensors whose coordinates are at \((x, y)\), \((0, 0)\), \((x_1 - v_x t_1, y_1 - v_y t_1)\), \((x_2 - v_x t_2, y_2 - v_y t_2)\) and \((x_3 - v_x t_3, y_3 - v_y t_3)\) respectively. The distances from the sensor \((x, y)\) to the other four stationary sensors are \(r_0, r_1, r_2, r_3\) respectively. Since we will have the same equations as (1) to (4), so the mobile case and the stationary case should have the same estimation bound with respect to the unknown parameters: the vector \((x, y, v_x, v_y)\). For the stationary case, we denote that \((x_i, y_i)\) is the position of a single sensor i. The actual distance \(d_{i,j}\) between sensor i and j is given by

\[ d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \]  

According to [1], the statistical model for the ensemble mean power at distance d is typically modelled as

\[ P = P_0 - 10n_p \log(d/d_0) \]
where $P_0$ is the received power at a short reference distance $d_0$ and $n_p$ is the path-loss exponent. The received power at sensor $i$ transmitted by $j$, $P_{i,j}$ has Gaussian distribution

$$f(P_{i,j} = p|\theta) = N(p; \bar{P}(d_{i,j}), \sigma^2_{P0}) ,$$

where $\sigma^2_{P0}$ is relatively constant with distance. Let $x_i, y_i$ be the 2i-1th and 2i-th parameters to be estimated respectively, $i=1,2,...M$. In our case, $x_i = x, 0, x_1 - v_s t_1, x_2 - v_s t_2, x_3 - v_s t_3$, and $y_i = y, 0, y_1 - v_p t_1, y_2 - v_p t_2, y_3 - v_p t_3$ respectively. The $10 \times 10$ Fisher Information Matrix $J$ is:

$$J_{2i-1,2i-1} = \gamma \sum (x_i - x_j)^2 / d_{i,j}^4$$
$$J_{2i,2i} = \gamma \sum (y_i - y_j)^2 / d_{i,j}^4$$
$$J_{2i-1,2i} = J_{2i,2i-1} = \gamma \sum (x_i - y_j)(y_i - y_j) / d_{i,j}^4 .$$

For nondiagonal entries $j \neq i$,

$$J_{2i-1,2j-1} = J_{2j-1,2i-1} = -\gamma (x_i - x_j)^2 / d_{i,j}^4$$
$$J_{2i,2j} = J_{2j,2i} = -\gamma (y_i - y_j)^2 / d_{i,j}^4$$
$$J_{2i-1,2j} = J_{2j,2i-1} = -\gamma (x_i - y_j)(y_i - y_j) / d_{i,j}^4 ,$$

where $\gamma = (\frac{10 n_p}{\sigma^2_{P0} d_{i,j}^4})^2$. Let’s define $\hat{x}, \hat{y}, \hat{v}_x, \hat{v}_y$ as unbiased estimators of the location and velocity of the moving object $x, y, v_x, v_y$. And define $\hat{\theta}$ as a vector $[\hat{x}, \hat{y}, \hat{v}_x, \hat{v}_y]$ and $\xi^2$ as the error of the whole estimation vector. Therefore, we have:

$$Var(\hat{x}) = E( (\hat{x} - x)^2 ) \geq J^{-1}_{1,1}$$
$$Var(\hat{y}) = E( (\hat{y} - y)^2 ) \geq J^{-1}_{2,2}$$
$$Var(\hat{v}_x) \geq J^{-1}_{5,5}$$
$$Var(\hat{v}_y) \geq J^{-1}_{6,6} .$$

Thus, the estimation bound for $v_x, v_y$ can be expressed as

$$Var(\hat{v}_x) \geq 1/t_1^2 + J^{-1}_{5,5}$$
$$Var(\hat{v}_y) \geq 1/t_1^2 + J^{-1}_{6,6} .$$

$$\xi^2 \equiv tr[cov(\hat{\theta})] = Var(\hat{x}) + Var(\hat{y})$$
$$+ Var(\hat{v}_x) + Var(\hat{v}_y) .$$

