The Capacity of the Interference Channel with a Cognitive Relay in Strong Interference

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***Abstract*—The interference channel with a cognitive relay consists of a classical interference channel with two source-destination pairs and with an additional cognitive relay that has *a priori knowledge* of the sources' messages and aids in the sources' transmission. We derive a new outer bound for this channel using an argument originally devised for the "more capable" broadcast channel, and show the achievability of the proposed outer bound for a class of channels where there is no loss in optimality if both destinations decode both messages. This result is analogous to the "very strong interference" capacity result for the classical interference channel and for the cognitive interference channel, and is the first capacity known capacity result for the general interference channel with a cognitive relay.**

***Index Terms*—Interference channel with a cognitive relay; Capacity; Outer bound; Strong interference**

I. INTRODUCTION

Cognition is a rapidly emerging new paradigm in wireless communication whereby a node changes its communication scheme to efficiently share the spectrum with other users in the network. Cooperation among smart and well-connected wireless devices has been recognized as a key factor in improving the spectrum utilization and throughput of wireless networks [1]. The information theoretic study of cognitive networks has focused mostly on the cognitive interference channel, a variation of the classical interference channel where one of the transmitters has *perfect, a priori knowledge* of both the messages to be transmitted. Albeit idealistic, this form of *genie-aided* cognition has provided precious insights on the rate advantages that can be obtained with transmitter cooperation with one cognitive encoder. In this paper we study a natural extension of the cognitive interference channel where the genie-aided cognition, instead of being provided to only one of the users of the interference channel, is rather provided to a third node, a *cognitive relay*, that aids the communication between both source-destination pairs.

Past work. Few results are available for the InterFerence Channel with a Cognitive Relay (IFC-CR) and the fully general information theoretic capacity of this channel remains an open problem. The IFC-CR was initially considered in [2] where the first achievable rate region was proposed, and was improved upon in [3], which also provided a sum-rate outer bound for the Gaussian channel. This outer bound is based on an outer bound for the MIMO Gaussian cognitive interference channel and, in general, has no closed form expression. In [4] an achievable rate region was derived that contains all previously known achievable rate regions[1]. The first outer bounds for a general (i.e., not Gaussian) IFC-CR were derived in [5] by using the fact that the capacity region only depends on the conditional marginal distribution of the channel outputs. The authors of [5] first derived an outer bound valid for any IFC-CR and successively tighten the bound for a class of semi-deterministic channels in the spirit of [6], [7]. In [5], the tightened bound was also shown to be capacity for a the high-SNR binary linear deterministic approximation of the Gaussian channel, a model originally proposed in [8] for the classical IFC, for the case where the sources do not interfere at the non-intended destinations. In [9], with the insights gained from the high-SNR binary linear deterministic channel, the authors showed capacity to within 3 bits/sec/Hz for any finite SNR.

Contributions. In this paper we determine:

1) **a new outer bound for the IFC-CR** inspired by an argument originally devised for the "more capable" broadcast channel [10], also utilized in deriving the capacity of the cognitive interference channel in "weak interference" [11].
2) **a new outer bound in the "strong interference" regime at receiver 1 and/or 2** which is defined as the regime where-loosely speaking-the non-intended destination can decode more information than the intended destination.
3) **capacity for the "very strong interference" regime**, a regime obtained by adding an additional constraint to the "strong interference" regime at receiver 1 and/or 2. In this regime, capacity is achieved by a superposition scheme in which both decoders, without rate loss, decode both messages.

Paper Organization. The rest of the paper is organized as follows: in Section II we formally introduce the channel model. In Section III we present a new outer bound for the general channel and an outer bound for the "strong interference" regime. In Section IV we show the achievability of the "strong interference at receiver 1 and/or 2" outer bound in the "very strong interference" regime. Section V specializes the results of the paper to the Gaussian interference channel with a cognitive relay. Section VI concludes the paper.

[1]The authors of [4] refer to the IFC-CR as broadcast channel with cognitive relays, arguing that the model can also be obtained by adding two partially cognitive relays to a broadcast channel.

