ROBUST CLOSED-FORM LOCALIZATION OF MOBILE TARGETS USING A SINGLE SENSOR BASED ON A NON-LINEAR MEASUREMENT MODEL

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ABSTRACT

In this paper, we propose a robust novel approach with closed-form estimator for object tracking based on a non-linear measurement model over time from a single sensor with arbitrary noise degradation. Relying on the widely-used dynamic motion model for arbitrary moving targets, tracking of moving objects can be formulated using Received Signal Strength (RSS) measurements. We provide a closed-form solution that integrates localization and filtering for both an ideal channel as well as noisy channel. We first derive an exact linear model from the non-linear system of equations provided by the RSS measurements. We subsequently present an iterative method to estimate the unknown parameters and the error covariance matrix. Moreover, we prove that the estimator gives more accuracy when the number of samples increases. The Cramer-Rao Bound (CRB) for the estimator are determined in Gaussian case. Computer simulation demonstrates that the proposed approach not only achieves more accuracy than traditional methods but also saves significant computation time.

1. INTRODUCTION

Successive localizations of a moving object using a sensor with a known reference point provides a method for object tracking. Most of the approaches used to address the localization problem rely on a triangulation scheme thus requiring a minimum of two or three reference points. In previous work, several techniques are proposed to solve the localization problem in wireless sensor networks, such as Newton's methods [1], non-linear least squares [2] and Extended Kalman filter (EKF) [3].

However, as the number of samples increase, the optimization of the location for the targets is still a challenging problem due to the high non-linearity of the systems with Received Signal Strength (RSS) measurements [4]. Moreover, it is also desirable to reduce the computation amount in the process of the localization and trajectory estimation of the mobile targets. In addition, the whiteness of the noise and the continuous samples are required for EKF. If some samples are lost, it is difficult to estimate the trajectory of the mobile targets. In this paper, we propose a novel and efficient approach to integrate the localization and filtering and thus improve the performance of the localization and trajectory estimation of mobile targets using a single moving sensor with arbitrary noise degradation. Specifically, we come up with a closed-form estimator based on linearization of non-linear systems. We discuss tracking of the trajectory assuming formulations based on RSS measurements transmitted over both ideal and noisy channels. The Cramer-Rao Bound (CRB) are derived in Gaussian cases. Finally, we use computer simulations to demonstrate the superiority of our approach over Newton's method, Extended Kalman Filters (EKF) and Multivariate Analysis of Variance (MANOVA) model [5] by comparing the accuracy and computation time. *Our key contributions are, (i) We use only a single sensor to solve the localization problem of mobile objects instead of using two or more sensors traditionally [6],(ii) as far as we know, it is the first time we propose a closed-form estimator from non-linear systems based on RSS measurements without assuming any Gaussianity, which provides more accuracy and less computation amount than traditional approaches, (iii) we have proved that our estimator gives more accuracy when the number of samples increases.*
We refer to the node with unknown position as a "target" and the node whose position is known as a "sensor". The rest of the paper is organized as follows. In Section 2, the mathematical models for the dynamics of moving targets is introduced. In Section 3, the tracking algorithm for ideal channel and noisy channel based on RSS measurements are presented. Moreover, we prove that the estimator gives more accuracy when the number of samples increases. In Section 4, the Cramer-Rao bound in Gaussian cases is determined. Section 5 demonstrates the superiority of our approach over traditional approaches with computer simulation. Finally, in section 6, we provide a brief summary of the results.

