Problem 1 (5 points)
The following lowpass filter has a 3dB frequency of 1kHz. Scale the circuit such that the 3dB frequency is moved up to 3kHz and the final capacitor values are 680pF.
You must draw the final circuit with the final component values.

⇒ To scale circuit from 1kHz to 3kHz, we use a frequency scaling factor of \( k_f = \frac{3kHz}{1kHz} = 3 \) 

⇒ To make the final capacitor values 680pF, we use a magnitude (impedance) scaling factor of \( k_m = \frac{11nF}{680pF} \approx 16.18 \). The final circuit is shown below.

![Original Circuit Diagram]

![Scaled Circuit Diagram]
Problem 2 (10 points)
Design a Chebyshev filter to meet the specifications shown below. Show all necessary steps including
determination of $n$, pole locations, the $\omega_0$ and $Q$ for each stage and frequency/magnitude scaling. Scale the
filter so that the resistors in the final circuit are all $10k\Omega$.
You must draw the final circuit with the final component values.

$$\eta = \frac{\cosh^{-1} \left( \frac{10^4 - 1}{10^{0.05} - 1} \right)}{\cosh^{-1}(3)} = \frac{6.6172}{1.7627} = 3.7539 \Rightarrow n = 4$$

Butterworth poles ($-\alpha \pm j\beta$)

$$\Psi_1 = \pm 22.5^\circ \Rightarrow -0.9239 \pm j0.3827$$
$$\Psi_2 = \pm 67.5^\circ \Rightarrow -0.3827 \pm j0.9239$$

$$a = \frac{1}{n} \sinh^{-1} \left( \frac{1}{\varepsilon} \right) = 0.5074 \Rightarrow \sinh(a) = 0.5294, \cosh(a) = 1.1315$$

Chebyshev poles ($-\sinh(a) \alpha \pm j\cosh(a) \beta$)

$$-0.4891 \pm j0.4330 \Rightarrow \omega_0 = 0.6532, \ Q_1 = 0.6678$$
$$-0.2026 \pm j1.0454 \Rightarrow \omega_2 = 1.0649, \ Q_2 = 2.6280$$

$$k_f = \frac{k_f \omega_0}{653.2}$$
$$k_m = 10,000$$

Gain reduction of $10^{0.3}$
Problem 2 (continued)

**Note:**

**Unscaled circuit**

\[
2Q_1 = 1.3356 \\
2Q_2 = 5.256 \\
0.7487 \\
k_{f1} = 653.2 \\
k_{f2} = 1064.9 \\
gain reduction of 1.0351
\]

**Frequency Scaled Circuit**

\[
2.0447 \text{mF} \\
1.1462 \text{mF} \\
k_{m1} = 10^4 \\
178.4 \mu\text{F} \\
k_{m2} = 10^4 \\
k_m = 10^4
\]

**Magnitude Scaled (Final circuit)**

\[
10k \text{ ohm} \\
10k \text{ ohm} \\
114.62 \text{nF} \\
10k \text{ ohm} \\
17842 \text{nF} \\
3512 \text{ ohm} \\
10k \text{ ohm}
\]
Problem 3 (10 points)
Design a Chebyshev filter to meet the specifications shown below. Show all necessary steps including determination of \( n \), the \( \omega_0 \) and \( Q \) for each stage and frequency/magnitude scaling. Scale the filter so that the capacitors in the final circuit are all 47nF.

You must draw the final circuit with the final component values.

\[
\eta = \frac{\cosh^{-1}\sqrt{\frac{10^{4.8} - 1}{10^{0.2} - 1}}}{\cosh^{-1}(3)} = 3.15 \Rightarrow \eta = 4
\]

\[
\varepsilon = \sqrt{10^{0.2} - 1} = 0.7648, \quad a = \frac{1}{\eta} \sinh^{-1}\left(\frac{1}{\varepsilon}\right) = 0.2708
\]

\[
\sinh(a) = 0.2741, \quad \cosh(c) = 1.0369
\]

\begin{align*}
\text{Butterworth poles} & : -0.19239 \pm jo.38277 \\
& \quad -0.38277 \pm jo.9239
\end{align*}

\begin{align*}
\text{Chebyshev LP proto poles} & : -0.2532 \pm jo.3968 \\
& \quad -0.1049 \pm jo.9580
\end{align*}