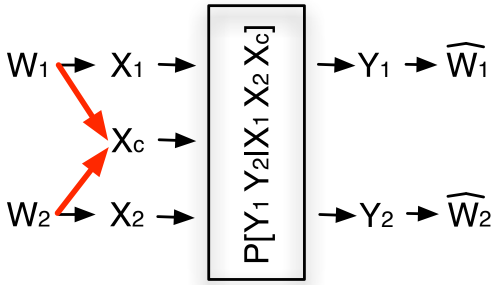

Fig. 1. The general InterFerence Channel with a Cognitive Relay (IFC-CR).

II. Channel Model

We consider the IFC-CR in which the transmission of the two independent messages $W_i \in [1 : 2^{NR_i}]$, $i \in \{1,2\}$, is aided by a single *cognitive* relay, whose input to the channel has subscript c. The memoryless channel has transition probability $P_{Y_1,Y_2|X_1,X_2,X_c}$. A rate pair is achievable if there exists a sequence of encoding functions

$$X_1^N = X_1^N(W_1),\ X_2^N = X_2^N(W_2),\ X_c^N = X_c^N(W_1,W_2),$$

and a sequence of decoding functions

$$\widehat{W}_1 = \widehat{W}_1(Y_1^N),\ \widehat{W}_2 = \widehat{W}_2(Y_2^N),$$

such that

$$\lim_{N\to\infty} \max_{i=1,2} \Pr\left[\widehat{W}_i \neq W_i\right] = 0.$$

The capacity region is defined as the closure of the region of all achievable (R_1,R_2)-pairs. The capacity of the general IFC-CR is open. The IFC-CR subsumes three well-studied channels as special cases: (a) InterFerence Channel (IFC): if $X_c = \emptyset$; (b) Broadcast Channel (BC): if $X_1 = X_2 = \emptyset$; and (c) Cognitive InterFerence channel (C-IFC): if $X_1 = \emptyset$ or $X_2 = \emptyset$.

III. Outer Bounds

The previously proposed outer bound for the general memoryless IFC-CR in [5, Th.3.1] equals capacity when the channel reduces to a Gaussian C-IFC in "weak interference" [11, Lem.3.6], in "very strong interference" [12, Th.6] and in "primary decodes cognitive" regimes [13, Th.3.1]. However, it does not reduce to the outer bound in [11, Th. 3.2], which is capacity for the C-IFC in the "very weak interference" regime [11, Th.3.4], and for the semi-deterministic C-IFC [14, Th.8.1]. For this reason we next derive a new outer bound inspired by the capacity of the "more capable" BC of [10] which does correspond to the outer bound of [11, Th.3.2] when the IFC-CR reduces to a C-IFC. We also derive a simple expression from this first outer bound for a specific class of channels: the "strong interference" regime, where one message is more favorably decoded at the non-intended receiver than at the intended receiver. This regime parallels the "strong interference" regime for the IFC [15] and the C-IFC [12, Th.6].

Remark 1. We note however that out notation is not entirely consistent with past uses of the term "strong interference". Here, as in our previous work on the C-IFC [14], [16], we use "strong interference" to denote regimes inspired by similar results in the IFC for which we may obtain either a tighter or simpler outer bound for the channel of interest, and use the terms "very strong interference" to denote regimes in which additional conditions, therefore forming subsets of the "strong interference" regimes, are imposed on top of the "strong interference" conditions for capacity.

Theorem 1. "More capable" BC-type outer bound. *If (R_1,R_2) lies in the capacity region of the IFC-CR, then the following must hold:*

$$\begin{aligned}
R_1 &\le I(Y_1;X_1,X_c|X_2,Q), && \text{(1a)}\\
R_1 &\le I(Y_1;U_2,X_1|Q), && \text{(1b)}\\
R_2 &\le I(Y_2;X_2,X_c|X_1,Q), && \text{(1c)}\\
R_2 &\le I(Y_2;U_1,X_2|Q), && \text{(1d)}\\
R_1{+}R_2 &\le I(Y_1;X_1,X_c|U_1,X_2,Q){+}I(Y_2;U_1,X_2|Q), && \text{(1e)}\\
R_1{+}R_2 &\le I(Y_2;X_2,X_c|U_2,X_1,Q){+}I(Y_1;U_2,X_1|Q), && \text{(1f)}\\
R_1{+}R_2 &\le I(Y_1;U_1|Q) + I(Y_2;U_2|Q), && \text{(1g)}\\
R_1{+}R_2 &\le I(Y_1;X_1,X_2,X_c|Q){+}I(Y_2;X_2,X_c|Y_1',X_1,Q), && \text{(1h)}\\
R_1{+}R_2 &\le I(Y_2;X_1,X_2,X_c|Q){+}I(Y_1;X_1,X_c|Y_2',X_2,Q), && \text{(1i)}
\end{aligned}$$

