An Integrated Modeling Framework for Exploring Network Reconfiguration of Distributed Controlled Homogenous Power Inverter Network using Composite Lyapunov Function Based Reachability Bound

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ABSTRACT:

We describe an integrated modeling framework for an interactive power network (IPN) consisting of a power network (PN) and a wireless communication network (WCN). The PN is modeled using a set of piecewise-linear (PWL) equations. The WCN is modeled using a Markov-chain based model that can capture the randomness of the communication channel. The impacts of the WCN are incorporated into the PN models using variable time delays. By formulating a convex optimization problem based on a composite Lyapunov function and solving this problem using linear-matrix-inequality solvers, we predict the reaching criteria for orbital existence. We investigate the impacts of time delays due to the wireless network and communication-channel disruptions on the reachability bound, mean-square stability, and performance of the IPN. Subsequently, using the integrated modeling framework, we demonstrate the efficacy of a scheme to jointly optimize control performance and network resource utilization. We demonstrate how communication fault-tolerant protocols can be implemented to ensure that the IPNs operate within their reachability and performance bounds, despite one or more disruptions in the communication channels. We further demonstrate that when the WCN is clustered due to communication disruptions, each cluster can optimize its control-communication network.

KEYWORDS:

Distributed control systems, piecewise linear systems, Markov-chain model, stability, joint optimization, switching power converters, electric power network, communication network, wireless, reaching conditions, Lyapunov stability, linear matrix inequality, optimization

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I. INTRODUCTION

Interactive power networks (IPNs), consisting of interconnected power-electronic converters have gained significant importance in the last several years due to their high reliability, better power quality, and superior dynamic performance. However, for effective control of such IPNs, control information has to be exchanged among the various modules [1], [2]. Recently, the authors have demonstrated the effectiveness of a wireless communication based network control system (NCS) for IPNs [3], [4]-[6], as shown in Fig. 1. Because the wireless NCS does not require physical connection among the modules, such a scheme can be used to implement redundant/reconfigurable control networks, like the droop method, while ensuring that the voltage regulation and load-sharing performance of the IPN is not compromised [7], [8].

However, because wireless communication networks (WCNs) are implemented over free-space, such NCS are susceptible to communication-channel disruptions due to physical changes in the channel, noise from other sources or due to rogue nodes. Such disruptions increase the delay (in information exchange among the modules) and could have a degrading effect on the stability and performance of the IPN [5], [9]. Depending on the status of the communication channels, the NCS can operate in three possible network states (as shown in Fig. 2(a)), namely (a) nominal network (NN), (b) rerouted network (RN), or (c) clustered network (CN). The timing diagram for the NN and RN is shown in Fig. 2(b). The transition from NN to RN occurs whenever a direct wireless link among one or more nodes is disrupted. If the time-delay bound is violated, there is a transition from RN to CN leading to clustered networks operating independently.

Due to multiple operating states of the NCS, we outline an integrated modeling framework for a homogenous inverter PN and a WCN. The PN is modeled using a set of piecewise-linear (PWL) equations. The WCN is modeled using a Markov-chain based model, which captures the randomness of the communication channel and is represented as variable time delay once incorporated into the PN models. Using the overall time-delayed PWL model of the IPN and by formulating a convex optimization problem based on a composite Lyapunov function and solving this problem using linear-matrix-inequality solvers, we outline a reaching criterion for orbital existence. We determine the effects of time delays due to the WCN and disruptions in the communication channel on the reachability bound, mean-square stability, and performance of the IPN.

While stability assures safe operation of the IPN, it does not guarantee performance. Optimization of physical systems (as well as some recent applications to power systems [3], [23]-[24]) using distributed control and optimal resource utilization of the WCN have been proposed separately in [10] and [11], respectively. A key observation from recent research is that the control and the communication networks do not always operate cooperatively if the communication protocol does not yield channel interference. For instance, in [3], [24], the authors demonstrated that to increase the stability margin of the power networks and attain high control performance, fast information exchange is desired. However, from the point of view of the communication network, progressively higher data rate cannot be sustained due to network resource limitations [22]. This problem is further aggravated as the number of nodes of the overall network increases because a progressively higher volume of information flow typically cannot be sustained at the same data rate without enhancing the probabilities of failed (end-to-end) transmissions.

To address this issue, recently, concepts of joint optimization of communication and control have been proposed in [12]-[14]. The control scheme is expected to yield an optimal compromise between the performance of the control system and the resource utilization of the WCN under constraints of power network stability and communication network capacity bounds. In this paper, using the models of the IPN described in Section II, we develop a joint optimization methodology in Section III, which ensures optimal performance of the inverter PN and optimal resource utilization of the WCN. We observe that, the joint optimization problem reflects a “non-cooperative” game since both control performance and communication network performance are dependent on the variable of time (for end-to-end information flow). Therefore, we outline an iterative methodology to yield optimal solution.

In Section V, detailed simulation results investigating the efficacy of the joint optimization framework for a homogeneous inverter network are presented. For the power network, the performance criteria include load-sharing among the inverter modules and the total-harmonic distortion (THD). While the load-sharing error is an indication of the distribution of currents among the inverter modules, the THD of a signal represents the harmonic distortion present in a signal and is defined as the ratio of the sum of all harmonic components to that of the fundamental frequency component. On the other hand, the WCN performance criterion is to maximize the network resource...
utilization, i.e. to operate as close as possible to the network capacity without violating the performance and stability requirements of the power network. The performance of the inverter control-communication network using the optimization and stability methodologies is found robust as well satisfactory even in the presence of channel disruptions.

II. DESIGN RATIONALE AND MODELING METHODOLOGY

Before providing the detailed models for WCN and PN we first discuss the operating conditions leading to the network states provided in Fig. 2. The operating state corresponding to the NN refers to the case when direct links exist among all the modules. This is illustrated in Fig. 3(a), where information is exchanged between nodes 1 and 4. Because we use a sequentially rotating master-slave control scheme, there is a constant time delay among the various states due to the information exchange among modules.

If the direct communication link is disrupted as shown in Fig. 3(b1), we attempt to route this information through other nodes of the NCS and corresponds to RN state of network operation. Fig. 3(b2) shows some possible single-hop routes to ensure that information exchange between nodes 1 and 4 is sustained. Rerouting information through other nodes increases the time delays among the states of the NCS. For the IPN under consideration, we use a protocol based on set-covering scheme [15] that aims to find a minimum set, such that its neighbors cover all other nodes of the network and the independent-transmission-set (IT-set) scheme [16] that aims to avoid collisions and reduce broadcasting latency. All nodes have knowledge about the current network topology and calculate the set of nodes required to cover all the other nodes of the network. The transmissions from these nodes are scheduled using IT-set scheme. Fig. 4 shows the implementation of the protocol used by the nodes to broadcast its information over the network, when it acts as a master. A node currently acting as a master uses the latest “first” and “second” neighbor sets $\beta_1$ and $\beta_2$, respectively, and runs the broadcast protocol shown in Fig. 4(a) to find the independent transmission set and their sequence of transmission. The protocol is terminated once the neighbor sets become null ($\emptyset$). This information is included in the packet header (as shown in Fig. 4(b)) when a broadcast transmission is made by the master node. Finally, we consider the clustered network state, where multiple communication channels are disrupted as shown in Fig. 3(c1) and the time-delay bound for information transfer between nodes 1 and 4 is violated despite routing. In order to maintain the proper operation of the system within the stability and performance bounds, we decompose the nominal network into smaller clusters, as shown in Fig. 3(c2).

Next we describe the model of the overall system consisting of the homogeneous inverter PN and a WCN. For the case-study system considered in this paper, the PN consists of parallel connected three-phase inverters, as illustrated in Fig. 7. The models of the inverter modules and overall IPN are described in Section II.B. A rotating master-slave closed-loop control, as illustrated in Fig. 1, is implemented. Such a control scheme requires information exchange through a WCN. The modeling approach for the WCN is described in Section II.A.

A. Wireless Communication Network Model

We model WCN as a graph with set $L$ representative of “currently-available” links for transmission. Every link $l \in L$ has an associated link capacity $c_l$ (bits/s), which is an upper bound on the information transfer rate. The information flow from the source to the destination node (possibly through the intermediate nodes in case of RN and CN) is represented by a transmission session and there can be multiple transmission sessions among different pairs of nodes. The set of all transmission sessions is denoted by $\Gamma$ and an individual transmission session $\mu$ s.t. $\mu \in \Gamma$ is a representative of an ongoing transmission between a source-destination pair through the intermediate nodes. Each transmission session $\mu$ is characterized by the following attributes:

- Shortest path for each transmission session $\mu$ consists of a subset of links $L(\mu) \subseteq L$ and can be obtained using Dijkstra’s shortest path algorithm [17];
- The parameter $r_{\mu}$ is the average information exchange rate achieved for $\mu$;
- An end-to-end delay requirement $d_{th}$ imposed by the application layer (in this case given by the delay depended stability criteria of the control system application layer);
The minimum rate $R_{\min}$ requirement based on the control system’s switching frequency;

The maximum transmit power $P_{\max}$ a node can use for transmitting the packet.

A transmission cycle being a sequence of time slots is represented by the set $H$, where each time slot has a corresponding transmission schedule $h_i$, s.t. $h_i \in H$. Each $h_i$ has an associated subset of simultaneously transmitting links denoted by $L(h_i) \subseteq L$. We also consider the WCN to be an interference limited network, where more than one transmission in each transmission schedule $h_i$ is permitted.

A.1 Link Capacity Model:

To model the channel behavior for link $l$ we use the signal-to-interference-and-noise ratio (SINR$_l$) \[1\] for that link, which is defined as

$$\text{SINR}_l(P) = \frac{\gamma_{ll} P_l}{n_l + \sum_{m \neq l} \gamma_{lm} P_m}. \quad (1)$$

In (1), $\gamma_{lm}$ is the channel gain from transmitter of link $m$ to the receiver of link $l$, $P_l$ is the power level used by the transmitter of link $l$ and is upper bounded by $P_{\max}$ and $n_l$ is the additive receiver noise, which is independent of the transmitter power levels. The accumulated flow rate assigned to link $l$ due to each $\mu$ (i.e. $\sum_{j \in L(\mu)} \delta_{ij}$) is upper bounded by the link capacity $c_l(P)$ \[1\] given by (for normalized bandwidth)

$$c_l(P) = \log(1 + \text{SINR}_l(P)). \quad (2)$$

A.2 End-to-End Delay Model:

We first describe the link delay using discrete time Markov-chain model, which will be used as building block for the end-to-end delay. For a given packet length $\lambda$, the packet-drop probability $p$ depends on the bit error probability $p_b$ and is given by $p = 1 - (1 - p_b)^{\lambda}$. The value of $p_b$ is a function of the transmitter power, modulation scheme used and the channel noise. The Markov-chain model for packet delay at link $l$ is shown in Fig. 5, where each state transition towards right represents an additional delay of one time slot and is given by

$$P_t(k+1) = p P_t(k) \quad k = \{0,1,2,\ldots\}. \quad (3)$$

Corresponding to the instantaneous probability $P_t(k)$, there is an associated steady state probability $\pi_k$ of being in state $k$ (in the long run) satisfying $\pi_{k+1} = p \pi_k$. For Markov-chain we know that the steady state probabilities also satisfy

$$\sum_{k=0}^{\infty} \pi_k = 1. \quad (4)$$

Expanding (4) and using $\pi_{k+1} = p \pi_k$, we obtain

$$\pi_0 = (1-p) \quad \pi_k = (1-p)p^k. \quad (5)$$

From (5) we observe that the link delay in time slots has a geometric distribution. Let $X$ is the random number of times slots a packet is delayed before successful transmission (with probability$(X=k) = (1-p)p^k$ from (5)), then the average number of time slots required for a successful transmission is given by $E[X] = (1-p)/p$. Now the end-to-end delay for the session $\mu$ is calculated by accumulating the delay at each link as follows:

$$t_{\text{d}}^{(\mu)} = \sum_{l \in L(\mu)} t_{l}^{(\mu)} \quad (6a)$$

where $t_{l}^{(\mu)}$ is the transmission delay and is bounded as

$$t_{l}^{(\mu)} \geq \frac{\lambda_l}{E[X] \sum_{\mu \in L(\mu)} r_{\mu}} \quad (6b)$$

where $\lambda_l$ is the packet length used at link $l$ and $E[X]$ represents the expected value of a set $X$. 


A.3 Network Clustering Distribution:

We model multiple communication channel failures leading to node isolation as decomposition of the nominal network into smaller clusters, as shown in Fig. 3(c). Depending on how the nodes are clustered, we define different network states by $s$ with the values of $s$ representing the number of clusters in the network. Clustering is initiated when the time-delay $\tau_d^{(\mu)} = kT_{\text{slot}} > \tau_{d_{\text{th}}}^{(\mu)}(s)$ and the corresponding probability $P_c(s)$ is given by

$$P_c(s) = P\left(k > \frac{\tau_{d_{\text{th}}}^{(\mu)}(s)}{T_{\text{slot}}}\right).$$  \hspace{1cm} (7)

Fig. 6 shows the Markov-chain model to describe the various clustered network states. In Fig. 6, the probability that the NCS stays at the same network state is given by $q_1 = (1 - P_c(\text{state }1))$, where $N$ is the number of nodes in the NCS. Also, for the nominal network, the probability that the NCS jumps to NS2, is given by $p_1 = (1 - q_1)$. The probability $q_2$ of the NCS remaining in state NS2 is given by

$$q_2 = NP_c(2)(1 - P_c(2))^{N-1}$$  \hspace{1cm} (8)

where $P_c$ is obtained from (7). Next, the state-transition probability $p_2$ is given by $p_2 = p_1 - q_2$ and can be rewritten as $p_2 = 1 - q_1 - q_2$. The state transition probabilities for the NCS being in state $s$ are given by

$$p_s = 1 - \sum_{i=1}^{s} q_i$$  \hspace{1cm} (9)

$$q_s = \left(\begin{array}{c} N \\ s - 1 \end{array}\right)p_{s-1}(s)(1 - P_c(s))^{N-s+1}.$$  \hspace{1cm} (10)

The state-transition matrix for the network clustering shown in Fig. 6 is given by

$$A = \begin{bmatrix} q_1 & p_1 & 0 & 0 & \cdots & 0 & 0 \\ 1 - q_2 - p_2 & q_2 & p_2 & 0 & \cdots & 0 & 0 \\ & & & & & & \\ 0 & 1 - q_3 - p_3 & q_3 & p_3 & \cdots & 0 & 0 \\ & & & & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 - q_N & q_N \end{bmatrix}.$$  \hspace{1cm} (11)

From the state-transition diagram in Fig. 6, it is observed that the Markov chain model is of class-one type and hence an equilibrium probability vector exists [18]. The steady-state vector $\bar{\pi} = [\pi_1 \ \pi_2 \ \cdots \ \pi_N]$ given by $\bar{\pi} = \lim_{n \to \infty} \bar{\pi}_0 A^n$ can be obtained by solving the equation $\bar{\pi} = \bar{\pi} A$ and is independent of initial state vector $\bar{\pi}_0$. This provides a steady state distribution of the number of clusters the network on average will have.

B. Model of the Homogeneous Inverter PN

In this section, we first describe the models of the power stage and feedback controller of the $r$th inverter module. The control scheme is based on a rotating master-slave architecture, as illustrated in Fig. 1. We note here that each controller model has to account for time delays due to exchange of information through the WCN, as outlined in Section II.A. By combining the power stage model and the controller model, we develop the closed-loop model of the $r$th inverter module. We note that for the homogeneous inverter PN considered in this paper, $r$ varies from 1 – 6. The overall model of the homogeneous inverter PN (Fig. 7) is obtained by integrating the models of all the inverter modules.

B.1 Power Stage Model of the $r$th Inverter:

The power stage of the $r$th inverter module is described in stationary (abc) frame by a set of PWL equation:

$$\frac{d\text{x}^{abc}_{pr}(t)}{dt} = A^{abc}_{pr} x^{abc}_{pr}(t) + B^{abc}_{pr} j$$  \hspace{1cm} (11)
where \( x_{\text{pr}}^{\text{abc}}(t) = \left[ \left( y_{\text{pr}}^{\text{abc}}(t) \right)^T \right]^T \) represents the power-stage states of the master module and \( y_{\text{pr}}^{\text{abc}}(t) \) represents the power-stage states of the slave modules. Here, any vector \( y_{\text{pr}}^{\text{abc}}(t) \) is represented as \( y_{\text{pr}}^{\text{abc}}(t) = \left[ y_{\text{pr}}(t) \ y_{\text{br}}(t) \ y_{\text{cr}}(t) \right]^T \). The subscript \( j \) depends on the switching functions of the inverter (\( S_{\text{ar}}, S_{\text{br}}, S_{\text{cr}} \)).

Using Park’s transformation [19], we convert the system of equations (11) in the stationary frame to the synchronous frame. The resulting state-space equation is given by

\[
\frac{dx_{\text{pr} \text{dqz}}(t)}{dt} = A_{\text{pr} \text{dqz}} x_{\text{pr} \text{dqz}}(t) + B_{\text{pr} \text{dqz}}
\]

where \( i \) represents the switching states of the synchronous-frame inverter model and \( x_{\text{pr} \text{dqz}}(t) \) represents the states of the power stage in the synchronous frame (\( x_{\text{pr} \text{dqz}}(t) = T \times x_{\text{pr} \text{abc}}(t) \), where \( T \) is the Park’s transformation matrix [19]). Matrices \( A_{\text{pr} \text{dqz}}, x_{\text{pr} \text{dqz}}(t), \text{and } B_{\text{pr} \text{dqz}} \) are defined as follows [20]:

\[
A_{\text{pr} \text{dqz}} = \begin{bmatrix}
-\frac{1}{L_r}(r_{Lr} + r_C) & 0 & -\frac{1}{L_r} + \delta N_{\text{inv}} & 0 \\
0 & -\frac{1}{L_r}(r_{Lr} + r_C) & 0 & -\frac{1}{L_r} + \delta N_{\text{inv}} \\
0 & 0 & -\frac{1}{C} & 0 \\
0 & 0 & 0 & -\frac{1}{C}
\end{bmatrix}
\]

\[
x_{\text{pr} \text{dqz}}(t) = \begin{bmatrix}
i_{dr}(t) \\
i_{qr}(t) \\
i_{pr}(t) \\
\delta v_d(t) \\
\delta v_q(t) \\
\delta v_z(t)
\end{bmatrix}
\]

\[
B_{\text{pr} \text{dqz}} = \begin{bmatrix}
S_{d_{\text{in}}} V_{\text{in}} \\
S_{q_{\text{in}}} V_{\text{in}} \\
S_{r_{\text{in}}} V_{\text{in}} \\
0 \\
0 \\
0
\end{bmatrix}
\]

Here, the value of \( \delta \) for the master module is 1, while for the slave modules it is 0.

B.2 Feedback Controller Model of the \( r \)-th Inverter:

The feedback controller for the master and the slave modules are implemented in the synchronous frame. For the d-axis control, the master module has a voltage loop, which generates the reference for its current loop as well as those for the slave modules. The reference for the q-axis current loop is generated internally depending on the type of load connected to the inverter. For all the control loops, conventional linear compensators are used. The compensators in the d- and q-axes control loops introduce additional states and are modeled as

\[
\frac{dx_{\text{cr}}(t)}{dt} = A_{0_{\text{cr}}} x_{\text{cr}}(t) + A_{1_{\text{cr}}} x_{\text{cr}}\left( 1 - e^{-\frac{t}{\tau_{d}}} \right) + H_{\text{pr}} x_{\text{pr} \text{dqz}}(t) + B_{\text{cr}}
\]

(13)
where \( x_{cr} (t) = [\xi_1(t) \xi_2(t) \xi_3(t) \xi_4(t) \xi_5(t) \xi_6(t)]^T \) are the states of the controller and \( \tau_d^{(1)} \) is the end-to-end time delay at the \( r \)th node due to the exchange of the d-axis current reference information from the master to the slave. Matrices \( A_{0cr} \), \( A_{1cr} \), and \( H_{pr} \) and the vector \( B_{cr} \) are defined as follows [20]:

\[
A_{0cr} = \begin{bmatrix}
-\delta \omega_p d_l & 0 & 0 & 0 & 0 & 0 \\
\delta & -\delta \omega_p d_l & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega_p q_i & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
A_{1cr} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
H_{pr} = \begin{bmatrix}
H_v d & 0 & 0 \\
0 & 0 & 0 \\
H_i q & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
B_{cr} = \begin{bmatrix}
\delta V_{ref} \\
0 \\
0 \\
0
\end{bmatrix}
\]

Here, the value of \( \delta \) for the master module is 1, while for the slave modules, its value is 0 and \( \tilde{\delta} = 1 - \delta \). A detailed description of how the time delays are obtained will be discussed in the following section. A conventional symmetrical space-vector modulation scheme [19] is used to generate the switching signals for the power stage based on the outputs of the d- and q-axes controllers.

**B.3 Closed-loop Model of an Inverter Module:**

Combining (12) and (13), the overall model of the \( r \)th inverter module is expressed as

\[
\frac{dx_{r}(t)}{dt} = A_{0r_i} x_{r}(t) + A_{1r} x_{r} \left( t - \tau_d^{(1)} \right) + B_{r_i}
\]

where

\[
x_{r}(t) = \begin{bmatrix}
\begin{align*}
\frac{dq}{dt} \\
\frac{dz}{dt} 
\end{align*}
\end{bmatrix}
\]

\[
A_{0r_i} = \begin{bmatrix}
A_{pr_i} & 0 \\
H_{pr} & A_{0cr}
\end{bmatrix}
\]

\[
A_{1r} = \begin{bmatrix}
0 & 0 \\
0 & A_{1cr}
\end{bmatrix}
\]

\[
B_{r_i} = \begin{bmatrix}
B_{q} \\
B_{c}
\end{bmatrix}
\]

Because of the switching action of the devices, the overall model (14) of the inverter is PWL in nature.
B.4 Overall IPN Model:

The overall model of the IPN under investigation, we combine the individual models of all of the inverters (14). The resultant state-space equation can be described as

\[
\frac{dX(t)}{dt} = A_{0i}X(t) + \sum_{\mu \in \Gamma} A_{1}(\mu)X\left(t - \tau_{d}^{(\mu)}\right) + B_{i}
\]

(15)

where \( i \) is an integer that denotes the switching state of the PN power devices, \( X(t) = \begin{bmatrix} x_{1}(t) \\ \vdots \\ x_{6}(t) \end{bmatrix} \), \( A_{0i} = \begin{bmatrix} A_{01i} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_{06i} \end{bmatrix} \), \( A_{1}(\mu) = \begin{bmatrix} A_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_{16} \end{bmatrix} \), \( B_{i} = \begin{bmatrix} B_{1i} \\ \vdots \\ B_{6i} \end{bmatrix} \), and \( \tau_{d}^{(\mu)} \) is the time delay of transmission session. Here, \( \Gamma \) represents the set of all transmission sessions (each corresponding to time delays \( \tau_{d}^{(\mu)} \)), which is obtained from the Markov-chain model described in Section II.A.

III. STOCHASTIC STABILITY ANALYSES

Using the model of the homogeneous inverter PN (15), in this section, we investigate the global stability of the system. Global stability analysis consists of first determining the reaching criteria for orbital existence (i.e. determining under what conditions the trajectories of the system converge to an orbit) and then predicting the stability of the nominal orbit (or steady-state stability). For the former, we use a composite Lyapunov function based approach that was proposed by the authors in [20].

A. Reaching Criteria for Orbital Existence

To determine the reaching criteria of the overall system, we first convert (15) to the error coordinates using

\[
e(t) = X^{*} - X(t),
\]

where \( X(t) \) and \( X^{*} \) are the actual and desired values of the states, respectively. The modified state-space equation is given by

\[
\frac{de(t)}{dt} = A_{0i}e(t) + \sum_{\mu \in \Gamma} A_{1}(\mu)e\left(t - \tau_{d}^{(\mu)}\right) + B_{i}
\]

(16)

where \( B_{i} = \left( A_{0i} + \sum_{\mu \in \Gamma} A_{1}(\mu)\right)X^{*} - B_{i} \). The time-delay threshold \( \tau_{d}^{(\mu)}\) is determined using a composite Lyapunov function based approach described in [20]. We first define a composite, positive-definite, quadratic Lyapunov function \( V_{n}(e) > 0 \)

\[
V_{n}(e(t)) = \sum_{i=1}^{h} \eta_{ni}e^{T}(t)p_{ni}e(t), \quad n = 1, 2, \cdots, R
\]

(17)

where \( R \) represents the number of feasible switching sequences, \( h \) is the number of switching states in a given sequence [20], \( 0 \leq \eta_{ni} \leq 1 \), \( \sum_{i=1}^{h} \eta_{ni} = 1 \), and \( p_{ni} = p^{T}_{ni} \) is a positive-definite matrix. The reaching criterion for orbital existence of (16) is given by the following matrix inequality [20]:

\[
\sum_{\mu \in \Gamma} \sum_{i=1}^{h} \eta_{ni} \begin{bmatrix}
C_{ni}^{(\mu)} & p_{ni}A_{1}(\mu)^{T}A_{0i} - p_{ni}A_{1}(\mu)^{T}A_{0i} & p_{ni}B_{i}^{T}
-A_{0i}A_{1}(\mu)^{T}p_{ni} - \varphi^{2}p_{ni} & 0 & 0
-A_{1}(\mu)^{T}p_{ni} & 0 & -\varphi p_{ni}
B_{i}^{T}p_{ni} & 0 & 0 & 0
\end{bmatrix} < 0
\]

(18)
where \( G_{ni}(\mu) = \frac{1}{\tau_{d_{th}}^{(\mu)}} \left[ \left( A_{0i} + A_{1i}^{(\mu)} \right)^{T} p_{ni} + p_{ni} \left( A_{0i} + A_{1i}^{(\mu)} \right) \right] + \varphi p_{ni} \) and \( \varphi > 0 \).

Using (18) and its dual condition, which is described in [20], we determine the time-delay bounds \( \tau_{d_{th}}^{(\mu)} \) for the inverter-network model (15). The probability for occurrence of such a delay is given by the Markov-chain model, shown in Fig. 5. However, these delay bounds have to be further modified to account for the time required to execute the WCN protocol, which have been described in Section II. The end-to-end delay for the protocol is the sum of the time to process the network protocol, packet transmission time including both header and data, and the protocol overhead and can be expressed as

\[
t_{\text{end-to-end}} = N_{\text{inv}}(t_{\text{slot}}) + t_{\text{routing}}
\]

(19a)

where \( N_{\text{inv}} \) is the number of modules in the IPN, \( t_{\text{routing}} = \frac{L}{R}, \) \( t_{\text{slot}} = t_{\text{trans}} + t_{\text{proc}}, \) \( t_{\text{trans}} = \frac{L + H_{f}}{R}, \) \( R \) is the data transmission rate of the wireless interface, \( H_{f} \) is the fixed packet header, \( l \) is the broadcast protocol overhead normalized to one node, as illustrated in Fig. 4(b) and \( t_{\text{proc}} \) is the protocol execution delay and is much smaller than the transmission delays for the network size under consideration. Using the above delay components, (19a) can be expressed as

\[
t_{\text{end-to-end}} = \frac{N_{\text{inv}}}{R} \left( L + H_{f} + l_{\text{inv}} + t_{\text{proc}} \right).
\]

(19b)

Using (19b), the time delay bound, where the IPN operating as NN or RN becomes unstable/unreachable is given by

\[
\tau_{d_{th}}^{(\mu)} = \tau_{d_{th}}^{*} - t_{\text{end-to-end}}.
\]

(20)

**B. Mean-square Stability**

If the time delay bounds \( \tau_{d_{th}}^{(\mu)} \), obtained in Section III.B are violated, the network is clustered into smaller clusters, where there exists no direct communication link among the two clusters. In a mathematical framework, the system equations during the clustering phenomena can be described by jumps from one set of state-space equations to a different set of state-space equations. The probability of the jumps can also be described by the Markov-chain models and is shown in Fig. 6. Because such jumps are random in nature, the deterministic stability analyses tools, discussed in Section III.B cannot be used. To determine whether the IPN is stable/reachable after clustering, we investigate the mean-square stability of the IPN described by (16).

**Definition of mean-square stability** [21]: A system described by (16) with probability of jumps, \( p_{ij} \) is stable in the mean-square sense, if the expected value of the Lyapunov function \( V_{n} \) satisfies

\[
E(V_{n+1} - V_{n}) < 0 \quad \text{for all possible } p_{ij}.
\]

(21)

To ascertain the mean-square convergence/stability of all possible jumps, we first define a composite Lyapunov function for each operating state of the network as

\[
V_{nk}(e(t)) = \sum_{i=0}^{h} \eta_{ki} e^{T}(t)p_{nki} e(t), \quad k = 1, 2, \ldots, M
\]

(22)

where \( n \) depends on the operating state of the IPN (defined by the Markov-chain model for clustering), \( h \) is the number of switching states in a given sequence, \( 0 \leq \alpha_{ki} \leq 1, \sum_{i=1}^{h} \alpha_{ki} = 1, \) and \( p_{nki} = p_{nn}^{T} \).

If the IPN is clustered, we define a composite-Lyapunov function for the new operating state as

\[
V_{n'k}(e(t)) = \sum_{i=0}^{h} \eta_{ki} e^{T}(t)p_{n'ki} e(t), \quad \text{where } n' \text{ corresponds to the new operating state of the IPN. If the converter}
\]

dynamics in both these states satisfy \( V_{n^{'k}}(t) < 0 \) and \( V_{nk}(t) < 0 \), we have to determine that during the transition
from one network state to the other, the Lyapunov function decreases, i.e. \( V_{n'k}(t) - \Pi V_{nk}(t) \leq 0 \) [21], where \( \Pi \leq 1 \). At the partition, the jump condition can be expressed as
\[
F_n e_-(t) = F_n e_+ (t).
\]  
(23)
where \( F_n \) and \( F_{n'} \) are matrices that describe each partition, and \( e_-(t) \) and \( e_+ (t) \) are the states of the network before and after the partition, respectively. Thus, we have
\[
V_{n'k}(t) - \Pi V_{nk}(t) = \sum_{i=0}^{h} \eta_{ki} e_i^T(t) p_{n'ki} e_+(t) - \Pi \sum_{i=0}^{h} \eta_{ki} e_i^T(t) p_{nki} e_-(t) \\
= \sum_{i=0}^{h} \eta_{ki} e_i^T(t) p_{n'ki} e_+(t) - \sum_{i=0}^{h} \eta_{ki} e_i^T(t) (\Pi F_n^T p_{nki} F_{n'}) e_+(t).
\]  
(24)
To satisfy \( V_{n'k}(t) - \Pi V_{nk}(t) \leq 0 \), one obtains the following condition:
\[
\sum_{i=0}^{h} \left\{ \eta_{ki} p_{n'ki} - \eta_{ki} \left( \Pi F_n^T p_{nki} F_{n'} \right) \right\} < 0.
\]  
(25)
Using (25), we determine how the dynamics of the system behave for every possible jump, when the converter is operating in a given network state. For a given network state, if all possible jumps satisfy (25), then that network state is stable for all possible jumps. However, if some jumps do not satisfy (25), then using the probabilities of jumps, \( p_{n,n'} \) (derived in Section II.A.3), we ascertain the mean-square stability of the network state. To determine the mean-square stability of the network state, we determine the expected values of (25) for all possible jumps from a given network state, \( n \). The expected value of the jump can be expressed as
\[
E(\Delta V_n) = \sum_{n'=1}^{S} p_{n,n'} \left\{ \sum_{i=0}^{h} \left( \eta_{ki} p_{n'ki} - \eta_{ki} \left( \Pi F_n^T p_{nki} F_{n'} \right) \right) \right\}
\]  
(26)
where \( S \) is the number of possible jumps from a given network state to an adjacent state. In the above expression, the value of \( p_{n,n'} \) for \( n = n' \) is the probability of the system remaining in the same network state. For the network state to be stable in the mean-square sense, the expected value given by (26) should satisfy \( E(\Delta V_n) < 0 \) [21]. Thus, the criteria for mean-square stability of a given network state is given by
\[
\sum_{n'=1}^{S} p_{n,n'} \left\{ \sum_{i=0}^{h} \left( \mu_{ki} p_{n'ki} - \eta_{ki} \left( \Pi F_n^T p_{nki} F_{n'} \right) \right) \right\} < 0.
\]  
(27)

IV. JOINT OPTIMIZATION OF WCN AND PN

Using the detailed models of the PN and the WCN, outlined in Section II, we describe a mechanism for jointly optimizing the performance of PN and resource utilization of WCN under constraints of PN stability and WCN delay and capacity. The goal of the optimization problem is to minimize the overall cost function \( J_o \) by using a weighted representation of the normalized optimal values of the PN and WCN cost functions (i.e. \( J_{PN}^* \) and \( J_{WCN}^* \)), as shown in Fig. 8.

We define an objective function for joint optimization of the PN performance and WCN resource utilization as
\[
J_o = \alpha J_{PN}^* + (1 - \alpha) J_{WCN}^*
\]  
(28)
where the parameter \( \alpha \) is a weighting coefficient (0 < \( \alpha < 1 \)). The value of \( \alpha \) under different operating scenarios can be pre-decided by the designer depending on the application and the number of nodes in a network. For instance, if a certain application requires a very high control performance, a high value of \( \alpha \) should be chosen, while if there are a large number of nodes in the WCN or if the WCN link is experiencing some disruptions, a low value of \( \alpha \) should be chosen. Here, the normalized values of the optimal WCN resource utilization \( J_{WCN}^* \) and optimal PN performance \( J_{PN}^* \) are defined as
\[ J_{\text{WCN}}^*(N) = 1 - \frac{J_{\text{WCN}}^*}{\max J_{\text{WCN}}} \] and \[ J_{\text{PN}}^*(N) = \frac{J_{\text{PN}}^*}{\max J_{\text{PN}}} \] (29)

In (29) the WCN resource utilization problem is formulated as:

\[
\begin{align*}
\text{maximize} & \quad J_{\text{WCN}} = \sum_{\mu \in \Gamma} U(r_{\mu}) \\
\text{s.t.} & \quad \sum_{l \in L(\mu)} \tau_l^{(\mu)} < \tau_{d_{\text{th}}} \quad \forall \mu, \quad \tau_l^{(\mu)} \geq \frac{\lambda_1}{E[X]\sum_{\mu \in L(\mu)} r_{\mu}} \quad \forall l \\
& \quad \sum_{\mu:l \in L(\mu)} r_{\mu} \leq c_l(P) \quad \forall l, \quad R_{\text{min}} \leq r_{\mu} \quad \forall \mu
\end{align*}
\] (30)

In the above expression \( U(r_{\mu}) \) is the network utility function and can be formulated as a concave function (e.g. \( U(r_{\mu}) = \log(r_{\mu}) \)). The above problem is a convex optimization problem and can be solved efficiently as described in [22].

The PN optimization problem consists of determining the gains of the controllers that minimizes a control-performance cost function. For the case of the inverter network considered in this paper, two critical performance parameters are the load-sharing error and the output voltage and current THDs. The THD of a signal represents the harmonic distortion present in a signal and is defined as the ratio of the sum of all harmonic components to that of the fundamental frequency component. To minimize these two control performance parameters, the cost function is expressed as a weighted second norm of the difference between the desired \( X_p^* \) and actual \( X_p \) values of the power-stage states of the inverter PN. Thus, optimization problem can be expressed as:

\[
\begin{align*}
\text{minimize} & \quad J_{\text{PN}} = \| Wf(X_p) \|_2^2 \\
\text{s.t.} & \quad \tau_d^{(\mu)} \leq \tau_{d_{\text{th}}} \\
& \quad e_{\text{LS}} = \frac{N}{m} \left( \frac{i_{\text{load}}}{N} - X_{m_i} \right) \leq \varrho_1 \\
& \quad X_{\text{THD}} = \left( \frac{\| X_p - X_p^* \|}{X_p^*} \right)^2 \leq \varrho_2
\end{align*}
\] (31)

where \( f(X_p) = \frac{X_p^* - X_p}{X_p^*} \), \( \| Wf(\cdot) \|_2 = \sqrt{f^T(\cdot)W(\cdot)} \), \( W \) is a positive-definite matrix, and \( \langle \cdot \rangle \) denotes the average of a function over a fixed time interval. In (31), \( N \) represents the number of nodes, \( X_{m_i} \) represents the output current of the \( m \)th inverter, \( i_{\text{load}} \) represents the total load current flowing through the homogeneous PN, and parameters \( \varrho_1 \) and \( \varrho_2 \) specify the acceptable performance bounds for the load-sharing error and THD, respectively that are pre-defined depending on the application.

V. SIMULATION RESULTS

In this section, we first present the results of the stability analysis technique described in Section III. Next, we evaluate the efficacy of the joint optimization framework (described in Section IV) using the models developed in Section II.

First, we determine the time-delay bounds for the unclustered network, which ensures that the IPN is stable and the load-sharing and THD of the system is within pre-defined performance bounds. For both these performance parameters, we consider a 5% limit. If either the stability bound or the performance bounds are violated, the communication network is clustered into smaller sub-networks as illustrated in Fig. 3(c2). Fig. 9(a), shows the stable
operating bounds of the unclustered network, which is obtained using the procedure described in Section III.A. We observe that, as the number of nodes increase, the time-delay bound reduces, while the actual delay for information exchange increases. Fig. 9(b), illustrates the variation of the performance time delay bound (based on the load-sharing error and harmonic distortion) of the unclustered network with variation of the number of nodes. In this case, because the time-delay bound based on the performance parameter is lower than that based on stability, this value is taken as the time delay bound of the IPN. The processing delay is specific to the hardware implementation of the NCS (comprising a TI TMS320C6713 DSP, Altera Flex10K Series FPGA and ML2722 wireless transceiver). The probability of occurrence of a certain time delay is governed by the Markov-chain model described in Fig. 5, and is illustrated in Fig. 10 for the network considered here. The probability of occurrence of large time delays is low when the signal to noise ratio is large and progressively increases with reduction in the signal to noise ratio. Hence, the possibility of the system violating its stability and performance bounds increases as the noise levels in the communication channel increase.

When the time-delay bounds of the system are violated, the communication network is clustered, as described in Section II.A.3. For such a clustered network, the stability analyses technique is described in Section III.B. Using the criteria (27) and taking into account the protocol-processing delays, the time-delay bounds for the IPN under consideration are illustrated in Fig. 11. We observe that, as the number of clusters increase, the range of time-delays that the system can handle increases. Also, because the number of nodes in each network reduces with clustering, the protocol processing delay decreases; hence, the region of stable operation increases.

To demonstrate the impacts of clustering on the performance of the IPN under study, we compute the expected values of the load-sharing errors and THD, using the clustering probability distribution described in Section II.A.3, as shown in Fig. 12. Using this probability distribution, we compute the mean of the $L_2$ norm of the load-sharing error as

$$\text{Mean} = \sum_{j=1}^{M'} P_j |L_2(e_{LS})|_j$$

where $P_j$ is the probability of occurrence of a cluster, $|L_2(e_{LS})|_j$ represents the $L_2$ norm of the load-sharing error of the $j^{th}$ cluster and $M'$ is the maximum number of clusters that are possible. The variance of the $L_2$ norm of the load-sharing error as

$$\text{Variance} = \left[ \sum_{j=1}^{M'} P_j |L_2(e_{LS}) - \text{Mean}|_j^2 \right]^{1/2}$$

where the value of the mean is given by (32). Figs. 13 and 14 show the variation of the mean and variance of the $L_2$ norm of the load-sharing error and the THD of the IPN, which is computed using (32) and (33), respectively, with variation of the distance between the nodes. The effect of the increase in distance was emulated by varying the reactances of the interconnections among the individual inverters [8]. Clearly the IPN operates within the performance limits in a “mean” sense. The variances indicate that while the THD of the IPN is within performance bounds, the load-sharing error may exceed the performance bounds for certain cluster configurations, as the distance among the nodes is increased. If all communication links fail within the IPN, the load-sharing performance of the system approaches that of the droop method. Although the probability of occurrence of such events is unlikely, the control strategies have to be modified, when such an event occurs to ensure proper operation of the IPN.

Next, we evaluate the efficacy of the joint optimization framework using the models and concepts outlined in Sections II and III. For the joint optimization problem, we first illustrate the variations of $J^*_W^{(N)}$ and $J^*_P^{(N)}$ with variation of time delay in Fig. 15. For three different values of $\alpha$, we illustrate the variations of the overall cost function $J_o$ (described in (28)) in Figs. 15(a) – 15(c). Clearly, for $\alpha = 0.1$, the overall cost function closely follows $J^*_P^{(N)}$, while for $\alpha = 0.9$, the overall cost function follows $J^*_W^{(N)}$. The value of $\alpha$ is pre-decided by the designer during planning and could also vary with the operating condition of the overall network.

Fig. 16 illustrates a time-domain simulation result (using the overall system model in Sections II and III) showing the variation of the overall cost function when there is a communication disruption and new communication links are established via the RN. After the communication link is disrupted as shown Fig. 16, new optimal paths, power
levels for the WCN and local controller gains for the PN are computed. From Fig. 16, we observe that during the transition from one optimal point to the next, the cost function exhibits a jump (corresponding to a jump in the states), which might not be acceptable from the PN performance point-of-view. Finally, Fig. 17(a) illustrates how the overall cost function (expressed as \( J_0 = J_0^{(1)} + J_0^{(2)} \)) varies when the WCN is divided into two clusters. Note that, in this case we assume that there is no communication among the individual clusters. Fig. 17(b) shows that the overall cost function can be reduced if there is a (slow) communication link among the two clusters to exchange the optimal overall costs of the individual clusters. This improvement in the performance also depends on the rate of communication among the two clusters and approaches the performance of Fig. 17(a) as the rate of inter-cluster communication progressively decreases.

**VI. SUMMARY AND CONCLUSIONS**

In this paper, we describe a methodology to model a distributed control scheme for an interactive power network (IPN). The model for the power network (PN) is used to determine the stability bounds of the system while the model for the wireless communication network (WCN) is used to determine time delays in information transfer among the nodes of a distributed control system. We investigate the impacts of time delay on the stability and performance of the IPN. We consider cases with one or more disruptions in the communication links among the nodes. Fault-tolerant network configuration schemes are investigated for rerouting information or clustering the network into smaller, independent clusters, which can ensure satisfactory network operation under communication disruptions without significantly degrading the network performance. Using the overall system model, we implement a framework for joint optimization of the network resource utilization of the WCN and control performance of the PN. The optimal operating points of the WCN and PN do not coincide under all operating conditions. Under such conditions, we demonstrate that the “joint” optimization strategy provides a compromise between the conflicting requirements of the two networks. We further demonstrate that when the WCN is clustered due to communication disruptions, each cluster can optimize its control-communication network.

**REFERENCES**


Fig. 1: Schematic and experimental setup of a homogeneous IPN consisting of parallel three-phase inverters. A rotating master-slave scheme is implemented for load-sharing and requires information exchange among the various nodes.
Fig. 2: (a) The state transition diagram for the three possible network operating states: nominal network (NN), rerouted network (RN), or clustered network (CN). (b) Timing diagram of delays experienced by the network operating in three different states.
Fig. 3: Illustration of the different scenarios of operation of the NCS showing (a) NN, (b) rerouting of information in the case of failure of one communication link, and (c) failure of multiple communication links leading to the isolation of the nodes from the network, in which case the network is decomposed into smaller and independent clusters.
Loop until $\beta_1 = \Phi, \beta_2 = \Phi$.

Initialize sets $\beta_1$ and $\beta_2$.

Find Set Cover for all neighbors.

Find Independent Transmission Set.

Update Transmission Sequence (TS).

Use TS in Packet header.

**Fig. 4:** (a) Schematic showing the implementation of the broadcast protocol to send data from the master node to the slave node, and (b) broadcast packet format used by the current master node where ID field is node identifier.
Fig. 5: Markov-chain based state-transition diagram for delay distribution.
Fig. 6: State transition diagram based on Markov-chain for the number of clusters in the network.
Fig. 7: Schematic of a parallel voltage source inverter network. For the results presented in this paper, $r$ (defined earlier as the number of inverter modules) varies from 1 through 6.
Fig. 8: Schematic illustrating the joint optimization framework for NN and RN states. In the CN state, initially each cluster determines its cost function $J_v$ (where $v$ represents the number of clusters), which is subsequently scaled once this information is shared among the clusters using a slow communication link.
Fig. 9: Time-delay bounds for $N$-module parallel VSI with variation of the number of nodes, obtained from (a) stability analyses and (b) performance measures.
Fig. 10: Probability distribution for the occurrence of time delays in the unclustered network with variation of the signal-to-noise ratio.
Fig. 11: Variation of time-delay bounds for N-module parallel VSI with variation of the number of clusters and the signal-to-noise ratio.
Fig. 12: Probability-density function of the number of clusters in the network for different bit error probabilities. Symbol $\frac{E_b}{N_0}$ represents the SINR of the signal at the receiver.
Fig. 13: Results illustrating the variation of the load sharing error with increase in the distance for the clustered network, with variation in the distance between the nodes.
Fig. 14: Results illustrating the variation of the THD with increase in the distance for the clustered network with variation in the distance between the nodes.
Fig. 15: Result illustrating the variation of the normalized PN and WCN cost function, as well as the joint cost function of the system for (a) $\alpha = 0.1$, (b) $\alpha = 0.5$, and $\alpha = 0.9$. 
Fig. 16: Variation in the optimal cost of the system due to WCN failures. Note that in this case the network structure after failure is where communication links among all the nodes can be established by routing.
Fig. 17: Optimal cost variation of the system due to WCN link failures when the network is clustered into two smaller clusters. (a) corresponds to the case where the two clusters are independent of each other, while (b) corresponds to the case, where there is a slow communication link between the two clusters.