Laplace Transform Pairs

\[ f(t) \leftrightarrow F(s) \]
\[ \delta(t) \leftrightarrow 1 \]
\[ u(t) \leftrightarrow \frac{1}{s} \]
\[ tu(t) \leftrightarrow \frac{1}{s^2} \]
\[ e^{-at} u(t) \leftrightarrow \frac{1}{s + a} \]
\[ te^{-at} u(t) \leftrightarrow \frac{1}{(s + a)^2} \]
\[ \cos(\omega_0 t) u(t) \leftrightarrow \frac{s}{s^2 + \omega_0^2} \]
\[ \sin(\omega_0 t) u(t) \leftrightarrow \frac{\omega_0}{s^2 + \omega_0^2} \]
\[ e^{-at} \cos(\omega_0 t) u(t) \leftrightarrow \frac{s + \alpha}{(s + \alpha)^2 + \omega_0^2} \]
\[ e^{-at} \sin(\omega_0 t) u(t) \leftrightarrow \frac{\omega_0}{(s + \alpha)^2 + \omega_0^2} \]
Problem 1 (15 points)
Consider the RLC circuit shown below driven by a voltage source \( v_{in}(t) = 10e^{-3t}u(t) \) where the initial conditions are \( v_C(0^-) = 5 \text{V} \) an \( i_L(0^-) = 0 \text{A} \). Consider the capacitor voltage \( v_C(t) \) as the response of the circuit and determine the natural response and the forced response.

![RLC Circuit Diagram](image-url)
Problem 1 (continued)
Problem 2 (10 points)
Draw the asymptotic magnitude Bode plot of $H(s)$ labeling all slopes, horizontal and vertical levels.

$$H(s) = 40 \frac{(0.1s + 10)(s + 1000)}{(1 + 200/s)(s + 4000)}$$
Problem 3 (10 points)
Consider a filter characterized by the transfer function $H(s)$ given below.

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{s^2}{(s + 1)(s^2 + 3s + 1)}$$

If the input to the filter is $v_{\text{in}}(t) = 2 \sin(2\pi t + \pi/6)$, determine the steady-state output voltage $v_{\text{out}}(t)$. 
Problem 4 (15 points)
Consider the transfer function \( H(s) \) given below.

\[
H(s) = \frac{s^2(1 + s/100)}{1000(1 + s/10)(1 + s/1000)(1 + s/10000)}
\]

Design a filter to implement transfer function \( H(s) \) using the opamp circuits from Tables 3.4, 4.2 and 4.3. You should try to use at most two opamps but you may use as many resistors and capacitors as you wish. All capacitors in the final circuit must equal 100nF. Using more than two opamps and/or unequal capacitors will result in point deductions.
Problem 4 (continued)