Problem 1

From Problem #4.1, we see that the solution requires a Butterworth filter with \( n = 7 \). The specifications also require a 3dB frequency of 7302 rad/s; therefore, if we use standard circuits with 3dB frequencies of 1 rad/s, we need a frequency scaling factor \( k_f = 7302 \).

Here's a summary of the functional blocks required, assuming a 3dB frequency of 1 rad/s.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Pole Location (°)</th>
<th>Coordinate ((-a+jb))</th>
<th>Qk = ( \frac{1}{2\cos\frac{k_f}{2}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0°</td>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>±25.71°</td>
<td>(-0.9011 ± j0.4339)</td>
<td>0.555</td>
</tr>
<tr>
<td>2</td>
<td>±51.43°</td>
<td>(-0.6235 ± j0.7818)</td>
<td>0.802</td>
</tr>
<tr>
<td>3</td>
<td>±77.14°</td>
<td>(-0.2225 ± j0.9749)</td>
<td>2.246</td>
</tr>
</tbody>
</table>

(a) There are 4 functional blocks: 3 2nd order plus a 1st order.

Stage 0 \( \rightarrow H_0(s) = \frac{1}{s+1} \)

Stage 1 \( \rightarrow H_1(s) = \frac{1}{s^2 + 1.8019s + 1} \frac{1}{s^2 + 2\cos\frac{k_f}{2}s + 1} \)

Stage 2 \( \rightarrow H_2(s) = \frac{1}{s^2 + 1.2470s + 1} \)

Stage 3 \( \rightarrow H_3(s) = \frac{1}{s^2 + 0.4950s + 1} \)

Overall \( H(s) = H_0(s)H_1(s)H_2(s)H_3(s) \)
(b) For the 1st order structure, use the following where the transfer function is

\[ \frac{G_m}{SC + G_m} = \frac{G_m/C}{S + G_m/C} = \frac{1}{S + 1} \]

If we require \( C = 1F \) then \( G_m = 1 \Omega \)

would give us the proper transfer function

For the 2nd order structures, use the following where the transfer function is

\[ \frac{G_m}{S^2 + \left( \frac{G_m G_2}{C_1 C_2} \right) S + \left( \frac{G_m G_2}{C_1 C_2} \right) \left( \frac{1}{Q} \right)} \]

The transfer functions to be implemented is of the form

\[ \frac{1}{S^2 + 2 \cos \psi \omega_0 S + 1} = \frac{1}{S^2 + \left( \frac{1}{Q} \right) S + 1} \]

Note that \( \omega_0 = \sqrt{\frac{G_m G_2}{C_1 C_2}} \) and \( Q = \frac{\omega_0 C_2}{G_m} = \sqrt{\frac{G_m C_2}{G_2}} \)

Since we require \( C_1 = 1F \) and \( C_2 = 1F \) \( \Rightarrow \omega_0 = \sqrt{G_m G_2} \) & \( Q = \sqrt{\frac{G_m}{G_2}} \)

To make a structure that has \( \omega_0 = 1 \) rad/s

\( \omega_0 = \sqrt{G_m G_2} = 1 \) or \( G_2 = \frac{1}{G_m} \)

Therefore \( Q = \sqrt{\frac{G_m}{G_2}} = \sqrt{G_m} = G_m \) (take positive value)

\( \Rightarrow G_m = Q \) and \( G_2 = \frac{1}{Q} = \frac{1}{Q} \)

The basic structure is shown next
(c) Frequency scaling the circuits will only require the 1F capacitors to be replaced by 1F/μF capacitors. 1F ⇒ 1F/7302 = 137μF. The transconductances remain unchanged.

(d) To make final capacitors equal 82nF we need magnitude (impedance) scaling factor \( k_m = \frac{137μF}{82nF} = 1670 \). This will result in all caps ⇒ 82nF. The new trans. \( g_m \) values will be \( g_m, new = g_m, old / k_m \). Here are the final values of each stage

Stage 0 ⇒ \( g_m = 0.6mV \), \( C = 82nF \)
Stage 1 ⇒ \( g_m = 0.332mV \), \( g_m = 1.079mV \), \( C_1 = C_2 = 82nF \)
Stage 2 ⇒ \( g_m = 0.480mV \), \( g_m = 0.747mV \), \( C_1 = C_2 = 82nF \)
Stage 3 ⇒ \( g_m = 1.345mV \), \( g_m = 0.266mV \), \( C_1 = C_2 = 82nF \)

(e) Simulation results shown separately.