Problem 1: Node voltages are circled.

(a) \[ V_+ = 0 \text{ (ground)} \]
\[ V_- = V_+ = 0 \text{ V (opamp assumption)} \]
\[ I_- = 0 \text{ (opamp assumption)} \]

KCL at (-) node \[ R_1 \rightarrow I_{R1} + I_{R2} = I_1 \]
\[ V_{R1} + \frac{V_{R2}}{R_2} = 0 \]
\[ V_{R1} = V_{in} - 0 \quad \& \quad V_{R2} = V_{out} - 0 \]

\[ \frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0 \implies \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \]

(b) \[ I_+ = 0, \text{ therefore the voltage across } R_3 \text{ is } I_+ R_3 = 0 \]
Since the bottom of \( R_3 \) is at 0V (ground) and there is no voltage difference across it, its top is also at 0V. \( V_+ = 0 \)
but \( V_- = V_+ \implies V_- = 0 \)

Now, we have the same situation as in (a):

KCL at (-) \[ \frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0 \implies \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \]

(c) Since \( I_+ = 0 \), \( R_3 I_+ = 0 \), therefore, there is no voltage across \( R_3 \).
As a result, \( V_+ = V_{in} \) and since \( V_- = V_+ \), \( V_- = V_{in} \) as well.

KCL at (-) \[ I_{R2} = I_{R1} \implies \frac{V_{R2}}{R_2} = \frac{V_{R1}}{R_1} \]
\[ V_{R2} = V_{out} - V_{in} \& \ V_{R1} = V_{in} \]
\[
\frac{V_{\text{out}} - V_{\text{in}}}{R_2} = \frac{V_{\text{in}}}{R_1} \Rightarrow \frac{V_{\text{out}}}{R_2} = \frac{V_{\text{in}}}{R_1} + \frac{V_{\text{in}}}{R_2} \Rightarrow V_{\text{out}} = R_2 \left( \frac{1}{R_1} + \frac{1}{R_2} \right) V_{\text{in}}
\]

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_2}{R_1}
\]

**Non-inverting amplifier**

Since \( I_+ = 0 \), \( I_+ R_3 = 0 \); therefore, \( V_+ = V_{\text{in}} \). Since \( V_+ = V_- \), \( V_- = V_{\text{in}} \).

Note that (-) is connected directly to the output.

\[ V_{\text{out}} = V_{\text{in}} \quad \text{and} \quad \frac{V_{\text{out}}}{V_{\text{in}}} = 1 \quad \text{buffer} \]

**Problem 2**

\[ V_+ = 0 \text{V (ground)} \quad V_- = V_+ = 0 \text{V} \quad I_- = 0 \text{A} \]

\[
\text{KCL at } (-) \Rightarrow \frac{V_{\text{in}1}}{R_1} + \frac{V_{\text{in}2}}{R_2} + \frac{V_{\text{out}}}{R_3} = 0
\]

\[
\frac{V_{\text{out}}}{R_3} = - \frac{V_{\text{in}1}}{R_1} - \frac{V_{\text{in}2}}{R_2} = - \left( \frac{V_{\text{in}1}}{R_1} + \frac{V_{\text{in}2}}{R_2} \right)
\]

\[ V_{\text{out}} = - \left[ \frac{R_3}{R_1} V_{\text{in}1} + \frac{R_3}{R_2} V_{\text{in}2} \right] \quad \text{Weighted, inverting summer (adder)} \]
\( V_{out} = \frac{V_{11} - \frac{R_4}{R_3} V_{in2}}{1 + \frac{R_4}{R_3}} \)
Problem 4

These solutions are not unique and the value of \( R \) can be any number \( > 0 \).

(a) \( V_{\text{out}} = -3V_{\text{in}} \) choose inverting amp with \( V_{\text{out}} = \frac{-R_2}{R_1}V_{\text{in}} \)

\[ R_1 = R \\ R_2 = 3R \]

(b) \( V_{\text{out}} = 4V_{\text{in}} \) choose non-inverting amp, with \( V_{\text{out}} = \left(1 + \frac{R_2}{R_1}\right)V_{\text{in}} \)

\[ R_1 = R \\ R_2 = 3R \]

(c) \( V_{\text{out}} = \frac{1}{2}V_{\text{in}} \) choose two inverting amps,

\[ V_{\text{in}} \]
\[ 2R \]
\[ R \]
\[ R \]
\[ \text{gain} = -\frac{1}{2} \]
\[ \text{gain} = -1 \]

(d) \( V_{\text{out}} = -5V_{\text{in1}} - 2V_{\text{in2}} \) choose inverting adder (2a)

\[ R_1 = 2R \]
\[ R_2 = 5R \]
\[ R_3 = 10R \]

\[ V_{\text{out}} = -\frac{R_3}{R_1}V_{\text{in1}} - \frac{R_3}{R_2}V_{\text{in2}} \]

if \( R_3 = 10R \), then

\[ R_1 = 2R \quad \text{and} \quad R_2 = 5R \]
(e) \( V_{out} = V_{in1} - V_{in2} \); easiest way is to choose the difference amplifier (2b) with \( R_1 = R_2 = R_3 = R_4 = R \).

Then, \( V_{out} = \left(1 + \frac{R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_{in1} - \left(\frac{R_4}{R_3}\right) V_{in2} = V_{in1} - V_{in2} \)

(f) \( V_{out} = 3V_{in1} - 4V_{in2} \), you can use the difference amplifier (2b) for this expression as well,

\[
V_{out} = \left(1 + \frac{R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_{in1} - \left(\frac{R_4}{R_3}\right) V_{in2}
\]

First choose \( \frac{R_4}{R_3} = 4 \) (for example, \( R_4 = 4R \) & \( R_3 = R \)).

This will result in the \( 1 + \frac{R_4}{R_3} \) factor equaling 5.

To make the coefficient of \( V_{in1} = 3 \), we need to make \( \frac{R_2}{R_1 + R_2} = \frac{3}{5} \).

\[ \Rightarrow \frac{R_2}{R_1 + R_2} = \frac{3}{5} \quad \text{Choose} \quad R_2 = 3R \quad \text{&} \quad R_1 = 2R \quad \text{for example} \]