for some input distribution $P_{Q,X_1,X_2,X_c,U_1,U_2}$ that factors as:

$$P_Q P_{X_1|Q} P_{X_2|Q} P_{X_c|X_1,X_2,Q} P_{U_1,U_2|X_1,X_2,X_c,Q}, \quad (2)$$

for and Y_1' and Y_2' having the same marginal distributions as Y_1 and Y_2, respectively, but otherwise arbitrarily correlated.

Proof: The bounds in (1a) and (1c), as well as the sum-rate bounds in (1h) and (1i), were originally derived in [5, Th. 3.1]. The bound in (1d) is obtained as follows:

$$\begin{aligned}
N(R_2-\epsilon_N) &\le I(Y_2^N;W_2)\\
&\overset{(a)}{\le} \sum_{i=1}^N H(Y_{2,i}|Y_{2,i+1}^N) - H(Y_{2,i}|Y_{2,i+1}^N,W_2,X_2^N,Y_1^{i-1})\\
&\overset{(b)}{\le} \sum_{i=1}^N I(Y_{2,i};U_{i,1},X_{2,i}),
\end{aligned}$$

where (a) follows from the "conditioning reduces entropy" [17] property and (b) from defining:

$$U_{1,i} = [Y_1^{i-1}, W_2, X_2^{i-1}, X_{2,i+1}^N, Y_{2,i+1}^N], \quad (3)$$

and letting $X^0 = X^{N+1} = \emptyset$. The bound in (1d) is obtained by introducing a time-sharing Random Variable (RV) Q uniformly distributed on the interval $[1:N]$ and independent of everything else. For the sum-rate bound in (1e):

$$\begin{aligned}
N(R_1+R_2-2\epsilon_N) &\le I(Y_1^N;W_1|W_2) + I(Y_2^N;W_2)\\
&\le \sum_{i=1}^N I(Y_{1,i};W_1,Y_{2,i+1}^N|Y_1^{i-1},W_2,X_2^N)\\
&\quad + I(Y_{2,i};W_2,X_2^N,Y_{2,i+1}^N)\\
&\le \sum_{i=1}^N I(Y_{1,i};Y_{2,i+1}^N|Y_1^{i-1},W_2,X_2^N)\\
&\quad - I(Y_{2,i};Y_1^{i-1}|W_2,X_2^N,Y_{2,i+1}^N)
\end{aligned}$$

$$+ I(Y_{1,i}; W_1 | Y_1^{i-1}, W_2, X_2^N, Y_{2,i+1}^N)$$
$$+ I(Y_{2,i}; W_2, X_2^N, Y_{2,i+1}^N, Y_1^{i-1})$$
$$\overset{(c)}{=} \sum_{i=1}^N I(Y_{1,i}; W_1 | U_{i,1}, X_{2,i}) + I(Y_{2,i}; U_{i,1}, X_{2,i})$$
$$\leq \sum_{i=1}^N I(Y_{1,i}; X_{1i}, X_{ci} | U_{1,i}, X_{2i}) + I(Y_{2,i}; U_{1,i}, X_{2i}),$$

where (c) follows from Csiszár's sum identity [18] and the definition of $U_{1,i}$ in (3). The bounds in (1b) and (1f) are obtained similarly to the bounds in (1d) and (1e), respectively, by swapping the role of the sources and by defining:

$$U_{2,i} = [Y_2^{i-1}, W_1, X_1^{i-1}, X_{1,i+1}^N, Y_{1,i+1}^N]. \quad (4)$$

Finally, the bound in (1g) is obtained as follows:

$$N(R_1 + R_2 - 2\epsilon) \leq I(Y_1^N; W_1) + I(Y_2^N; W_2)$$
$$\leq \sum_{i=1}^N H(Y_{1,i}) + H(Y_{2,i})$$
$$- H(Y_{1,i} | Y_2^{i-1}, Y_{1,i+1}^N, X_1^{i-1}, X_{1,i+1}^N, W_1)$$
$$- H(Y_{2,i} | Y_1^{i-1}, Y_{2,i+1}^N, X_2^{i-1}, X_{2,i+1}^N, W_2)$$
$$= \sum_{i=1}^N H(Y_{1,i}) - H(Y_{1,i} | U_{2,i}) + H(Y_{2,i}) - H(Y_{2,i} | U_{1,i}).$$

