Problem 1
Consider the biquad filter circuit shown below.

(a) Find the transfer function $H_{LP}(s) = V_2(s)/V_1(s)$.

(b) Show that $H_{LP}(s)$ has the standard form of a second-order lowpass filter given by

$$H_{LP}(s) = \frac{K\omega_0^2}{s^2 + \frac{\omega_0}{Q} \omega_0^2}$$

Find expressions for $K$, $\omega_0$ and $Q$ in terms of the circuit variables.

(c) Now let $C_1 = C_2 = 1\, \text{F}$, $R_4 = 1\, \Omega$ and $\omega_0 = 1\, \text{rad/s}$ in the expressions for $K$, $\omega_0$ and $Q$ from part (b) and solve for $R_1$, $R_2$ and $R_3$. These values should be in terms of $K$ and $Q$.

Figure 1: The Tow-Thomas biquad filter. *The last stage has two resistors of equal value – this is not a typo.*

Problem 2
Consider the same biquad filter circuit.

(a) Find the transfer function $H_{BP}(s) = V_3(s)/V_1(s)$.

(b) Show that $H_{BP}(s)$ has the standard form of a second-order bandpass filter given by

$$H_{BP}(s) = \frac{K\omega_0}{s^2 + \frac{\omega_0}{Q} \omega_0^2}$$

Find expressions for $K$, $\omega_0$ and $Q$ in terms of the circuit variables.

(c) Now let $C_1 = C_2 = 1\, \text{F}$, $R_4 = 1\, \Omega$ and $\omega_0 = 1\, \text{rad/s}$ in the expressions for $K$, $\omega_0$ and $Q$ from part (b) and solve for $R_1$, $R_2$ and $R_3$. These values should be in terms of $K$ and $Q$. 
Problem 3
Sketch the asymptotic Bode plots of the following transfer functions. Make sure that you indicate all break frequencies and amplitude levels. You may use Matlab or other software to verify your results.

(a) \( H(s) = 100 \frac{s+100}{s+10} \)

(b) \( H(s) = -10 \frac{s+200}{s+2000} \)

(c) \( H(s) = \frac{s+10}{(s+100)^2} \)

(d) \( H(s) = \frac{s+400}{s(s+4000)} \)

(e) \( H(s) = \frac{100}{s^2+2s+100} \)

(f) \( H(s) = \frac{s+100}{s^2+10s+10000} \)

Problem 4
Derive the transfer function \( H(s) = \frac{V_{out}(s)}{V_{in}(s)} \) for the following circuit and put it in standard form given by the expression below for \( H(s) \). Also write the parameters \( H_0, \omega_0 \) and \( Q \) in terms of \( L, C, R_1 \) and \( R_2 \).

\[
H(s) = H_0 \frac{1}{1 + \left( \frac{s}{\omega_0} \right) \frac{1}{Q} + \left( \frac{s}{\omega_0} \right)^2}
\]