\begin{align*}
\hat{\omega}_0 & : 0.4707 \\
\hat{Q} & : 0.9295
\end{align*}

Chebyshev HP proto poles produce \( \omega_0 = \sqrt{\hat{\omega}_0} \) and \( Q = \hat{Q} \) as a result, we have the following implementation:

\[
\begin{align*}
\omega_0 & = 2.1245 \\
Q & = 0.9295
\end{align*}
\]

\[
\begin{align*}
\omega_0 & = 1.3777 \\
Q & = 4.5935
\end{align*}
\]

\[
\begin{align*}
k_f & = k_f \omega_0 = 6373.5 \\
k_f & = k_f \omega_0 = 3113.1
\end{align*}
\]

Gain reduction of \( 10^{2.5\circ} = 1.2589 \)
Problem 3 (continued)

**Unscaled Circuit**

\[ \frac{1}{2Q_1} = 537.92 \text{ mS} \]
\[ \frac{1}{2Q_2} = 108.85 \text{ mS} \]
\[ k_{f1} = \frac{1}{373.5} \]
\[ k_{f2} = \frac{1}{313.1} \]

**Frequency Scaled Circuit**

\[ \frac{537.92}{156.9 \mu F} = \frac{108.85}{321.22 \mu F} = \frac{0.25895}{1} \]
\[ k_m = \frac{156.9 \mu F}{47 \mu F} = 3.338, 3 \]
\[ k_m = \frac{321.22 \mu F}{47 \mu F} = 6.83 \times 10^4 \]
\[ k_m = 10^4 \]

**Magnitude Scaled (final) Circuit**

\[ \frac{1.7957}{47 \mu F} = \frac{743.9482}{47 \mu F} = \frac{2.589}{10 k \Omega} \]
\[ 6.2059 k \Omega = \frac{\frac{47 \mu F}{47 \mu F}}{6.2789 k \Omega} \]
Problem 4 (10 points)
The standard form of a transfer function $H(s)$ for a bandpass filter is shown below and is characterized by parameters $K$, $Q$ and $\omega_0$. The Friend circuit also shown below (with component values shown) is one circuit to implement a bandpass filter and results in the second expression for $H(s)$.

$$H(s) = \frac{Ks}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2} = \frac{-2Qs}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

$\omega_0 = 1 \text{ rad/s}$

![Circuit Diagram]

Based on the above information, design a bandpass filter with a center frequency of 10kHz where the gain at the center frequency is 12dB and all final capacitors are 820pF.
You must draw the final circuit with the final component values.

Let's start with the circuit above which has a center frequency of $\omega_0 = 1 \text{ rad/s}$. Later in the problem we can use $K_f = 2\pi 10000$ to put the center frequency at its desired location.
The magnitude of the gain is given by

$$|H(j\omega)| = \frac{2Q\omega}{\sqrt{(1-\omega^2)^2 + (\frac{\omega}{Q})^2}}$$

Evaluating the gain at the normalized center frequency of 1 rad/s gives

$$|H(j1)| = \frac{2Q(1)}{\sqrt{(1-1^2)^2 + (\frac{1}{Q})^2}} = \frac{2Q}{\frac{1}{Q}} = 2Q^2$$

Since the gain at the center frequency is specified as 12 dB (which is 10^{12/20} ≈ 4 linear gain),
Problem 4 (continued)

we have \( 2Q^2 = 4 \Rightarrow Q = \sqrt{2} \)

\( \Rightarrow \) using \( Q = \sqrt{2} \), the unscaled circuit for \( \omega_0 = 1 \text{ rad/s} \) is:

\[ \begin{array}{c}
1 \Omega \\
\hline
2Q = 353.55 \text{ mf} \\
\hline
\hline
2Q = 353.55 \text{ mf} \\
\hline
\hline
1 \Omega \\
\hline
\hline
\Rightarrow \text{ since we want the center frequency to be } 10 \text{ kHz, we frequency scale the circuit by } k_f = 2\pi \cdot 10000 \\
\Rightarrow \text{ since the final caps are required to be } 820 \text{ pF, we use a magnitude scaling factor of } k_m = \frac{5.627 \text{ mf}}{820 \text{ pF}} \\
\Rightarrow k_m = \frac{54.898 \text{ k} \Omega}{820 \text{ pF}} \\
\end{array} \]
Problem 5 (10 points)
The standard form of a transfer function $H(s)$ for highpass filter is shown below and is characterized by parameters $K$, $Q$ and $\omega_0$.