■

Remark 2. Th. 1 is the tightest known outer bound for a general IFC-CR and it reduces to the capacity region of the "more capable" BC when $X_1 = X_2 = \emptyset$ in which case (1b) and (1e) are tight. Th. 1 also reduces to the outer bound of [11, Th. 3.2] when either $X_2 = \emptyset$ or $X_1 = \emptyset$ in which case (1b), (1d) and (1e) are tight. However, Th. 1 does not reduce to the capacity region of the class of deterministic IFCs studied in [8] and to the outer bound for the semi-deterministic IFC in [7] when $X_c = \emptyset$. The difficulty in deriving outer bounds for the IFC-CR that are tight when the IFC-CR reduces to an IFC is also noted in [5]. The authors of [5, Th. 3.2] are able to derive tight bounds in this scenario by imposing additional constraints on the effect of interference on the channel outputs.

Theorem 2. "Strong interference at Rx 1" outer bound. *If*

$$I(Y_2; X_2, X_c | X_1) \leq I(Y_1; X_2, X_c | X_1) \quad (5)$$

for all distributions

$$P_{X_1,X_2,X_c} = P_{X_1} P_{X_2} P_{X_c|X_1,X_2}, \quad (6)$$

then, if (R_1, R_2) *lies in the capacity region of the IFC-CR, the following must hold:*

$$R_1 \leq I(Y_1; X_1, X_c | X_2, Q), \quad (7a)$$
$$R_2 \leq I(Y_2; X_2, X_c | X_1, Q), \quad (7b)$$
$$R_1 + R_2 \leq I(Y_1; X_1, X_2, X_c | Q), \quad (7c)$$

for some distribution

$$P_{Q,X_1,X_2,X_c} = P_Q P_{X_1|Q} P_{X_2|Q} P_{X_c|X_1,X_2,Q}. \quad (8)$$

Proof: Since

$$I(Y_1; X_2, X_c | X_1, U) = \sum_u P(u) I(Y_1; X_2, X_c | X_1, U = u)$$
$$\geq \sum_u P(u) I(Y_2; X_2, X_c | X_1, U = u) = I(Y_2; X_2, X_c | X_1, U)$$

we see that $I(Y_2; X_2, X_c | X_1, U) \leq I(Y_1; X_2, X_c | X_1, U)$, for all $P_{X_1,X_2,X_c,U} = P_{X_1} P_{X_2} P_{X_c|X_1,X_2} P_{U|X_1,X_2,X_c}$. From this, it follows that when condition (5) holds, we can upper bound the bound in (1f) as:

$$I(Y_1; U_2, X_1 | Q) + I(Y_2; X_2, X_c | X_1, U_2, Q)$$
$$\leq I(Y_1; U_2, X_1 | Q) + I(Y_1; X_2, X_c | X_1, U_2, Q)$$
$$\leq I(Y_1; X_1, X_2, X_c, U_2 | Q) = I(Y_1; X_1, X_2, X_c | Q),$$

where the last equality follows from the Markov chain $Y_1 - (X_1, X_2, X_c) - U_2$ which is readily established by using the memoryless property of the channel. We drop all remaining bounds involving the auxiliary random variable U, which loosens the outer bound. ■

Remark 3. Given the symmetry of the channel model, Th. 2 holds when the role of the sources is reversed, which we then term the "strong interference at Rx 2" outer bound. Although not valid for a general IFC-CR, Th. 2 is expressed only as a function of the channel inputs and does not contain auxiliary RVs as in Th. 1.

