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Lattice strategies for a multi-pair bi-directional relay network

Sang Joon Kim, Besma Smida, Natasha Devroye

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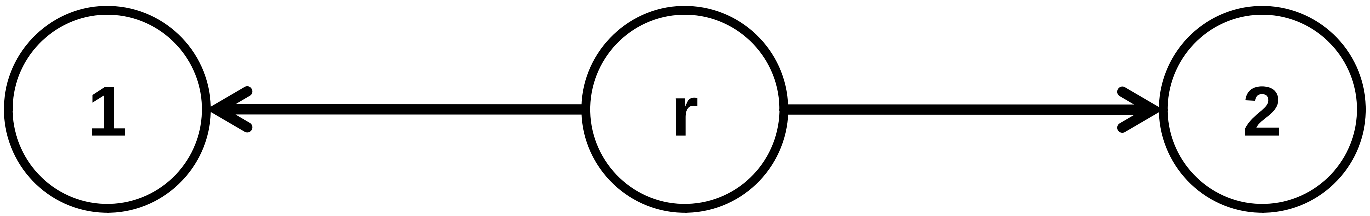
creation+

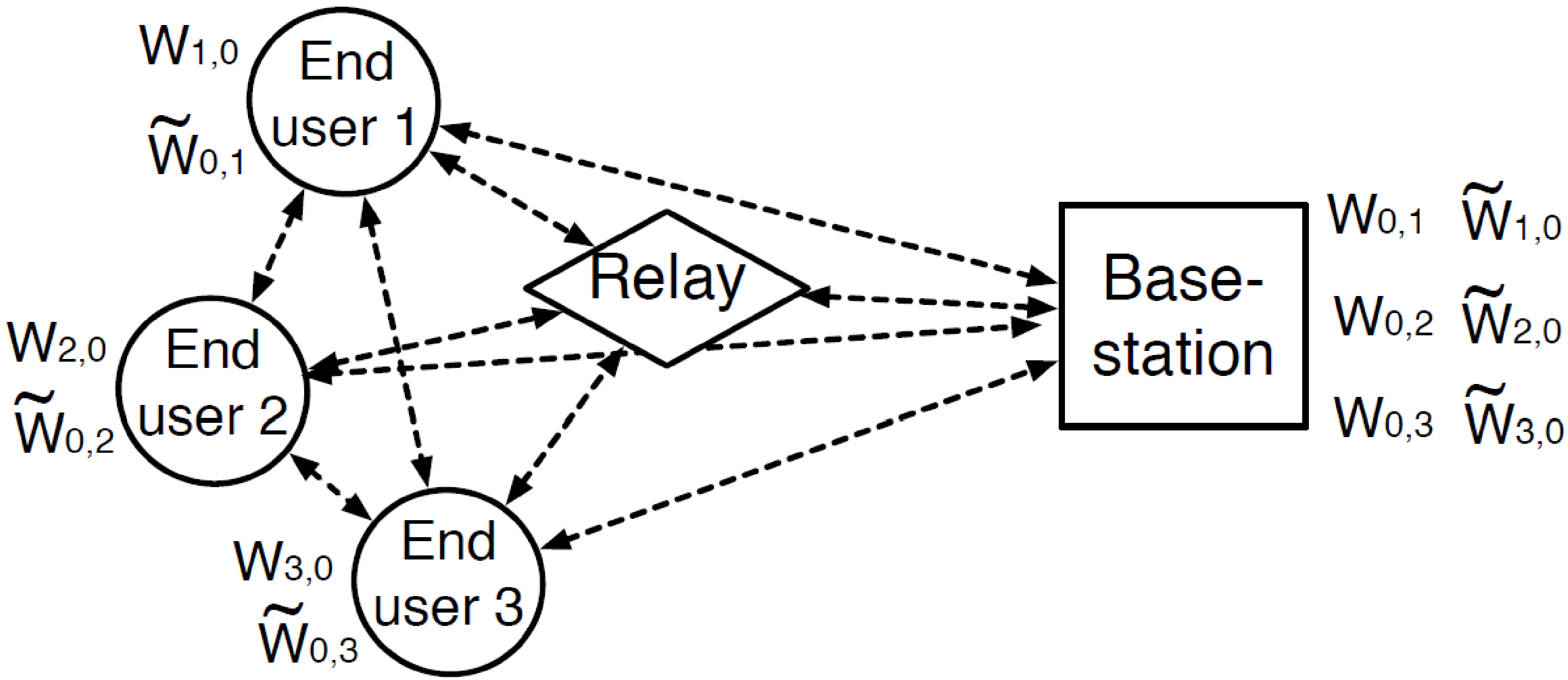
Channel model



- **Half-duplex AWGN channel**