2. MATHEMATICAL MODEL FOR THE DYNAMICS OF MOVING TARGETS

For an arbitrary moving target, let $X_k, Y_k, V_{x,k}, V_{y,k}$ denote the coordinates and velocities in x and y directions at kth sample. We assume that the acceleration during two adjacent samples is invariant provided that we sample very fast. Based on Newton's laws of motion, the motion of the kth sample in x direction is governed by:

$$\begin{pmatrix} X_{k+1} \\ V_{x,k+1} \end{pmatrix} = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_k \\ V_{x,k} \end{pmatrix} + G_{x,k} \begin{pmatrix} T^2/2 \\ T \end{pmatrix}, \quad (1)$$

where the acceleration in x direction during the interval of two adjacent samples $G_{x,k}$ is assumed to be IID with zero mean and the variance δ_g^2. Increasing the index from k to k+1 in (1), we obtain:

$$X_{k+2} = X_{k+1} + TV_{x,k+1} + G_{x,k+1}T^2/2 . \quad (2)$$

Combining (1) and (2) to cancel X_{k+1} and $V_{x,k+1}$:

$$X_{k+2} = X_k + 2TV_{x,k} + G_{x,k}3T^2/2 + G_{x,k+1}T^2/2 . \quad (3)$$

Similarly, X_n can be expressed as:

$$X_n = \begin{cases} X_k + (n-k)TV_{x,k} + [(n-k) - \frac{1}{2}]T^2 G_{x,k} \\ +... + \frac{1}{2}T^2 G_{x,n-1} (n > k) \\ X_k - (k-n)TV_{x,k} + [(k-n) - \frac{1}{2}]T^2 G_{x,k-1} \\ +... + \frac{1}{2}T^2 G_{x,n} (n < k) . \end{cases}$$

Denoting the noise term in X_n as $\epsilon_{x,n}$, we have:

$$\epsilon_{x,n} = \begin{cases} [(n-k) - \frac{1}{2}]T^2 G_{x,k} + [(n-k) - \frac{3}{2}]T^2 G_{x,k+1} \\ +... + \frac{1}{2}T^2 G_{x,n-1} (n > k) . \\ T^2 G_{x,k-1}[(k-n) - \frac{1}{2}] + [(k-n) - \frac{3}{2}]T^2 G_{x,k-2} \\ +... + \frac{1}{2}T^2 G_{x,n} (n < k) . \end{cases}$$

Thus,

$$X_n = X_k + (n-k)T + \epsilon_{x,n} . \quad (4)$$

So $\epsilon_{x,n}$ is also a zero mean random variable with the variance:

$$Var(\epsilon_{x,n}) = \frac{|n-k|(4(n-k)^2 - 1)}{12} T^4 \delta_g^2 . \quad (5)$$

In equation (5), larger $|n-k|$ implies that X_n contains larger variance of the noise. Due to symmetry, similar expression and the statistics of $\epsilon_{y,n}$ can be obtained in y direction.

3. TRACKING ALGORITHM

3.1. Ideal Channel

We consider two sources of noise in this paper: (1) the dynamic motion noise, which arises from the assumption of constant acceleration during two samples, and (2) channel noise, which refers to multiplicative fading and additive noise effects. When we assume an ideal channel, it implies that we only consider dynamic motion noise. We assume δ_g^2 as prior knowledge. In RSS measurements, the distances between the target and the sensor are obtained as $r_1, r_2 ... r_n$. The ith sampling location of the sensor is denoted as (a_i, b_i). So we have:

$$(Y_n - b_n)^2 + (X_n - a_n)^2 = r_n^2 , \quad (6)$$

where a_n, b_n, r_n are known and Y_n and X_n are unknown. Relying on the equation (4), cancel X_n, Y_n to obtain:

$$(Y_k + (n-k)TV_{y,k} + \epsilon_{y,n} - b_n)^2 + (X_k + (n-k)TV_{x,k} + \epsilon_{x,n} - a_n)^2 = r_n^2 . \quad (7)$$

Expand equation (7) and denote the noise terms as $Z_{k,n}$:

$$Z_{k,n} = -2\epsilon_{x,n}(X_k + (n-k)TV_{x,k} - a_n) - 2\epsilon_{y,n}(Y_k + (n-k)TV_{y,k} - b_n) - \epsilon_{x,n}^2 - \epsilon_{y,n}^2 . \quad (8)$$

Traditionally, in order to solve the non-linear estimation problem, Newton's method or non-linear least squares are proposed. However, the limitations of Newton's method are threefold: (1) the nonlinear function to be optimized should be convex, (2) the initial value for Newton's method should be in the convergence region which is difficult to define in general, and (3) if the Hessian in Newton's approach is close to a non-invertible matrix, the inverted Hessian can be numerically unstable and the solution may diverge. In view of these problems, we propose a novel approach to first linearize the system and then rely on best linear unbiased estimator to (BLUE) to estimate the unknown parameters and the covariance matrix iteratively. Specifically, we denote the nonlinear equations in the matrix form as $V = A\beta + Z_k$, where β=$[Y_k^2, X_k^2, Y_k V_{y,k}, X_k V_{x,k}, V_{y,k}^2, V_{x,k}^2, Y_k, X_k, V_{y,k}, V_{x,k}]^T$, A is the coefficient matrix, Z_k is the noise. We linearize the equations by

$$SV = SA\beta + SZ_k , \quad (9)$$

where

$$S = \begin{pmatrix} 1 & -3 & 3 & -1 & 0 & 0 & ... & 0 \\ 0 & 1 & -3 & 3 & -1 & 0 & ... & 0 \\ ... \\ 0 & 0 & ... & 0 & 1 & -3 & 3 & -1 \end{pmatrix} . \quad (10)$$

In the equation (9) and (10), it is easy to verify that the coefficients of the quadratic terms all turn out to be zero. Therefore, the unknown parameters after linearization $\beta = [Y_k, X_k, V_{y,k}, V_{x,k}]^T$. The covariance matrix $\Sigma_{\tilde{Z}_k}$ for the noise after linearization $\tilde{Z}_k = [\tilde{Z}_{k,1}, ...\tilde{Z}_{k,n}]$ is computed as:

$$\Sigma_{Z_k} = \begin{cases} 4[(X_k + T(i-k)V_{x,k} - a_i)^2 \\ +(Y_k + T(i-k)V_{y,k} - b_i)^2]Var(\epsilon_{x,i}) \\ +4Var(\epsilon_{x,i})^2 & i = j \\ 4\{[(X_k + T(i-k)V_{x,k} - a_i)(X_k + \\ T(j-k)V_{x,k} - a_j)] + [(Y_k + \\ T(i-k)V_{y,k} - b_i)(Y_k + T(j-k)V_{y,k} \\ -b_j)]\}E(\epsilon_{x,i}\epsilon_{x,j}) + 4E(\epsilon_{x,i}\epsilon_{x,j})^2 & i \neq j \end{cases}$$

$$\Sigma_{\tilde{Z}_k} = S\Sigma_{Z_k}S^T, \quad (11)$$

Tracking Algorithm:

1. Subtract the mean of the noise $Z_{k,n}$ from both sides of the n equations to make the noise zero mean.

2. Linearize the nonlinear equations and reduce them to n-3 linear equations $V = \tilde{A}\beta + \tilde{Z}_k$ by $\tilde{A} = SA, \tilde{Z}_k = SZ_k$, where the transformation matrix S is given in the equation (10).

3. Based on the linear equations obtained from step 2, the parameter β and the covariance matrix of the error $\Sigma_{\tilde{Z}_k}$ which also is a function of β are estimated iteratively.
   3a) Initialize the estimation for β by ordinary least square estimator: $\hat{\beta}^{(0)} = (\tilde{A}'\tilde{A})^{-1}\tilde{A}'V$.
   3b) At the m+1 cycle, since the covariance matrix is a function of the unknown parameter $[Y_k, X_k, V_{y,k}, V_{x,k}]$, update the estimated covariance matrix of error $\Sigma_{\tilde{Z}_k}^{(m+1)}$ based on $\hat{\beta}^{(m)}$.
   3c) Conduct BLUE estimation for $\hat{\beta}^{(m+1)}$ with estimated covariance matrix of the error evaluated at step 2: $\hat{\beta}^{(m+1)} = (\tilde{A}'\Sigma_{\tilde{Z}_k}^{(m+1)^{-1}}\tilde{A})^{-1}\tilde{A}'\Sigma_{\tilde{Z}_k}^{(m+1)^{-1}}V$.
   3d) Repeat steps 3b-3d until the estimates have all stabilized or until there is no improvements in the residual sums of squares

Linearizing the nonlinear functions in RSS measurements and using best linear unbiased estimator circumvents the requirement of the good initial guess in the convergence region and computation of the gradient and Hessian matrix. Moreover, fast computation speed is guaranteed with our approach by best linear unbiased estimator.