Remark 4. When condition (5) holds, it also implies:

$$0 \leq I(Y_2; X_2, X_c | X_1, Y_1') \leq I(Y_1; X_2, X_c | X_1, Y_1') = 0 \quad (9)$$

since Y_1' must have the same conditional marginal distribution of Y_1 but can otherwise be arbitrarily correlated with the channel outputs; we can thus choose $Y_1 = Y_1'$ and such that Y_2 is a degraded version of Y_1' conditioned on X_1. Given (9), sum rate bound (1h) coincides with (7c). The bound (7c) is derived in [5] using the fact that the capacity region does not depend on the conditional joint distribution of the channel outputs but only on their conditional marginal distributions. As for the C-IFC of [14], the sum rate bound derived using Csiszár's sum identity coincides with the bound derived using Sato's idea in the "strong interference" regime.

IV. Capacity in "very strong interference at Rx 1"

In this section we show the achievability of the outer bound of Th. 2 in the "very strong interference at Rx 1" regime (to be defined later), which is a subset of the "strong interference at Rx 1" regime defined by (5). This result parallels the capacity results under "strong interference at RX 1" for the IFC [15] and the C-IFC [12], where the channel reduces to a compound two-user multiple access channel. [2] For this class of channels the interfering signal at each receiver can be decoded without imposing any additional rate penalty and thus successively stripped from the received signal. Since the interference can

[2]We again note that our terminology of "strong" and "very strong" does not exactly correspond to that for IFC and C-IFC channels besides the authors' previous work. We use "very strong" to denote that we need to satisfy the "strong" conditions as well as additional constraints.

always be distinguished from the intended signal, there is no need to perform interference pre-coding at the cognitive relay. This greatly simplifies the achievable scheme required to match the outer bound in Th. 2. We will show in fact that a simple superposition coding schemes achieves Th. 2.

Theorem 3. Capacity in "very strong interference at Rx 1". *If* (5) *holds together with*

$$I(Y_1;X_1,X_2,X_c) \le I(Y_2;X_1,X_2,X_c) \tag{10}$$

for all distributions in (6)*, then the region in* (7) *is capacity.*

Proof: Under the assumption of the theorem, the region in (7) is an outer bound for the considered IFC-CR. The achievability of the outer bound the region in (7) can be shown by considering a transmission scheme that employs two common messages, U_{1c}, U_{2c} for source 1 and source 2, respectively, that are encoded in the channel inputs according to the distributions $P_{X_1|U_{1c}}$, $P_{X_2|U_{2c}}$ and $P_{X_c|U_{1c},U_{2c}}$. This scheme achieves the region:

$$\begin{aligned} R_1 &\le I(Y_1;U_{1c}|U_{2c},Q), &(11a)\\ R_2 &\le I(Y_2;U_{2c}|U_{1c},Q), &(11b)\\ R_1+R_2 &\le I(Y_1;U_{1c},U_{2c}|Q), &(11c)\\ R_1+R_2 &\le I(Y_2;U_{1c},U_{2c}|Q), &(11d)\end{aligned}$$

for some input distribution that factors as:

$$P_Q P_{U_{1c},X_1|Q} P_{U_{2c},X_2|Q} P_{X_c|U_{1c},U_{2c},X_1,X_2,Q}, \tag{12}$$

where Q is a time-sharing random variable defined as in Th. 1. Let now $U_{1c}=X_1$, $U_{2c}=X_2$ and X_c be a deterministic function of X_1, X_2. Under the condition in (10) the bound in (11d) can be dropped from the region in (11) and the resulting region coincides with the one in (7). ■

Following the proof of Th. 3 one can also show:

Theorem 4. Capacity in "strong interference at Rx 1 and at Rx 2". *If the condition in* (5) *holds for all distributions in* (6) *together with the equivalent of* (5) *with the role of the users swapped, then the region in* (11) *with* $U_{1c}=X_1$ *and* $U_{2c}=X_2$ *is capacity.*

V. The Gaussian Case

In the following we evaluate Th. 2 and Th. 3 for the Gaussian IFC-CR. Without loss of generality we restrict our attention to the Gaussian IFC-CR in *standard form* given by:

$$\begin{aligned} Y_1 &= |h_{11}|X_1+|h_{1c}|X_c+h_{12}X_2+Z_1, &(13a)\\ Y_2 &= h_{21}X_1+|h_{2c}|X_c+|h_{22}|X_2+Z_2, &(13b)\end{aligned}$$

where $h_i \in \mathbb{C}$, $i\in\{11,1c,12,22,2c,21\}$, are constant and known to all terminals, $Z_i \sim \mathcal{N}_{\mathbb{C}}(0,1)$, $i\in\{1,2\}$, and $\mathbb{E}[|X_i|^2]\le 1$, $i\in\{1,2,c\}$. The channel links $h_i, i\in\{11,22,1c,2c\}$ can be taken to be real-valued without loss of generality because receivers and transmitters can compensate for the phase of the signals. The correlation among the noises is irrelevant because the capacity of the channel without receiver cooperation only depends on the noise marginal distributions.