Substituting (17), (18), (21), (22) into (23), we obtain

$$\xi^2 \geq J^{-1}_{1,1} + J^{-1}_{2,2} + J^{-1}_{5,5} + J^{-1}_{6,6} .$$

\section{Localization Performance and Unscented Kalman Filtering}

In our nonlinear model, unscented Kalman filter can be used to improve the accuracy for localization. We assume the motion of the $i^{th}$ sample is governed by following state equation:

$$\begin{pmatrix}
    x_{k+1} \\
    y_{k+1} \\
    v_{x,k+1} \\
    v_{y,k+1}
\end{pmatrix} = F
\begin{pmatrix}
    x_k \\
    y_k \\
    v_{x,k} \\
    v_{y,k}
\end{pmatrix} +
\begin{pmatrix}
    w_{x,k} \\
    w_{y,k} \\
    w_{v_{x,k}} \\
    w_{v_{y,k}}
\end{pmatrix} ,$$

where $F =
\begin{pmatrix}
    1 & 0 & \Delta_i(k) & 0 \\
    0 & 1 & 0 & \Delta_i(k) \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{pmatrix}$, and $\Delta_i(k)$ is used to denote the time between the $k+1$ sample and $k$ sample, and $w(k) = [w_{x,k}, w_{y,k}, w_{v_{x,k}}, w_{v_{y,k}}]$ is a random noise process that accounts for random perturbations. We model $w(k)$ as a zero-mean white-noise process with covariance given by $E[w(k)w(k)^T] = Q$ .

The observation equation is given by

$$y(k) = g(x(k)) + \nu(k) ,$$

where $g(.)$ is provided by equations (1)-(4). The unscented Kalman filter embeds the unscented transform into the recursive prediction and update structure of the Kalman filter [2]. We provide the prediction and update steps for our approach to localization based on a minor reformulation of the state and observation models in [2].

\textbf{Tracking Algorithm:}

\textbf{Predict:}

$$X^u_{k-1|k-1} = ( \hat{X}_{k-1|k-1} E[w_k] )$$

$$P^u_{k-1|k-1} = \begin{pmatrix}
    P_{k-1|k-1} & 0 \\
    0 & Q_k
\end{pmatrix} .$$

generate $(2L+1)$ sigma points:

$$X^\beta_{k-1|k-1} = X^u_{k-1|k-1} + (\sqrt{(L+\lambda)P^u_{k-1|k-1}})_i$$

$$i = 1,\ldots,L$$

$$X^\alpha_{k-1|k-1} = X^u_{k-1|k-1} - (\sqrt{(L+\lambda)P^u_{k-1|k-1}})_i.$$ 

$$i = L,\ldots,2L .$$

propagated sigma points through state equation f:

$$X^\beta_{k|k-1} = f(X^\alpha_{k-1|k-1})$$

$$i = 0,1,\ldots,2L .$$

the mean and covariance of predicted state:

$$X_{k|k-1} = \sum_{i=0}^{2L} W_i X^\beta_{k|k-1}$$

$$P_{k|k-1} = \sum_{i=0}^{2L} W_i (X^\beta_{k|k-1} - \hat{X}_{k|k-1})(X^\beta_{k|k-1} - \hat{X}_{k|k-1})^T .$$
the weight is computed as:

\[ W_0 = \frac{\lambda}{L + \lambda} \]  
\[ W_i = \frac{1}{2(L + \lambda)} \]  
\[ W_{i+n} = \frac{1}{2(L + \lambda)} \]  

