1. Convert the following complex numbers from rectangular form to polar form.

(a) $3 + j4$
Note that this complex number is in the 1st quadrant. The magnitude is $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$. The absolute value of the angle with respect to the $+x$-axis is $\tan^{-1}(4/3) = 53.13^\circ$. Therefore, the polar representation is $5\angle 53.13^\circ$, which is a shorthand for $5e^{j53.13^\circ}$.

(b) $-5 + j10$
Note that this complex number is in the 2nd quadrant. The magnitude is $\sqrt{5^2 + 10^2} = \sqrt{125} = 11.18$. The absolute value of the angle measured from the $-x$-axis is $\tan^{-1}(10/5) = 63.43^\circ$; therefore, the angle as measured from the $+x$-axis is $180^\circ - 63.43^\circ = 116.57^\circ$. As a result, the polar representation is $11.18\angle 116.57^\circ$, which is a shorthand for $11.18e^{j116.57^\circ}$.

(c) $-10 - j10$
Note that this complex number is in the 3rd quadrant. The magnitude is $\sqrt{10^2 + 10^2} = \sqrt{200} = 14.14$. The absolute value of the angle measured from the $-x$-axis is $\tan^{-1}(10/10) = 45^\circ$; therefore, the angle as measured from the $+x$-axis is $180^\circ - 45^\circ = 135^\circ$. As a result, the polar representation is $14.14\angle 135^\circ$, which is a shorthand for $14.14e^{j\pi/2}$.

(d) $3 - j4$
Note that this complex number is in the 4th quadrant. The magnitude is $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$. The absolute value of the angle as measured from the $+x$-axis is $\tan^{-1}(4/3) = 53.13^\circ$. As a result, the polar representation is $5\angle -53.13^\circ$, which is a shorthand for $5e^{-j53.13^\circ}$.

2. Simplify the following expressions, then convert the result to polar form.

(a) $(-3 + j4)(-10 - j10)$
Expanding the two factors (using the FOIL method) we get, $30 + j30 - j40 - j^240 = 30 + j30 - j40 - (-1)40 = 70 - j10$. Note that this complex number is in the 4th quadrant. The magnitude is $\sqrt{70^2 + 10^2} = \sqrt{5000} = 70.71$. The absolute value of the angle measured from the $-x$-axis is $\tan^{-1}(10/70) = 8.13^\circ$; therefore, the angle as measured from the $+x$-axis is $-8.13^\circ$. As a result, the polar representation is $70.71\angle -8.13^\circ$.

Another method is to convert each of the factors in the product to polar form; therefore, we get $(-3 + j4)(-10 - j10) = (5e^{j126.87^\circ})(14.14e^{j225^\circ}) = 70.7e^{j351.87^\circ} = 70.7e^{-j8.13^\circ}$.

(b) $10 + j10$
$-5 - j5$
We can simplify this quotient by multiplying the top and the bottom by the complex conjugate of the denominator ($-5 + j5$). We can then proceed by applying the FOIL method to the numerator and denominator as shown below.

$$\frac{10 + j10}{-5 - j5} \cdot \frac{-5 + j5}{-5 + j5} = \frac{-50 + j50 - j50 + j^250}{-25 - j25 + j25 - j^225} = \frac{-100}{50} = -2 + j0 = 2\angle 180^\circ = 2\angle -180^\circ$$

Another method is to convert each of the factors in the quotient to polar form first. We get $(10 + j10)/(-5 - j5) = (14.14e^{j45^\circ})/(7.07e^{j225^\circ}) = (14.14/7.07)e^{j45^\circ} e^{-j225^\circ} = 2e^{-j180^\circ}$.

3. Convert the following complex numbers from polar form to rectangular form.

(a) $10e^{j\pi/2}$
$10e^{j\pi/2} = 10\cos(\frac{\pi}{2}) + j10\sin(\frac{\pi}{2}) = 10(\sqrt{2}/2) + j10(\sqrt{2}/2) = 5\sqrt{2} + j5\sqrt{2}$.