Theorem 5. The "strong interference at Rx 1" outer bound for the Gaussian IFC-CR. *If*

$$\left| |h_{22}| + \widetilde{\beta}_2^* \ |h_{2c}| \right|^2 \le \left| h_{12} + \widetilde{\beta}_2^* \ |h_{1c}| \right|^2 \tag{14}$$

for

$$\measuredangle\widetilde{\beta}_2 = \measuredangle\big(|h_{22}||h_{2c}| - h_{12}|h_{1c}|\big), \tag{15a}$$

$$|\widetilde{\beta}_2|^2 = \begin{cases} 1 & \text{if } |h_{2c}|\ge|h_{1c}| \\ \min\left\{1, \frac{\left||h_{2c}||h_{22}|-|h_{1c}|h_{12}\right|}{\left||h_{2c}|^2-|h_{1c}|^2\right|}\right\} & \text{if } |h_{2c}|<|h_{1c}| \end{cases} \tag{15b}$$

the capacity of a Gaussian IFC-CR is contained in the set:

$$\begin{aligned} R_1 &\le \mathcal{C}\left(\left||h_{11}|+|h_{1c}|\beta_1\right|^2\right), &(16a)\\ R_2 &\le \mathcal{C}\left(\left||h_{22}|+|h_{2c}|\beta_2\right|^2\right), &(16b)\\ R_1+R_2 &\le \mathcal{C}\left(\left||h_{11}|+|h_{1c}|\beta_1\right|^2 + \left|h_{12}+|h_{1c}|\beta_2\right|^2\right), &(16c)\end{aligned}$$

taken over the union of all $(\beta_1,\beta_2): |\beta_1|^2+|\beta_2|^2\le 1$.

Proof: Given the "Gaussian maximizes entropy" property [17] we have that the union over all the distributions in (8) of the region in (7) is equal to the union over all the zero-mean complex-valued proper Gaussian random vectors $[X_1,X_2,X_c]$ with covariance matrix

$$\mathrm{Cov}(X_1,X_2,X_c) = \begin{bmatrix} 1 & 0 & \beta_1 \\ 0 & 1 & \beta_2 \\ \beta_1^* & \beta_2^* & 1 \end{bmatrix} \tag{17}$$

for some $|\beta_1|^2+|\beta_2|^2\le 1$. With this factorization we can rewrite the condition in (5) as

$$\max_{|\beta_1|^2+|\beta_2|^2\le 1} \left||h_{22}|+\beta_2^*|h_{2c}|\right|^2 - \left|h_{12}+\beta_2^*|h_{1c}|\right|^2 + \left(|h_{2c}|^2-|h_{1c}|^2\right)\left(1-|\beta_1|^2-|\beta_2|^2\right)\le 0$$

The solution of this maximization problem is (15). ■

Theorem 6. Capacity in "very strong interference at RX 1" for the Gaussian IFC-CR.

If in addition to condition (14) *the following also holds:*

$$(|h_{11}|^2+|h_{1c}|^2+|h_{12}|^2) - (|h_{21}|^2+|h_{2c}|^2+|h_{22}|^2) + 2\sqrt{\left||h_{11}||h_{1c}|-h_{21}|h_{2c}|\right|^2 + \left|h_{12}|h_{1c}|-|h_{22}||h_{2c}|\right|^2} \le 0 \tag{18}$$

the region of (16) *is capacity.*

Proof: With the factorization in (17) the condition in (10) can be rewritten as

$$(|h_{11}|^2+|h_{1c}|^2+|h_{12}|^2) - (|h_{21}|^2+|h_{2c}|^2+|h_{22}|^2) + \max_{|\beta_1|^2+|\beta_2|^2\le 1} 2\mathrm{Re}\left\{\beta_1\ \left(|h_{1c}||h_{11}|-|h_{2c}|h_{21}\right)+\beta_2\left(|h_{1c}|h_{12}-|h_{2c}||h_{22}|\right)\right\}\le 0$$

