Two-Port Networks

Gyrator (with gyration resistance $r$): $V_1 = -rI_2$, $V_2 = rI_1$
Gyrator (with gyration conductance $g$): $I_1 = gV_2$, $I_2 = -gV_1$
Transformer ($n:1$, step down): $V_2 = V_1/n$, $I_2 = -nI_1$
Transformer ($1:n$, step up): $V_2 = nV_1$, $I_2 = -I_1/n$
OTA: $I_{out} = g_m(V_+ - V_-)$, $I_+ = 0, I_- = 0, V_+ \neq V_-$

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \quad Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

$I_1 = y_{11}V_1 + y_{12}V_2$
$I_2 = y_{21}V_1 + y_{22}V_2$

$V_1 = z_{11}I_1 + z_{12}I_2$
$V_2 = z_{21}I_1 + z_{22}I_2$

$Z = Y^{-1}$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \quad G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$V_1 = h_{11}I_1 + h_{12}V_2$
$I_2 = h_{21}I_1 + h_{22}V_2$

$I_1 = g_{11}V_1 + g_{12}I_2$
$V_2 = g_{21}V_1 + g_{22}I_2$

$G = H^{-1}$

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$V_1 = AV_2 - BI_2$
$I_1 = CV_2 - DI_2$

$T_{series} = \begin{bmatrix} 1 & Z_{series} \\ 0 & 1 \end{bmatrix} \quad T_{parallel} = \begin{bmatrix} 1 & 0 \\ Y_{parallel} & 1 \end{bmatrix}$
Problem 1 (10 points)

(a) Determine the transmission \((T)\) matrix model for the ideal 1 : \(n\) step-up transformer shown below.

(b) Determine the transmission \((T)\) matrix model for the ideal \(n : 1\) step-down transformer shown below.
(c) Determine the transmission (T) matrix model for the cascade connection of an ideal $n_1 : 1$ step-down transformer and an ideal $1 : n_2$ step-up transformer shown below.
Problem 2 (10 points)
Find $V_{out}/V_{in}$ and $Z_{out}$ for the following network.
Problem 2 (continued)
Problem 3 (10 points)
(a) Determine the transmission ($T$) matrix for the ideal gyrator (with gyration resistance $r$) shown below.

(b) Determine the transmission ($T$) matrix model for the cascade connection of an ideal gyrators (with gyration resistances $r_1$ and $r_2$) shown below.
(c) Show that two gyrators with a capacitor $C$ between them as shown below can be used to emulate a series (floating inductor) if gyration resistances are chosen to be equal ($r_1 = r_2 = r$). What is the value of the emulated inductance?