Open and closed loop systems respond in different ways to disturbances and effects of unmodeled dynamics. In this exercise you will compare open and closed systems using Matlab and Simulink. The purpose of the exercise is to show you how having numerical tools allows a designer to quickly evaluate different design alternatives.

A remotely controlled vehicle has the transfer function:

\[ G(s) = \frac{1}{s^2 + 2s + 4} \]

We want to compare an open loop controller with a closed loop controller.

1. Design a proportional open loop controller:

\[ H(s) = K. \]

Figure 1 shows the diagram of the overall system. On the figure, \( R(s) \) is the input and \( D(s) \) is a disturbance.

(a) Plot the response of the system to a step input (assuming \( D(s) = 0 \)) for \( K = 1, 4, 10 \). How does a change in \( K \) affect the response? Which \( K \) would you choose to get a steady-state error equal to zero?

(b) Plot the response of the system to a unit step disturbance (assuming \( R(s) = 0 \)). Evaluate the steady-state error for a disturbance numerically (from the plot) and analytically. Does \( K \) affect the disturbance response of the system?

(c) What is the sensitivity of the open-loop transfer function to changes in \( K \)?

2. Now design a closed-loop controller:

\[ H(s) = \frac{K(s + 2)}{(s + 1)}. \]

The overall system is shown in Figure 2.
(a) Plot the response of the system to a step input (assuming $D(s) = 0$) for $K = 4.5, 10, 20$. Create a table and compare the three responses with respect to the:

- The time at which the response reaches its peak.
- The steady-state error.
- Percentage the system overshoots the steady-state value. Compute this value as $\frac{M_p - F_s}{F_s}$ where $M_p$ is the peak value and $F_s$ is the steady-state value.

Which $K$ would you choose for the system based on your table?

(b) Plot the response of the system to a unit step disturbance (assuming $R(s) = 0$) for the same values of $K$. In each case, determine the steady-state error of the system. Which $K$ gives the smallest error?

(c) Evaluate the sensitivity of the closed-loop transfer function to changes in $K$. Using the nominal value $K = 10$, plot a graph of the sensitivity function on the complex plane for frequencies $\omega = 0.01\,\text{rad/s}$ to $\omega = 100\,\text{rad/s}$.

*Hint:* Figure 4.31 in Dorf & Bishop shows you how to do this.

3. Based on your analysis, suggest the best design for the system.

Print all the plots and submit them with your report.