(b) $-2e^{j90^\circ}$
$-2e^{j90^\circ} = -2\cos(90^\circ) - j2\sin(90^\circ) = -2(1/2) - j2(\sqrt{3}/2) = -1 - j\sqrt{3}$.

(c) $-5e^{-j\pi/3}$
$-5e^{-j\pi/3} = -5\cos(-\pi/3) - j5\sin(-\pi/3)$. Recall that $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$. Therefore, we have $-5\cos(\pi/3) + j5\sin(\pi/3) = -5/2 + j5\sqrt{3}/2$.

(d) $10e^{j225^\circ}$
$10e^{j225^\circ} = 10\cos(225^\circ) + j10\sin(225^\circ)$. Therefore, $-10\cos(225^\circ) - j10\sin(45^\circ) = -10(\sqrt{2}/2) - j10(\sqrt{2}/2) = -5\sqrt{2} - j5\sqrt{2}$.
4. Simplify the following expressions, then convert the result to rectangular form.

(a) \((20 e^{j\pi/6})(-4 e^{j60^\circ})\)
\((20 e^{j\pi/6})(-4 e^{j60^\circ}) = -80 e^{j30^\circ} e^{j60^\circ} = -80 e^{j90^\circ} = -j80.\)

(b) \((-2 e^{-j\pi/2})(4 e^{j225^\circ})\)
\((-2 e^{-j\pi/2})/(4 e^{j225^\circ}) = (-2/4) e^{-j270^\circ}/(e^{j225^\circ}) = (-1/2)(-e^{-j45^\circ}) = (1/2)(e^{j45^\circ}).\)
Our result can now be written as \(-j(1/2) [-\cos(45^\circ) - j\sin(45^\circ)] = (1/2)(\sqrt{2}/2) + j(1/2)(\sqrt{2}/2) = 1/(2\sqrt{2}) + j1/(2\sqrt{2}).\)

5. Find the total complex impedance (in Ω) for the following circuit elements or combinations at an angular frequency \(\omega = 2000\) rad/s.

(a) 100Ω resistor
\(Z = R = 100\) Ω

(b) 10mH inductor
\(Z = j\omega L = j(2000)(10 \times 10^{-3}) = j20\) Ω

(c) 100µF capacitor
\(Z = 1/(j\omega C) = 1/(j(2000)(100 \times 10^{-6})) = 1/(j0.2) = -j5\) Ω

(d) 200Ω resistor in series with a 5mH inductor
\(Z = R + j\omega L = 200 + j(2000)(5 \times 10^{-3}) = 200 + j10\) Ω

(e) 500Ω resistor in series with a 50µF capacitor
\(Z = R + 1/(j\omega C) = 500 + 1/(j(2000)(50 \times 10^{-6})) = 500 + 1/(j0.1) = 500 - j10\) Ω

(f) 1kΩ resistor in series with a 2mH inductor in series with a 20µF capacitor
\(Z = R + j\omega L + 1/(j\omega C) = 1000 + j(2000)(2 \times 10^{-3}) + 1/(j(2000)(20 \times 10^{-6})) = 1000 + j4 + 1/(j0.04) = 1000 + j4 - j25 = 1000 - j21\) Ω

(g) 200Ω resistor in parallel with a 5mH inductor
\(Z = R||j\omega L = \frac{j\omega LR}{R + j\omega L} = \frac{j(2000)(5 \times 10^{-3})(200)}{200 + j(2000)(5 \times 10^{-3})} = \frac{j20000}{200 + j10}\)

(h) 10mH inductor in parallel with a 100µF capacitor
\(Z = (j\omega L)||1/(j\omega C) = \frac{(j\omega L)[1/(j\omega C)]}{(j\omega L) + [1/(j\omega C)]} = \frac{j\omega L}{1 - \omega^2 LC} = \frac{j(2000)(10 \times 10^{-3})}{1 - (2000)^2(10 \times 10^{-3})(100 \times 10^{-6})}
\) Simplifying, we get \(-j20/3\).
6. Given the following circuit with complex impedances, find the complex current $\bar{I}$ and the complex voltage $\bar{V}$.