$$Y_i[k] = \sum_{j \neq i} h_{ji} X_j[k] + Z_i[k]$$

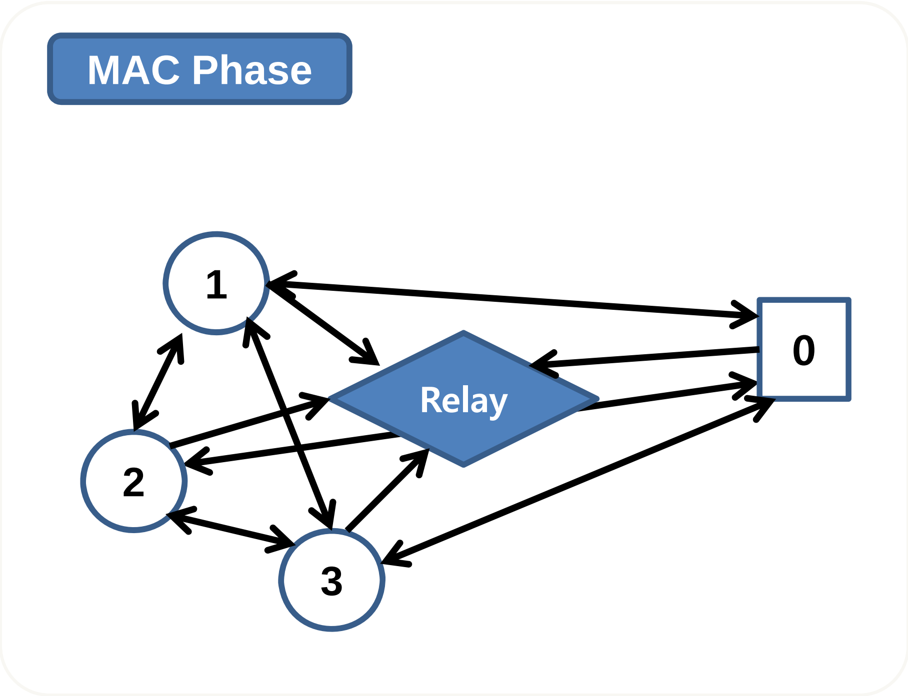
Channel model

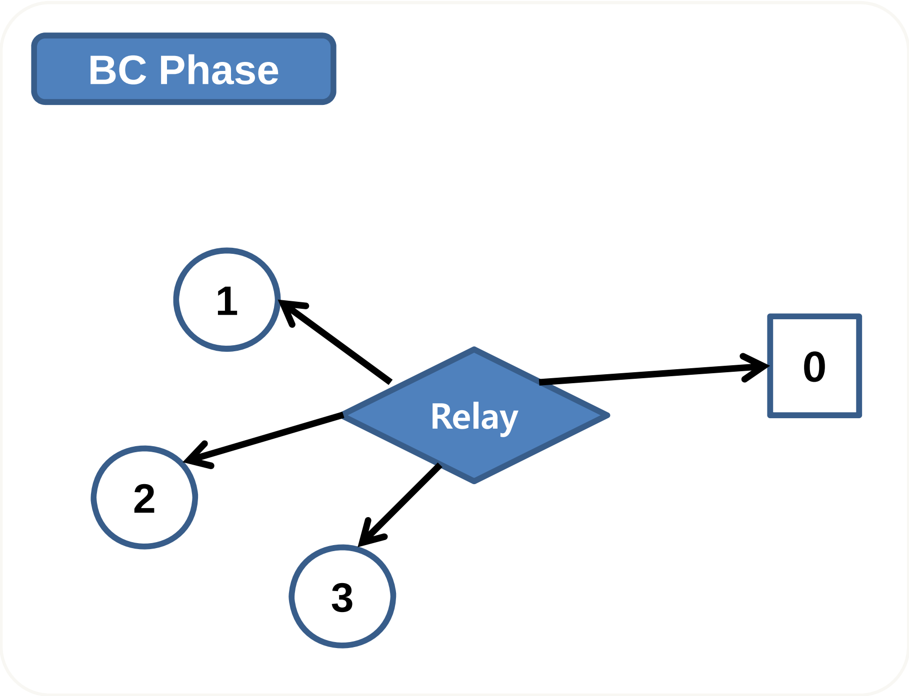


- One base station (**0**), "m" terminal nodes (**{1,2,…,m}**), and one relay node (**r**)
- 2m messages between base station and "m" terminal nodes, $\{W_{0,i}\}$'s and $\{W_{i,0}\}$'s where $i \in \{1,2,\ldots,m\}$.