$$H(s) = \frac{Ks^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

The OTA circuit also shown below (with component values shown) is one circuit to implement a highpass filter and has the transfer function given by:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{s^2C_1C_2}{s^2C_1C_2 + sC_1g_{m2} + g_{m1}g_{m2}} = \frac{s^2}{s^2 + \left(\frac{9m_2}{C_2}\right)s + \frac{9m_1g_{m2}}{C_1C_2}}$$

$$K = 1, \quad \omega_0 = \sqrt{\frac{9m_1g_{m2}}{C_1C_2}}, \quad \omega_c = \frac{9m_2}{C_2}$$

$$\Rightarrow Q = \frac{C_2}{g_{m2}}, \quad \omega = \frac{C_2}{g_{m2}}\sqrt{\frac{9m_1g_{m2}}{C_1C_2}} = \sqrt{\frac{C_2g_{m1}}{C_1g_{m2}}}$$

Based on the above information, design a 4th order Butterworth highpass filter with a 3dB frequency of 2kHz where all final capacitors are 68nF.

You must draw the final circuit with the final component values.

Easiest approach is the one presented in HW#7. Start with $\omega_0 = 1 \text{ rad/s}$ ($Kf = 2\pi 2000$)

Butterworth poles for $n=4$ are (lowpass prototype)

$$\gamma_1 = \pm 22.5^\circ \Rightarrow -0.9239 + j0.3827, \quad \omega_1 = 1, \quad Q_1 = 0.5412$$
$$\gamma_2 = \pm 67.5^\circ \Rightarrow -0.3827 + j0.9239, \quad \omega_2 = 2, \quad Q_2 = 1.3066$$

Recall the fact that HP poles and LP poles are the same for Butterworth.

For the circuit above, let’s choose $C_1 = C_2 = 1F$ since we eventually want the final capacitors to be equal. Then, $\omega_0 = \sqrt{g_{m1}g_{m2}} = 1 \Rightarrow g_{m2} = \frac{g_{m1}}{\sqrt{2}}$

$$\Rightarrow Q = \sqrt{\frac{g_{m1}}{g_{m2}}} = \sqrt{\frac{g_{m1}}{\frac{g_{m1}}{\sqrt{2}}}} = g_{m1}.$$ The unscaled circuit is shown next.
Problem 5 (continued)  \( Q_1 = Q_2 = 1 \), \( g_{m1} = Q \), \( g_{m2} = \frac{1}{Q} \).

![Circuit Diagram]

\[ \Rightarrow \] Frequency scaling will scale the capacitors from 1F to \( \frac{1}{k_f} = \frac{1}{2\pi f \cdot 2000} = 79.58 \mu F \). The \( g_m \) values remain unchanged.

\[ \Rightarrow \] To make final cap values to equal 68nF we need a magnitude scaling factor of \( k_m = \frac{79.58 \mu F}{68 \mu F} = 1170.3 \).

This will result in final cap values of 68nF, while all the transconductances will be reduced by 1170.3.
Problem 6 (5 points)
Assuming that the OTAs are ideal, determine the transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$.

Using the s-domain version of the circuit, we apply OTA assumptions to determine currents that are summarized above.

**KCL at the OTA #2 output node gives**

$$g_{m1} V_{in} + sC(V_{in} - V_{out}) - g_{m2} V_{out} = 0$$

$$V_{in} (g_{m1} + sC) = V_{out} (g_{m2} + sC)$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{g_{m1} + sC}{g_{m2} + sC} = \frac{g_{m1}}{g_{m2}} \cdot \frac{1 + \frac{s}{g_{m1}/C}}{1 + \frac{s}{g_{m2}/C}}$$

<table>
<thead>
<tr>
<th>Case I</th>
<th>( g_{m1} &lt; g_{m2} )</th>
<th>( (\omega_z &lt; \omega_p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{g_{m1}}{C} )</td>
<td>( \frac{g_{m2}}{C} )</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Case II</th>
<th>( \frac{g_{m2}}{g_{m1}} \cdot \frac{\omega_z}{\omega_p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{g_{m2}}{C} )</td>
<td>( \frac{g_{m1}}{C} )</td>
</tr>
</tbody>
</table>

\( H(s) = H_0 \cdot \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \)