When working with connected systems (such as electrical circuits), we often encounter two types of fundamental variables: the through variable and the across variable. Therefore, we say that an element \( X \) has an associated through and across variable as shown, with the direction and polarity as given, respectively.

\[
\begin{align*}
\text{through} & \\
+ \text{ across} & -
\end{align*}
\]

The through variable is also known as the flow variable while the across variable is also known as the effort variable.

For electrical circuits, the flow or through variable is current in Amperes [A], and the effort or across variable is voltage in Volts [V].

\[
\begin{align*}
+ v & - \\
\text{flow} & \text{effort} \\
\text{current} & \text{voltage}
\end{align*}
\]

The choice of having current flow in the direction of voltage drop (from + to -) is known as passive sign convention (PSC).

Ohm's Law

Ohm's Law states the relationship between voltage and current for a resistor and follows PSC:

\[
+ v \rightarrow R \rightarrow - \\
\text{current} \quad \text{resistance} \quad \text{voltage}
\]

\[
V_R = R \cdot i_R
\]

\( V_R \) is in Volts [V] 
\( i_R \) is in Amperes [A] 

\( R \) is in Ohms [Ω]
The amount of power dissipated in a resistor as heat is given by

\[ P_{\text{diss}, R} = V_R i_R = Ri_R^2 = \frac{V_R^2}{R} \]

\( P_{\text{diss}, R} \) is in Watts [W]

For a resistor \( P_{\text{diss}, R} \geq 0 \) (since \( Ri_R^2 \) and \( \frac{V_R^2}{R} \) are always \( \geq 0 \))

**Independent voltage sources**

An independent voltage source has a known (given) across variable; however, its through variable can be anything (is independent of the voltage).

Voltage \( V_S \) can be constant (e.g., a battery) or a function of time (e.g., the wall outlet).

**Independent current sources**

An independent current source has a known through variable; however, its across variable can be anything (is independent of the current).

**Kirchhoff's Current Law (KCL)**

KCL states that the algebraic sum of currents into a node must equal the algebraic sum of currents out of that node. This means that we cannot accumulate charge at a node (with current defined as a rate change/time). KCL implies the fundamental law of conservation of charge.

Compact statement of KCL \( \Rightarrow \sum_{\text{in}} i_{\text{in}} = \sum_{\text{out}} i_{\text{out}} \) ("\( \sum \) means sum)
Kirchhoff’s Voltage Law (KVL)

KVL states that the algebraic sum of voltage rises must equal the algebraic sum of voltage drops around a closed loop, where the direction of the loop may either be clockwise (CW) or counterclockwise (CCW). KVL implies the fundamental law of conservation of energy.

Compact statement: \[ \sum \text{v}_{\text{rises}} = \sum \text{v}_{\text{drops}} \text{ of KVL} \]

Equivalent Resistance Calculations

In general, the equivalent resistance of an electrical network is defined as \[ \text{R}_{\text{eq}} = \frac{\text{V}}{\text{I}} \]
(basically a statement of Ohm’s Law). \( \text{R}_{\text{eq}} \) is simply the constant of proportionality between current \( \text{I} \) and voltage \( \text{V} \). To determine \( \text{R}_{\text{eq}} \), we can either apply a known current \( \text{I} \) and calculate (or measure) the resulting voltage \( \text{V} \) or we can apply \( \text{V} \) and calculate (or measure) \( \text{I} \).

\[ \text{R}_{\text{eq}} = \frac{\text{V}}{\text{I}} \]

\[ \text{V} = \text{R}_{\text{eq}} \cdot \text{I} \]
Resistors in series

\[ R_1 R_2 \rightarrow \sum R_n \Rightarrow i_t \]

\[ v_1 = R_1i_1 = R_1it \]
\[ v_2 = R_2i_2 = R_2it \]
\[ v_n = R_ni_n = R_nit \]

\[ \text{n resistors in series} \]

\[ \text{series circuit } \Rightarrow \text{ all currents same} \]
\[ i_t = i_1 = i_2 = \ldots = i_n \] (KCL)

\[ \text{(KVL)} \Rightarrow \Sigma v_{\text{risers}} = \Sigma v_{\text{drops}} \]
\[ v_t = v_1 + v_2 + \ldots + v_n \]
\[ v_t = R_1it + R_2it + \ldots + R_nit \]
\[ v_t = (R_1 + R_2 + \ldots + R_n)it \]

\[ R_0 = \frac{v_t}{it} = R_1 + R_2 + \ldots + R_n \]

Resistors in parallel

\[ R_0 \Rightarrow \frac{1}{R_1} \parallel \frac{1}{R_2} \parallel \frac{1}{R_n} \]

\[ \text{n resistors in parallel} \]

\[ i_1 = \frac{v_t}{R_1} = \frac{v_t}{R_1} \]
\[ i_2 = \frac{v_t}{R_2} = \frac{v_t}{R_2} \]
\[ \ldots \]
\[ i_n = \frac{v_t}{R_n} = \frac{v_t}{R_n} \]

\[ \text{parallel circuit } \Rightarrow \text{ all voltages same} \]
\[ v_t = v_1 = v_2 = \ldots = v_n \] (KVL)

\[ \text{KCL } \Rightarrow \Sigma i_{\text{in}} = \Sigma i_{\text{out}} \]
\[ i_t = i_1 + i_2 + \ldots + i_n \]
\[ i_t = \frac{v_t}{R_1} + v_t/R_2 + \ldots + v_t/R_n \]
\[ i_t = (\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n})v_t \]

\[ R_0 = \frac{v_t}{i_t} = \frac{1}{(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n})} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \right)^{-1} \]