Protocols

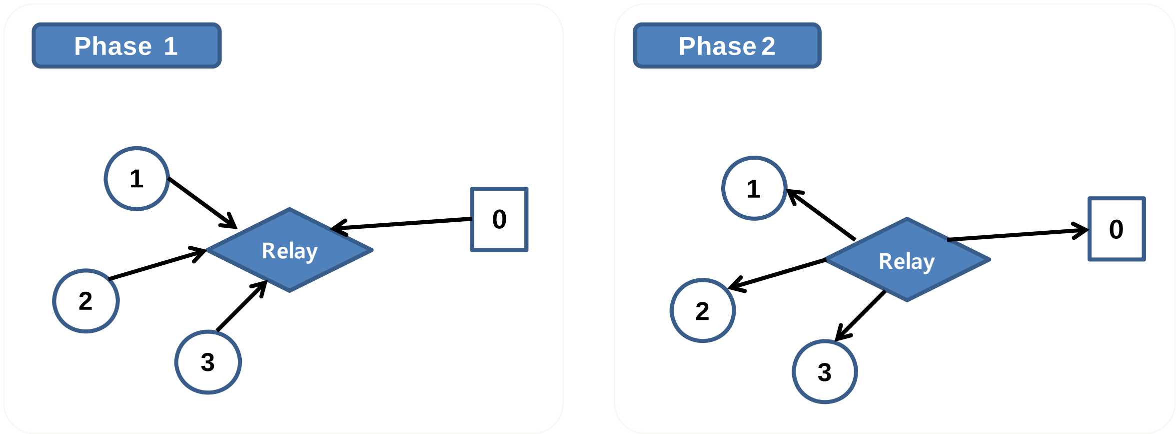
Half-duplex → time "phases" = protocols

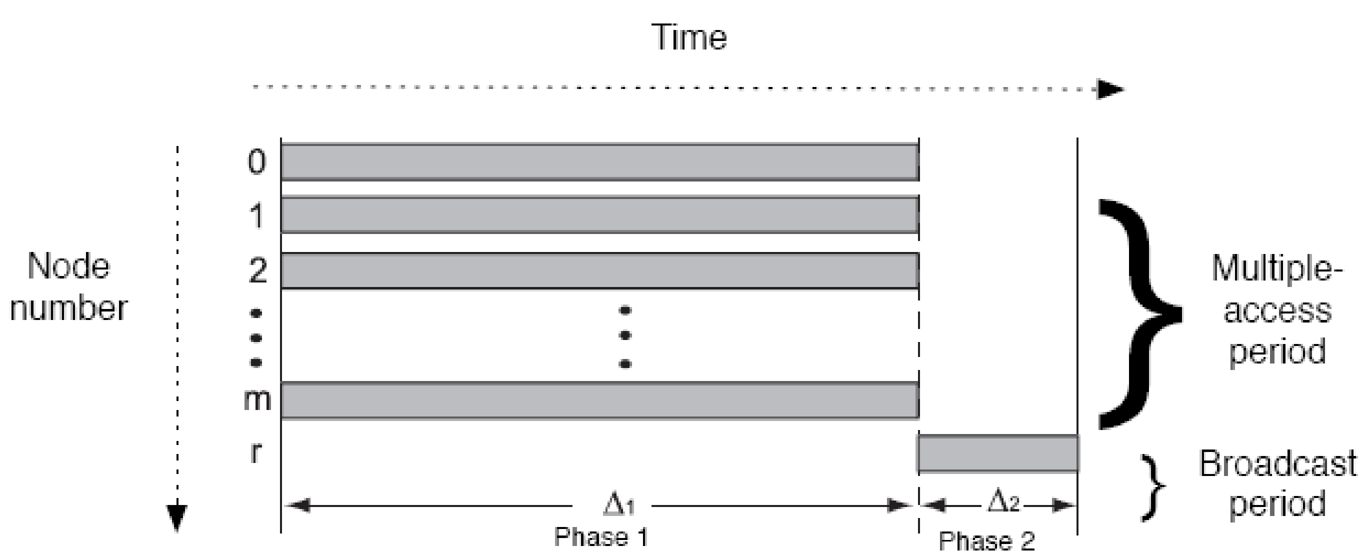




Protocols

I. FMABC protocol (Full Multiple Access Broadcast)

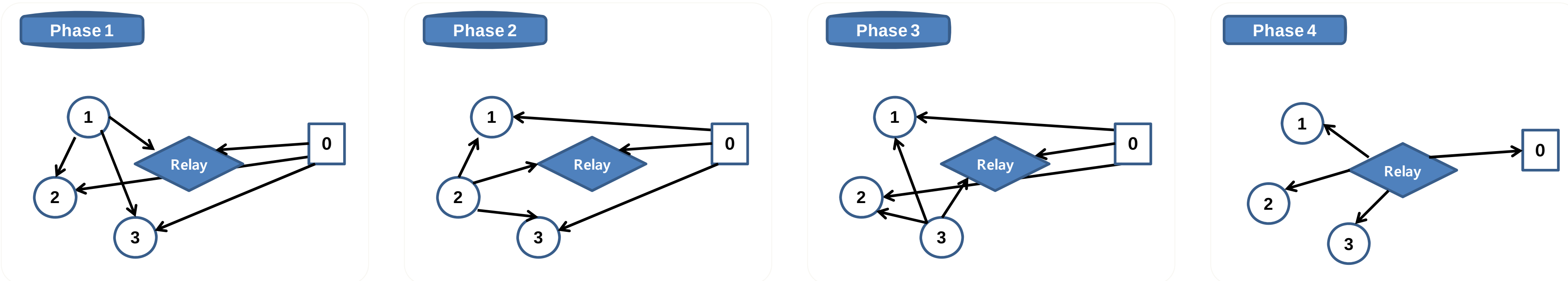


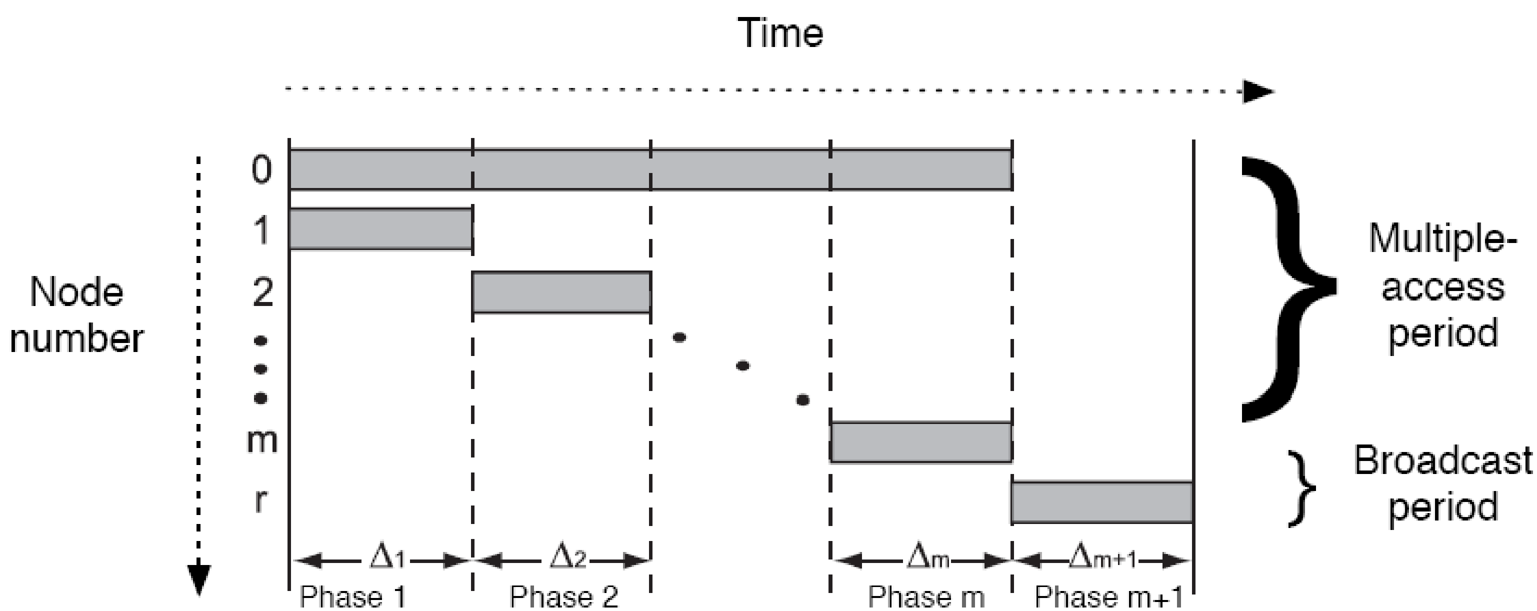


(a) FMABC protocol

Protocols