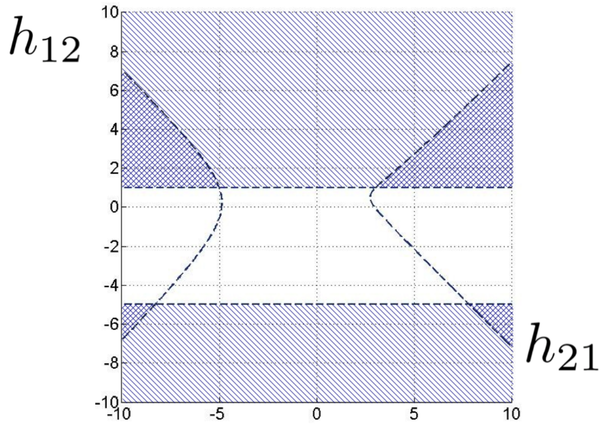

Fig. 2. The "strong interference at Rx 1" regime of Th. 5 (light grey) and the "very strong interference at Rx 1" regime of Th. 6 (dark grey) for the real-valued Gaussian IFC-CR with $|h_{11}| = |h_{22}| = |h_d| = 1$ and $|h_{1c}| = |h_{2c}| = |h_c| = 2$ in the plane $(h_{12}, h_{21}) \in \mathbb{R}^2$.

The solution of this maximization problem is (18). ■

Remark 5. When the IFC-CR reduces to an IFC, i.e., $h_{1c} = h_{2c} = 0$, the condition in (14) reduces to the well-known "strong interference at Rx 1" $|h_{22}|^2 \leq |h_{12}|^2$, and the condition in (18) to $|h_{11}|^2 + |h_{12}|^2 \leq |h_{21}|^2 + |h_{22}|^2$ (larger total received power at Rx 2 than at Rx 1).

When the IFC-CR reduces to a C-IFC with user 1 as primary user, i.e., $h_{22} = h_{12} = 0$, the condition in (14) reduces to $|h_{2c}|^2 \leq |h_{1c}|^2$ (strong interference at the primary receiver) and the condition in (18) to

$$|h_{11}|^2 + |h_{1c}|^2 - |h_{21}|^2 - |h_{2c}|^2 + 2\big||h_{11}||h_{1c}| - h_{21}|h_{2c}|\big| \leq 0,$$

which is the same as the condition in [19, Th.II.3].

When the IFC-CR reduces to a C-IFC with user 2 as primary user, i.e., $h_{11} = h_{21} = 0$, the conditions in (14) in (18) are equivalent to $I(Y_1; X_2, X_c) = I(Y_2; X_2, X_c)$ for all input distributions, that is,

$$|h_{1c}|^2 + |h_{12}|^2 = |h_{2c}|^2 + |h_{22}|^2, \quad h_{12}|h_{1c}| = |h_{22}||h_{2c}|.$$

When the IFC-CR reduces to a BC. i.e., $h_{11} = h_{21} = h_{22} = h_{12} = 0$ the conditions in (14) and in (18) are equivalent to $I(Y_1; X_c) = I(Y_2; X_c)$ for all input distributions, that is, a BC with statistically equivalent Rx's, i.e., $|h_{2c}|^2 = |h_{1c}|^2$.

A representation of the "strong interference at Rx 1" condition of Th. 5 and the "very strong interference at Rx 1" condition of Th. 6 for the real-valued Gaussian IFC-CR in the plane $(h_{12}, h_{21}) \in \mathbb{R}^2$ is shown in Fig. 2 for $|h_{1c}| = |h_{2c}| = |h_c|$ and $|h_{11}| = |h_{22}| = |h_d|$.

VI. Conclusion and Future Work

We introduce a new outer bound for the interference channel with a cognitive relay and show the achievability of this outer bound in the "very strong interference" regime by having both decoders decode both messages as in a compound multiple access channel. Although significant, the contributions of this paper are only the first step to a better understanding of the capacity region of the cognitive interference channel with a cognitive relay which remains largely undiscovered.

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