3.2. Noisy Channel

For non-ideal channels including multiplicative fading and additive noise effects, let's assume:

$$\hat{r_i} = r_i + \omega_i \ , \tag{12}$$

for RSS measurements, where r_i is denoted as the true values of the distance at ith sample. The additive zero mean noise ω_n is uncorrelated with the dynamic motion noise. The covariance matrix of the channel noise satisfies $\Sigma_{\omega_i\omega_j} = 0$ for $i \neq j$ and $\Sigma_{\omega_i\omega_i} = \xi_i$ which is available and can be estimated via sample averaging as [7]. Thus, the noise vector $Q_{k,n}$ and its covariance matrix $\Sigma_{\tilde{Q}_k}$ after linearization can be computed as:

$$Q_{k,n} = Z_{k,n} - 2\omega_n\hat{r_n} + \omega_n^2 \ . \tag{13}$$

$$\Sigma_{Q_k} = \begin{cases} Var(Z_{k,i}) + 2\xi_i^2 + 4\hat{r_i^2}\xi_i & i = j \\ cov(Z_{k,i}, Z_{k,j}) - 2\xi_j Var(\epsilon_{x,i}) - \\ 2\xi_i Var(\epsilon_{x,j}) & i \neq j \end{cases}$$

$$\Sigma_{\tilde{Q}_k} = S\Sigma_{Z_k}S^T, \ , \tag{14}$$

where the notation for the transformation matrix S is the same as the case in RSS ideal channel. The case of noisy channel can be solved with the same approach as that in RSS ideal channel.

Lemma 1: For both of the ideal and noisy channels, the error of the estimator monotonically decrease when the number of samples increases.

Proof: Let us denote the covariance matrix of the estimator as $Cov(\beta_1)$ for k+1 samples and $Cov(\beta_2)$ for k samples. We need to prove $Cov(\beta_1) \leq Cov(\beta_2)$, where

$$Cov(\beta_1) = (A_1'S_1'(S_1\Sigma_1S_1')^{-1}S_1A_1)^{-1} \tag{15}$$

$$Cov(\beta_2) = (A_2'S_2'(S_2\Sigma_2S_2')^{-1}S_2A_2)^{-1} \ . \tag{16}$$

We partition the matrices as $A_1 = \begin{pmatrix} A_2 \\ a \end{pmatrix}, S_1 = \begin{pmatrix} S_2 & 0 \\ P & 1 \end{pmatrix}$, $\Sigma_1 = \begin{pmatrix} \Sigma_2 & T' \\ T & C \end{pmatrix}$ so that A_1 is $n \times 4$, A_2 is $(n-1) \times 4$, a is 1×4, S_1 is $(n-3) \times n$, Σ_1 is $n \times n$, Σ_2 is $(n-1) \times (n-1)$ and P is $1 \times (n-1)$. So we have

$$\begin{aligned} (S_1\Sigma_1S_1')^{-1} &= [\begin{pmatrix} S_2 & 0 \\ P & 1 \end{pmatrix}\begin{pmatrix} \Sigma_2 & T' \\ T & C \end{pmatrix}\begin{pmatrix} S_2' & P' \\ 0 & 1 \end{pmatrix}]^{-1} \\ &= [\begin{pmatrix} S_2\Sigma_2 & S_2T' \\ P\Sigma_2 + T & PT' + C \end{pmatrix}\begin{pmatrix} S_2' & P' \\ 0 & 1 \end{pmatrix}]^{-1} \\ &= [\begin{pmatrix} S_2\Sigma_2S_2' & S_2\Sigma_2P' + S_2T' \\ P\Sigma_2S_2' + TS_2' & P\Sigma_2P' + TP' + PT' + C \end{pmatrix}]^{-1} \\ &= [\begin{pmatrix} X_1 & X_2 \\ X_2' & X_3 \end{pmatrix}]^{-1} = X^{-1} \end{aligned} \tag{17}$$

$$S_1A_1 = \begin{pmatrix} S_2A_2 \\ PA_2 + a \end{pmatrix} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = Y \tag{18}$$