II. PMABC protocol (Partial Multiple Access Broadcast)

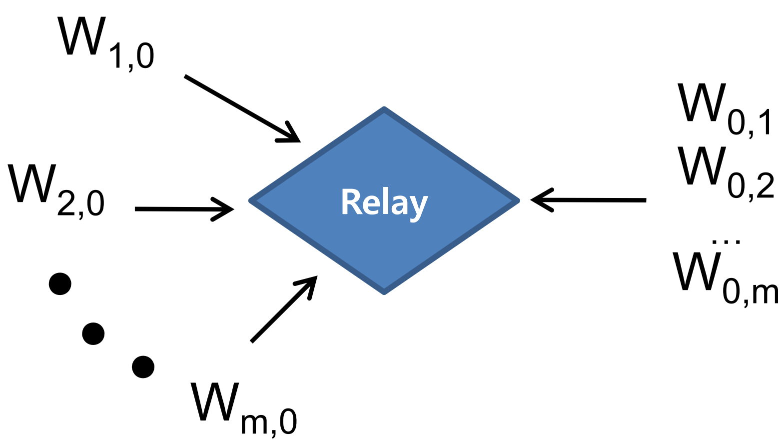




(b) PMABC protocol

Relaying scheme

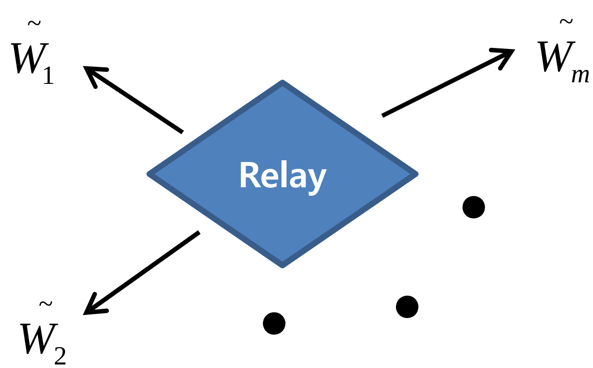
- **Message sum Decode and forward using Lattice structures**



Decode message sum w/ lattice structures (this work)

$$\tilde{W}_1 = W_{1,0} + W_{0,1}$$
$$\tilde{W}_2 = W_{2,0} + W_{0,2}$$
$$\vdots$$
$$\tilde{W}_m = W_{m,0} + W_{0,m}$$

- **Broadcast "m" messages w/ side information**



Marton's coding with side information at receivers ([kim, Smida, Devroye 2010])

- Node 0 decodes all $\{W_{i,0}\}$'s
- Node i decodes $W_{0,i}$ only

Coding scheme (Lattice code)