Since the $3\Omega$ resistor and the $j4\Omega$ are in series, the total impedance is $(3 + j4)\Omega$. The current $\bar{I}$ is simply the voltage source divided by the impedance.

$$\bar{I} = \frac{50\angle 60^\circ}{3 + j4} = \frac{50\angle 60^\circ}{5\angle 53.1^\circ} = 10\angle 6.9^\circ$$

Note that the same current $\bar{I}$ flows through the resistor and the inductor (since they are in series). Therefore, the voltage across the inductor (marked as $\bar{V}$) is given by Ohm’s Law.

$$\bar{V} = Z_{\text{inductor}}\bar{I} = (j4)(10\angle 6.9^\circ) = (4\angle 90^\circ)(10\angle 6.9^\circ) = 40\angle 96.9^\circ$$

7. Find the complex current $\bar{I}$ using current division.

First denote the current source as $\bar{I}_s$ and the resistor and the capacitor as impedances $Z_1 = 10\Omega$ and $Z_2 = -j20\Omega$, respectively. Based on this, we can apply current division as we had done for resistors.

$$\bar{I} = \frac{Z_1}{Z_1 + Z_2}20\angle 30^\circ = \frac{10}{10 - j20}20\angle 30^\circ = \frac{(10\angle 0^\circ)(20\angle 30^\circ)}{22.36\angle -63.4^\circ} = 8.94\angle 93.4^\circ$$

Note that $93.4^\circ$ is in the 2nd quadrant; therefore, the cosine of this angle will be negative while the sine will be positive. The base angle that we can use for this calculation is $60^\circ$. As a result, we get

$$\bar{I} = 8.94\angle 93.4^\circ = 8.94\cos(93.4^\circ) + j8.94\sin(93.4^\circ) = -0.53 + j8.92$$
8. Find the complex current $\bar{I}$ and complex voltage $\bar{V}$, then find the corresponding time-domain current $i(t)$ and voltage $v(t)$. *Hint:* Start by converting the circuit shown below to its phasor equivalent to determine $\bar{I}$ and $\bar{V}$.

First draw the phasor equivalent circuit corresponding the problem using an angular frequency of $\omega = 200\text{rad/s}$. In the equivalent circuit, the voltage source becomes $\bar{V}_s = 100\angle45^\circ$. The impedances for the resistor, inductor and capacitor are $50$, $j50$ and $-j25$ (all in $\Omega$), respectively. The complete equivalent circuit is shown below, where the current and the voltage are denoted by their complex counterparts, $\bar{I}$ and $\bar{V}$, respectively.

\[
\begin{align*}
\bar{V}_s &= 100\angle45^\circ \\
\bar{I} &= \frac{\bar{V}_s}{Z_{\text{total}}} = \frac{100\angle45^\circ}{50 + j25} = 100\angle45^\circ / 55.9\angle26.6^\circ = 1.79\angle18.4^\circ
\end{align*}
\]

Since $\bar{I}$ represents the magnitude and phase of the current time function $i(t)$, we can write it as

\[i(t) = 1.79\angle18.4^\circ \cos(200t + 18.4^\circ)\text{ A}\]

Getting back to the phasor circuit, we see that voltage $\bar{V}$ across the capacitor can be given by Ohm’s Law as $\bar{V} = Z_c\bar{I}$, where $Z_c = -j25$ is the impedance of the capacitor.

\[\bar{V} = (-j25)\bar{I} = (-j25)(1.79\angle18.4^\circ) = (25\angle-90^\circ)(1.79\angle18.4^\circ) = 44.75\angle-71.6^\circ\]

Since $\bar{V}$ represents the magnitude and phase of the voltage time function $v(t)$, we can write it as

\[v(t) = 44.75\angle-71.6^\circ \Rightarrow v(t) = 44.75\cos(200t - 71.6^\circ)\text{ V}\]