Problem 1 (8 points total – 2 points per equation, 2 points for standard form)

For the circuit shown below, determine the nodal analysis equations in terms of the unknown node voltages $V_1$, $V_2$ and $V_3$ and the known current source $I_s$. Put the equations in standard form.

DO NOT SOLVE.

Write KCL equations at $V_1$, $V_2$ and $V_3$ ($E_{\text{out}} = 0$, for example)

\[
\begin{align*}
\frac{V_1 - 0}{R_1} + \frac{V_1 - V_2}{R_5} + \frac{V_1 - V_3}{R_2} &= 0 \\
\frac{V_2 - V_1}{R_5} + \frac{V_2 - 0}{R_3} + \frac{V_2 - V_3}{R_4} &= 0 \\
-I_s + \frac{V_3 - V_1}{R_2} + \frac{V_3 - V_2}{R_4} &= 0
\end{align*}
\]

Standard form

\[
\begin{align*}
\begin{bmatrix}
\frac{1}{R_1} + \frac{1}{R_5} + \frac{1}{R_2}
\end{bmatrix} V_1 + \begin{bmatrix}
-\frac{1}{R_5}
\end{bmatrix} V_2 + \begin{bmatrix}
-\frac{1}{R_2}
\end{bmatrix} V_3 &= 0 \\
\begin{bmatrix}
-\frac{1}{R_5}
\end{bmatrix} V_1 + \begin{bmatrix}
\frac{1}{R_5} + \frac{1}{R_3} + \frac{1}{R_4}
\end{bmatrix} V_2 + \begin{bmatrix}
-\frac{1}{R_4}
\end{bmatrix} V_3 &= 0 \\
\begin{bmatrix}
-\frac{1}{R_2}
\end{bmatrix} V_1 + \begin{bmatrix}
-\frac{1}{R_4}
\end{bmatrix} V_2 + \begin{bmatrix}
\frac{1}{R_2} + \frac{1}{R_4}
\end{bmatrix} V_3 &= I_s
\end{align*}
\]
Problem 2 (8 points total – 2 points per equation, 2 points for standard form)
For the circuit shown below, determine the loop (mesh) analysis equations in terms of the unknown loop currents \( I_1, I_2 \) and \( I_3 \) and the known voltage source \( V_s \). Put the equations in standard form.

**DO NOT SOLVE.**

\[
\begin{align*}
R_4 (I_1 - I_3) + R_2 (I_1 - I_2) + R_3 I_1 &= 0 \\
R_1 I_2 + R_2 (I_2 - I_1) + R_5 (I_2 - I_3) &= 0 \\
-V_s + R_5 (I_3 - I_2) + R_4 (I_3 - I_1) &= 0
\end{align*}
\]

**Standard Form**

\[
\begin{align*}
(R_4 + R_2 + R_3) I_1 + (-R_2) I_2 + (-R_4) I_3 &= 0 \\
(-R_2) I_1 + (R_1 + R_2 + R_5) I_2 + (-R_5) I_3 &= 0 \\
(-R_4) I_1 + (-R_5) I_2 + (R_5 + R_4) I_3 &= V_s
\end{align*}
\]
Problem 3 (8 points, 1 point each)
Consider the circuit shown below where the resistances and the current source value are given as shown. Assume that the node voltages have been calculated and are boxed.

\[
\begin{align*}
V_1 &= 12 - 8 = 4 \text{ V} \\
V_2 &= 8 - 4 = 4 \text{ V} \\
V_3 &= 12 - 4 = 8 \text{ V} \\
V_4 &= 8 - 0 = 8 \text{ V} \\
V_5 &= 4 - 0 = 4 \text{ V} \\
\end{align*}
\]

(a) Determine \(V_1\).
(b) Determine \(V_2\).
(c) Determine \(V_3\).
(d) Determine \(V_4\).
(e) Determine \(V_5\).

(f) Determine the power dissipated in the 2Ω resistor (marked with voltage \(V_2\)).
\[
P_{2\Omega} = V_2^2 / 2\Omega = 16 / 2 = 8 \text{ W}
\]
(g) Determine the power dissipated in the 4Ω resistor (marked with voltage \(V_3\)).
\[
P_{4\Omega} = V_3^2 / 4\Omega = (8)^2 / 4 = 16 \text{ W}
\]
(h) Determine the power delivered to the circuit by the current source.
\[
P_{del} = (12 \text{ V}) (2 \text{ A}) = 24 \text{ W}
\]

Currents through resistors can be computed by using Ohm’s Law once voltages \(V_1 - V_5\) have been determined by KVL in parts (a) – (e).
Problem 4 (8 points)
Consider the digital circuit shown below.

(a) (4 points) Determine the truth table for the circuit with inputs $x$ and $y$ and output $z$.

(b) (2 points) Write a logic expression for $z$ as a function of $x$ and $y$.

(c) (2 points) Given the waveforms for $x$ and $y$, determine the waveform for $z$.

\[
\begin{array}{ccc}
 x & [0011] & [1100] \\
 y & [0101] & [0100] \\
 & [0111] & [0010] \\
 & [0101] & [1010] \\
\end{array}
\]

(a) 
\[
\begin{array}{ccc}
 x & y & z \\
 0 & 0 & 0 \\
 0 & 1 & 1 \\
 1 & 0 & 1 \\
 1 & 1 & 0 \\
\end{array}
\]

(b) using the sum-of-products form
\[
z = x'y + xy'
\]

(c) 
Use truth table to determine $z$ for each $x, y$ combination
\[
z = 1 \text{ if } x, y \text{ are different and } 0 \text{ if } x, y \text{ are the same}
\]
Problem 5 (8 points)
In the circuit below, the current through the 2Ω resistor is 2A. Determine the voltage $V_s$.

*Hint: Remember Kirchhoff and Ohm.*

\[ I_4 = 4Ω \quad I_4 = 2A \quad I_3 = 3Ω \]

\[ V_1 = V_2 - 48V \quad V_3 - 6V \]

\[ V_4 = 10Ω \quad V_1 = 10V \]

\[ V_2 = 4V \]

Here are the steps working right to left.

1. **Ohm's Law** $\Rightarrow V_2 = (2A)(2Ω) = 4V$
2. **KCL** $\Rightarrow I_3 = 2A$
3. **Ohm's Law** $\Rightarrow V_3 = (2A)(3Ω) = 6V$
4. **KVL** $\Rightarrow V_1 = V_3 + V_2 = 6V + 4V = 10V$
5. **Ohm's Law** $\Rightarrow I_1 = \frac{V_1}{1Ω} = \frac{10V}{1Ω} = 10A$
6. **KCL** $\Rightarrow I_4 = I_1 + I_3 = 10A + 2A = 12A$
7. **Ohm's Law** $\Rightarrow V_4 = (12A)(4Ω) = 48V$
8. **KVL** $\Rightarrow V_5 = V_4 + V_1 = 48V + 10V = 58V$
Extra Workspace