Carrier statistics

\[ n_i = 2 \left( \frac{2\pi kT}{\hbar^2} \right)^{3/2} \left( m_n^* m_p^* \right)^{3/4} e^{-E_g/2kT} \]

\[ n_0 = n_i e^{(E_F-E_i)/kT} \]

\[ p_0 = n_i e^{(E_i-E_F)/kT} \]

\[ n_0 p_0 = n_i^2 \]

Drift velocity

\[ v_d = \frac{qE}{\eta m^*} \tau_c = \left( \frac{q \tau_c}{\eta m^*} \right) E \]

Conductivity

\[ \sigma = q n \mu_n + q p \mu_p \]

Resistivity

\[ \rho = \frac{1}{\sigma} = \frac{1}{q n \mu_n + q p \mu_p} \]

Diffusion current

\[ J_{n,\text{diff}} = q D_n \frac{dn}{dx} \]

\[ J_{p,\text{diff}} = -q D_p \frac{dp}{dx} \]

Drift current

\[ J_n = q n \mu_n E \]

\[ J_p = q p \mu_p E \]
\[ R_n = \frac{n}{\tau_n}; R_p = \frac{p}{\tau_p} \]  

Recombination

\[ \frac{\partial n}{\partial t} = G_n - R_n + \mu_n \left( \frac{\partial n}{\partial x} E + \frac{\partial E}{\partial x} n \right) + D_n \frac{\partial^2 n}{\partial x^2} \]

\[ \frac{\partial p}{\partial t} = G_p - R_p + \mu_p \left( \frac{\partial p}{\partial x} E + \frac{\partial E}{\partial x} p \right) + D_p \frac{\partial^2 p}{\partial x^2} \]

Continuity equations

\[ \nabla \cdot \vec{E} = \frac{q}{\varepsilon} (p - n + N_D - N_A) \]

Poisson's equation

\[ \nabla \equiv \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \]

Differential operator

\[ L_n = \sqrt{D_n \tau_n} \]

\[ L_p = \sqrt{D_p \tau_p} \]

Diffusion length

\[ \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q} \]

Einstein's equation
Example - 1

EXAMPLE 4

A silicon ingot is doped with $10^{16}$ arsenic atoms/cm$^3$. Find the carrier concentrations and the Fermi level at room temperature (300 K).

**SOLUTION** At 300 K, we can assume complete ionization of impurity atoms. We have

$$n = N_D = 10^{16} \text{ cm}^{-3}.$$ 

From Eq. 20,

$$p = n_i^3/N_D = (9.65 \times 10^3)^3 / 10^{16} = 9.3 \times 10^3 \text{ cm}^{-3}.$$ 

The Fermi level measured from the bottom of the conduction band is given by Eq. 25:

$$E_C - E_F = kT \ln(N_c/N_D)$$

$$= 0.0259 \ln(2.86 \times 10^{19}/10^9) = 0.205 \text{ eV}.$$ 

The Fermi level measured from the intrinsic Fermi level is given by Eq. 28:

$$E_F - E_i = kT \ln(N_D/n_i) = kT \ln(N_D/n_i)$$

$$= 0.0259 \ln(10^{16}/0.65 \times 10^9) = 0.358 \text{ eV}.$$ 

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Fig. 27  Band diagram showing Fermi level $E_F$ and intrinsic Fermi level $E_i$. 

EXAMPLE 2

Find the room-temperature resistivity of an \( n \)-type silicon doped with \( 10^{16} \) phosphorus atoms/cm\(^3\).

**SOLUTION**  At room temperature we assume that all donors are ionized; thus,

\[
n \approx N_D = 10^{16} \text{ cm}^{-3}.
\]

From Fig. 7 we find \( \rho \approx 0.5 \) \( \Omega \)-cm. We can also calculate the resistivity from Eq. 15a:

\[
\rho = \frac{1}{qn\mu_n} = \frac{1}{1.6 \times 10^{-19} \times 10^{16} \times 1300} = 0.48 \quad \Omega\text{-cm}.
\]

The mobility \( \mu_n \) is obtained from Fig. 3.
EXAMPLE 4

Assume that, in an $n$-type semiconductor at $T = 300$ K, the electron concentration varies linearly from $1 \times 10^{18}$ to $7 \times 10^{17}$ cm$^{-3}$ over a distance of 0.1 cm. Calculate the diffusion current density if the electron diffusion coefficient is $D_n = 22.5$ cm$^2$/s.

SOLUTION  The diffusion current density is given by

$$J_{n, \text{diff}} = qD_n \frac{dn}{dx} \approx qD_n \frac{\Delta n}{\Delta x}$$

$$= (1.6 \times 10^{-19})(22.5) \left( \frac{1 \times 10^{18} - 7 \times 10^{17}}{0.1} \right) = 10.8 \text{ A/cm}^2.$$
Example 5

Minority carriers (holes) are injected into a homogeneous n-type semiconductor sample at one point. An electric field of 50 V/cm is applied across the sample, and the field moves these minority carriers 1 cm in 100 $\mu$s. Find the drift velocity and the diffusivity of the minority carriers.

**SOLUTION**

$$v_p = \frac{1 \text{ cm}}{100 \times 10^{-6} \text{ s}} = 10^4 \text{ cm/s};$$

$$\mu_p = \frac{v_p}{E} = \frac{10^4}{50} = 200 \text{ cm}^2/\text{V-s};$$

$$D_p = \frac{kT}{q \mu_p} = 0.0259 \times 200 = 5.18 \text{ cm}^2/\text{s}.$$
3.4.1 Steady-State Injection from One Side

Figure 16a shows an n-type semiconductor where excess carriers are injected from one side as a result of illumination. It is assumed that light penetration is negligibly small (i.e., the assumptions of zero field and zero generation for \( x > 0 \)). At steady state there is a concentration gradient near the surface. From Eq. 59 the differential equation for the steady-state minority carriers inside the semiconductor is

\[
\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{\infty}}{\tau_p}.
\]

The boundary conditions are \( p_n(x = 0) = p_n(0) = \text{constant value} \) and \( p_n(x \to \infty) = p_{\infty} \). The solution of \( p_n(x) \) is

\[
p_n(x) = p_{\infty} + [p_n(0) - p_{\infty}] e^{-x/L_p}.
\]

The length \( L_p \) is equal to \( \sqrt{D_p \tau_p} \) and is called the diffusion length. Figure 16a shows the variation of the minority carrier density, which decays with a characteristic length given by \( L_p \).

If we change the second boundary condition as shown in Fig. 16b so that all excess carriers at \( x = W \) are extracted, that is, \( p_n(W) = p_{\infty} \), then we obtain a new solution for Eq. 61:

\[
p_n(x) = p_{\infty} + [p_n(0) - p_{\infty}]
\left[
\frac{\sinh \left( \frac{W - x}{L_p} \right)}{\sinh (W/L_p)}
\right].
\]

The current density at \( x = W \) is given by the diffusion current expression, Eq. 32 with \( \varepsilon = 0 \):

\[
J_p = -q D_p \frac{\partial p_n}{\partial x} \bigg|_{x=W} = q \left[ p_n(0) - p_{\infty} \right] \frac{D_p}{L_p} \frac{1}{\sinh (W/L_p)}.
\]