Thus, using the formula for inversion of a partitioned matrix [8] to get:

$$\begin{aligned} Y'X^{-1}Y &= Y_1'X_1^{-1}Y_1 + (Y_2 - X_2'X_1^{-1}Y_1)' \\ &(X_3 - X_2'X_1^{-1}X_2)^{-1}(Y_2 - X_2'X_1^{-1}Y_1) \ , \end{aligned} \tag{19}$$

$$Y'X^{-1}Y \geq Y_1'X_1^{-1}Y_1 \ , \tag{20}$$

from which the assertion of the lemma 1 follows easily.

4. CRAMER-RAO BOUND

The Cramer-Rao Bound is widely used to evaluate the fundamental hardness of an estimation problem [6]. Noticed that our system does not rely on any assumption of distribution and the assumption of Gaussianity is only needed for computing the lower bound. Let us assume the noise $\tilde{Z}_k$ has Gaussian distribution. Thus, $\tilde{V}_k = \tilde{A}\beta + \tilde{Z}_k$ is distributed as $N_n(\tilde{A}\beta, \Sigma_{\tilde{Z}_k})$. The likelihood function for the random variable $\tilde{V}_k$ can be represented as:

$$\begin{aligned} P(\tilde{V}_{k,i}) &= \frac{1}{\sqrt{(2\pi)^n|\Sigma_{\tilde{Z}_k}|}} exp[-\frac{1}{2}(\tilde{V}_{k,i} - \tilde{A}\beta)'\Sigma_{\tilde{Z}_k}^{-1} \\ &(\tilde{V}_{k,i} - \tilde{A}\beta)] \ . \end{aligned} \tag{21}$$

The 4×4 Fisher information matrix J can be computed as:

$$\begin{aligned} J &= E\{[\frac{\partial lnP(\tilde{V}_k)}{\partial \beta}][\frac{\partial lnP(\tilde{V}_k)}{\partial \beta}]'\} \\ &= \tilde{A}'\Sigma_{\tilde{Z}_k}^{-1}\tilde{A} \\ &= A'S'(S\Sigma_{Z_k}S')^{-1}SA\,. \end{aligned} \quad (22)$$

So we have:

$$CRB(\beta) \geq (A'S'(S\Sigma_{Z_k}S')^{-1}SA)^{-1}\,. \quad (23)$$

Lemma 2: In Gaussian case, the Cramer-Rao lower bound monotonically decreases when the number of samples increases. Proof: Since mathematically the Cramer-Rao lower bound in Gaussian case has the same form as the covariance matrix with our approach, therefore, with similar proof as in Lemma 1, the assertion of lemma 2 follows easily.

5. SIMULATION AND COMPARISON

5.1. Discussion of Accuracy

In this section, we first make comparison of our approach with Newton's method, MANOVA and EKF in terms of estimation accuracy. In order to implement and evaluate the proposed estimation system, we have used trajectories from the the Australian Sign language (ASL) data set obtained from University of California at Irvine's Knowledge Discovery in Databases archive [9]. The trajectories in the data set are obtained by registration of the hand coordinates at each successive instant of time by using a Power Glove interfaced to the system. In our simulations, we used 40 different classes representing signing of 40 different words in the data set. Each class has 69 trajectories recorded at different instances.