For \( n=2 \), \[ R_0 = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} = \left( \frac{R_1 + R_2}{R_1R_2} \right)^{-1} = \frac{R_1R_2}{R_1 + R_2} \]
Voltage Division

(Determine resistor voltages as fraction of \( V_g \))

Voltage source \( V_g \) connected to \( n \) resistors in series

Series circuit \( \Rightarrow \) all currents are equal (KCL)

Let's define \( i_1 = i_2 = \ldots = i_n \)

Ohm's Law \( \Rightarrow \)

\[
\begin{align*}
V_1 &= R_1 i_1 = R_1 i_s \\
V_2 &= R_2 i_2 = R_2 i_s \\
&\vdots \\
V_n &= R_n i_n = R_n i_s \\
\end{align*}
\]

KVL \( \Rightarrow \)

\[
\begin{align*}
V_g &= V_1 + V_2 + \ldots + V_n \\
&= R_1 i_s + R_2 i_s + \ldots + R_n i_s \\
&= (R_1 + R_2 + \ldots + R_n) i_s \\
\end{align*}
\]

Resistor voltages current \( \Rightarrow \)

\[
\begin{align*}
V_1 &= R_1 i_s = R_1 \left( \frac{V_g}{R_{eq}} \right) = \left( \frac{R_1}{R_{eq}} \right) V_g = \left( \frac{R_1}{R_1 + R_2 + \ldots + R_n} \right) V_g \\
V_2 &= R_2 i_s = R_2 \left( \frac{V_g}{R_{eq}} \right) = \left( \frac{R_2}{R_{eq}} \right) V_g = \left( \frac{R_2}{R_1 + R_2 + \ldots + R_n} \right) V_g \\
&\vdots \\
V_n &= R_n i_s = R_n \left( \frac{V_g}{R_{eq}} \right) = \left( \frac{R_n}{R_{eq}} \right) V_g = \left( \frac{R_n}{R_1 + R_2 + \ldots + R_n} \right) V_g \\
\end{align*}
\]

For \( n=2 \), \( V_1 = \left( \frac{R_1}{R_1 + R_2} \right) V_g \) and \( V_2 = \left( \frac{R_2}{R_1 + R_2} \right) V_g \)

Current Division

(Determine resistor currents as fraction of \( i_s \))

Current source \( i_s \) connected to \( n \) resistors in parallel

Parallel circuit \( \Rightarrow \) all voltages equal (KVL)

Therefore \( V_1 = V_2 = \ldots = V_n \)

Define \( V_p \triangleq V_1 = V_2 = \ldots = V_n \)

(parallel voltage)
Ohm's Law \[ \begin{align*}
    i_1 &= \frac{V_1}{R_1} = \frac{V_P}{R_1} \\
    i_2 &= \frac{V_2}{R_2} = \frac{V_P}{R_2} \\
    \vdots \\
    i_n &= \frac{V_n}{R_n} = \frac{V_P}{R_n}
\end{align*} \]

KCL \[ \begin{align*}
    i_s &= i_1 + i_2 + \ldots + i_n \\
    i_s &= \frac{V_P}{R_1} + \frac{V_P}{R_2} + \ldots + \frac{V_P}{R_n} \\
    i_s &= V_P \left( \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \right)
\end{align*} \]

Parallel voltage \[ V_P = \frac{i_s}{\left( \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \right)} \]

Resistor currents \[ i_s = R \bar{E} \cdot i_s \]

\[ \begin{align*}
    i_1 &= \frac{V_P}{R_1} = R \bar{E} \frac{i_s}{R_1} = \left( \frac{R \bar{E}}{R_1} \right) i_s = \left( \frac{1}{R_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \right)} \right) i_s \\
    i_2 &= \frac{V_P}{R_2} = R \bar{E} \frac{i_s}{R_2} = \left( \frac{R \bar{E}}{R_2} \right) i_s = \left( \frac{1}{R_2 \left( \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \right)} \right) i_s \\
    \vdots \\
    i_n &= \frac{V_P}{R_n} = R \bar{E} \frac{i_s}{R_n} = \left( \frac{R \bar{E}}{R_n} \right) i_s = \left( \frac{1}{R_n \left( \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \right)} \right) i_s
\end{align*} \]

For \( n=2 \), \[ \begin{align*}
    i_1 &= \left( \frac{1}{R_1 \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \right) i_s = \left( \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right) i_s = \left( \frac{R_2}{R_1 + R_2} \right) i_s \\
    i_2 &= \left( \frac{1}{R_2 \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \right) i_s = \left( \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right) i_s = \left( \frac{R_1}{R_1 + R_2} \right) i_s
\end{align*} \]