**Update:** Augment the predicted state and covariance with the mean and covariance of the measurement noise:

\[ X_{k|k-1}^\alpha = \begin{pmatrix} \hat{X}_{k|k-1} & E[v_k] \end{pmatrix} \]  
\[ P_{k|k-1}^\alpha = \begin{pmatrix} P_{k|k-1} & 0 \\ 0 & R_k \end{pmatrix} \]

as before generate sigma points:

\[ \chi_{0k|k-1} = X_{k|k-1}^\alpha \]  
\[ \chi_{ik|k-1} = X_{k|k-1}^\alpha + \sqrt{(L + \lambda)P_{k|k-1}^\alpha} \]  
\[ i = 1 \ldots L \]  
\[ \chi_{ik|k-1} = X_{k|k-1}^\alpha - \sqrt{(L + \lambda)P_{k|k-1}^\alpha} \]  
\[ i = L + 1 \ldots 2L \]

sigma points are projected through nonlinear observation equations \( g \)

\[ \gamma_k^i = g(\chi_{ik|k-1}) \]  
\[ i = 0, 1 \ldots 2L \]

recombine sigma points to produce the predicted measurement, covariance and cross-covariance:

\[ \hat{y}_k = \sum_{i=0}^{2L} W_i \gamma_k^i \]  
\[ P_{yy_k} = \sum_{i=0}^{2L} W_i (\gamma_k^i - \hat{y}_k)(\gamma_k^i - \hat{y}_k)^T \]  
\[ P_{X_ky_k} = \sum_{i=0}^{2L} W_i (\chi_{ik|k-1} - \hat{X}_{k|k-1})(\gamma_k^i - \hat{y}_k)^T \]

the cross-covariance is used to compute Kalman gain:

\[ K_k = P_{X_ky_k} P_{y_ky_k}^{-1} \]

\[ \hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k(\hat{y}_k - \hat{y}_k) \]  
\[ P_{k|k} = P_{k|k-1} - K_k P_{y_ky_k} K_k^T \]

Initial values: \( E[x(0|0)] \) and \( P(0|0) \).

In computer simulations, the performance of the localiza-
Fig. 6. Trajectory estimation for RSS measurement when SNR=100 and sampling frequency f=10Hz. (Solid line denotes real track, × represents estimated track).

The tracking algorithm is illustrated using an example of tracking three moving targets in a four-node sensor network with trajectories which are denoted as: 

\[ y = 3 \sin(2x), \]

\[ y = \sin(3x), \]

\[ y = -2 \sin(2x). \]

The tracking is performed over the sampling interval 0.2s. We select \( \lambda = -2 \) according to the useful heuristic \( L + \lambda = 3 \) for the case of Gaussian distribution [2], where \( \lambda \) provides an extra degree of freedom to "fine tune" the higher order moments of the approximation, and can be used to reduce the overall prediction errors. The performance is evaluated when SNR is 20dB and 100dB with sampling frequency is 5Hz and 10Hz respectively. Note that SNR is the ratio of the mean of the power to the variance of the noise. The process noise \( w(k) \) and measurement noise \( v(k) \) are zero mean with the variance 0.1. The initial value for the covariance matrix is set to be \( P[0|0] = 10^4 I \), I is identity matrix.

**Evaluation:** Compared with Monte Carlo-type methods, although it seems that UKF has something similar to them, the difference is that the samples are not drawn at random but according to a specific, deterministic algorithm and high order distribution can be captured with a small number of points [2]. As it can be seen from simulation results, by increasing SNR, the estimated trajectory is much more accurate generally. The localization performance is improved with higher sampling frequency especially when the moving objects make turns.

### 5. CONCLUSIONS

In this paper, we presented a novel approach to identify spatial location of a moving object and its trajectory estimation from successive samples of a single moving sensor. We derived Cramer-Rao bound on estimation error from moving sensors by extending existing results for stationary models. We also proposed to use unscented Kalman filter for improved localization estimation and prediction from noisy measurements. We finally provided numerical simulation results that illustrate the effectiveness of our approach to localization problem and improved accuracy of the estimation obtained by using unscented Kalman filters. The approach presented here for localization from a single moving sensor based on RSS can be easily extended to TOA and AOA measurements.

### 6. REFERENCES