In the cases of noisy channels for RSS measurement, we take 60 noisy samples from the different trajectories respectively with the sampling rate 10Hz. The simulation is evaluated over the channel noise and the dynamic motion noise $G_{x,k}, G_{y,k}$ zero mean IID uniform distribution with the variance 0.1, 0.15 respectively. Quantitively, we compute the root mean square error (RMSE) for estimated trajectories and true trajectories on average and the results are shown in Fig.1. It can be seen from Fig 1 that our approach gives the best estimation accuracy among the four approaches. Specifically, the performance of MANOVA model is the worst because our approach gives precise weight to each measurement for non-Gaussian cases and MANOVA model is only valid in Gaussian cases. Meanwhile, the reasons for the poor performance of EKF in this case are two fold: 1) when the noise is not white noise, EKF provides a biased estimator for tracking of mobile targets. 2) EKF uses Taylor approximation for non-linear systems. Therefore, it induces great error. The performance of Newton's method is slightly better than MANOVA and EKF. However, the estimation based on Newton's method (NM)

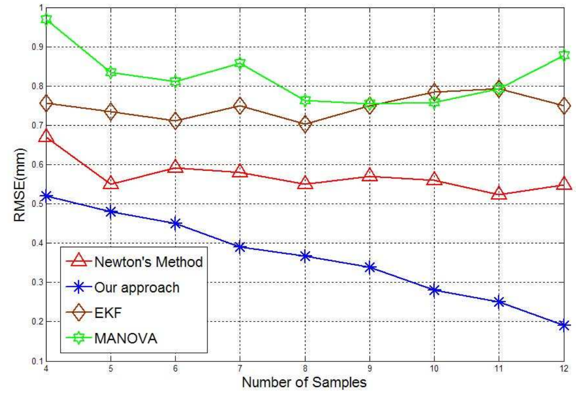

Fig. 1. Comparison of the performance for estimation accuracy with our approach and traditional methods for RSS noisy channel quantitively.

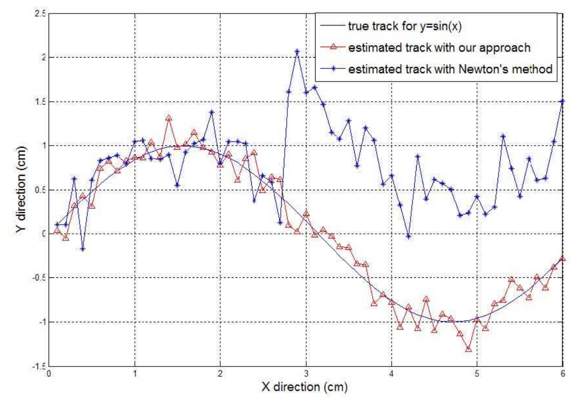

Fig. 2. Comparison of the performance for estimation accuracy with our approach and Newton's method for RSS noisy channel qualitively.

does not converge to the true value due to either the poor initial guess or the non-positive definite Hessian matrix. While our approach works gracefully. Moreover, the estimation error decrease dramatically when the number of samples increase, which we proved in Lemma 1 in Section 3. While the other three approaches does not have this property.

Qualitively, we provide an example for the comparison of Newton's method and our approach. We choose the true trajectory with non-convex function $y = sinx$ and estimate the trajectory with our approach and Newton's method respectively in Fig.2. It can be seen that Newton's method works nicely at the first few samples, however, when the initial value is outside the converge region, Newton's method completely fails. While in this case, our approach outperforms Newton's method due to the closed-form estimator.

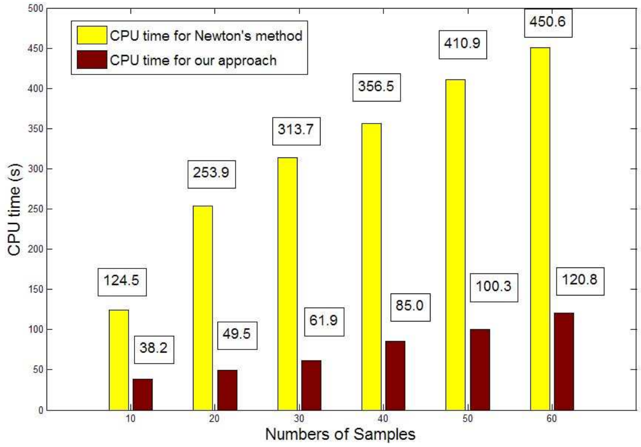

Fig. 3. Comparison of the performance for computation time with our approach and Newton's methods for RSS noisy channel

5.2. Discussion of Computation Time

Since the MANOVA model and EKF already give great estimation error, here we only compare the computation time of our approach and Newton's method using the same database [9]. We choose the same threshold of accuracy for terminating both of two iterative approaches: the residual sums of squares $\Delta = (\beta - \hat{\beta})'(\beta - \hat{\beta}) \leq 0.1$. We compute the CPU time of estimating the 10 to 60 positions in the trajectory respectively. It can be seen from Fig.3 that our approach saves approximately 70 percent of computation time compared to Newton's method due to three reasons: (1) using smaller dimension matrix inversion for each iterations (i.e. 1a) we only inverse 4 by 4 matrices instead of 10 by 10 matrices by computing $(A'S'\Sigma_{Z_k}^{-1}SA)^{-1}$ instead of $(A\Sigma_{Z_k}^{-1}A)^{-1}$, 1b) we only inverse n-3 by n-3 matrices instead of n by n matrices by computing $\Sigma_{\tilde{Z}_k}^{-1}$ instead of $\Sigma_{Z_k}^{-1}$), (2) avoiding computing the gradient and Hessian matrix, (3) reducing times of iterations with linear estimation.

6. CONCLUSION

In this paper, we proposed a novel and robust closed-form framework for localization and trajectory estimation of moving targets using a single sensor with arbitrary noise degradation. By applying the widely-used dynamic motion model for arbitrary moving targets, we come up with a closed-form estimator by obtaining an exact linear model from a system of non-linear equations based on RSS measurements. Subsequently, we iteratively estimate the unknown parameters and the covariance matrix of the noise. Our systems can be applied to both ideal and noisy channels. Moreover, we proved that our estimator gives more accuracy when the number of samples increases. The Cramer-Rao Bound (CRB) are derived for Gaussian cases. Furthermore, we prove that the CRB monotonically decreases as the number of samples increases. Numerical results demonstrate the superiority of our approach over traditional methods in terms of both the accuracy and computation time. Our scheme can also be easily extended to other localization methods such as Angle of Arrival (AOA).

7. REFERENCES

[1] B.Cheng, R.Hudson, F.Lorenzelli, L.Vandenberghe, and K.Yao, "Distributed gauss-newton method for node localization in wireless sensor networks," IEEE 6th Workshop on Signal Processing Advances in Wireless Communication, 2005.

[2] R.Moses, D.Krishnamurthy, and R.Patterson, "A self-localization method for wireless sensor networks," EURASIP Journal on Applied Signal Processing, 2003, vol. 4, pp. 345–348.

[3] J.Leonard and H.Durrant-Whyte, "Mobile robot localization by tracking geometric beacons," IEEE Transactions on Robotics and Automation, 2002, vol. 7, pp. 376–382.

[4] N.Patwari and A.Hero, "Relative location estimation in wireless sensor networks," IEEE Transactions on Signal Processing, Special Issue on Signal Processing in Networking, 2003, pp. 2137–2148.

[5] E.F.Vonesh and V.M.Chinchilli, *Linear And Nonlinear Models For The Analysis Of Repeated Measurements*.

[6] C.Chang and A.Sahai, "Estimation bounds for localization," IEEE SECON 2004, First Annual IEEE Communications Society Conference, Sensor and Ad Hoc Communications and Networks, 2004.

[7] I.D.Schizas, G.B.Giannakis, and Z. Luo, "Distributed estimation using reduced dimensionality sensor observations," IEEE Transactions on Signal Processing, August 2007.

[8] T.Soderstron and P.Stoica, *System Identification*, 1989.

[9] S.Hettich and S.D.Bay, "The uci kdd archive," University of California, Department of Information and Computer Science, Irvine, California, 1999.