- **Nested lattice chain**

$$\Lambda \subseteq \Lambda_C, \sigma^2(\Lambda) = P$$

$$R = \frac{1}{n}\log\left(\frac{|\Lambda|}{|\Lambda_C|}\right)$$

$$R \le \log\left(\frac{P_C}{P_{\hat{z}}}\right)$$

When α = 1 $\quad R \le \log\left(\frac{P}{N}\right)$

When $\alpha = \frac{P}{P+N}$ $\quad R \le \log\left(1 + \frac{P}{N}\right)$

- **Dithering**
  - "u" is uniformly distributed in Λ
  - Both TX and RX know "u"

$$x = (c - u) \bmod \Lambda$$

- **Inflated lattice decoding**
  - partially remove dither to minimize effective noise

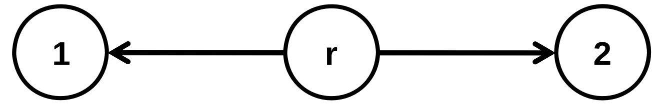
$$\hat{y} = (\alpha y + u) \bmod \Lambda$$
$$= (c + \hat{z}) \bmod \Lambda$$

$$\hat{z} = (1 - \alpha)x + \alpha z$$

U. Erez and R. Zamir, "Achieving 1/2 log(1+snr) on the awgn channel with lattice encoding and decoding," IEEE Trans. Inform. Theory, vol. IT-48, no. 6, pp. 1250–1276, 2004.

Related work

Two-way lattice coding



- **Encoding**

$$x_1 = (c_1 - u_1) \bmod \Lambda_1$$
$$x_2 = (c_2 - u_2) \bmod \Lambda_2$$

- **Decoding**

$$y_r = x_1 + x_2 + z_r$$
$$y_{r1} = (\alpha y_r + (u_1 + u_2)) \bmod \Lambda_1$$
$$= (c + z_{r1}) \bmod \Lambda_1$$
$$c = Q_C(y_{r1})$$

Euclidian lattice decoding

- **Variables and conditions**

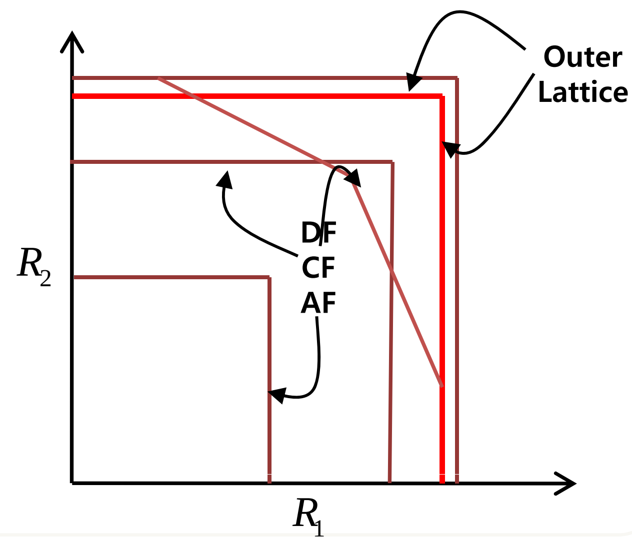
$$\Lambda_1 \subseteq \Lambda_2 \subseteq \Lambda_C, \sigma^2(\Lambda_i) = P_i$$
$$C_i = \{\Lambda_C \bmod \Lambda_i\} \quad u_i : dither$$
$$c = c_1 + c_2$$
$$z_{r1} = -(1-\alpha)(x_1 + x_2) + \alpha z_r$$

Q_C : Lattice quantizer in Λ_C

- **Bounds**

	Achievability	Outer bound
R_1	$\left[\log_2\left(\frac{P_1}{P_1+P_2} + \frac{P_1}{N_r}\right)\right]^+$	$\log_2\left(1 + \frac{P_1}{N_r}\right)$
R_2	$\left[\log_2\left(\frac{P_2}{P_1+P_2} + \frac{P_1}{N_r}\right)\right]^+$	$\log_2\left(1 + \frac{P_2}{N_r}\right)$

Const. gap



** W. Nam, S. Y. Chung, and Y. Lee, "Capacity of the Gaussian Two-Way Relay Channel to Within 1/2 Bit," *IEEE Trans. Inform. Theory, vol. 56,* no. 11, pp. 5488–5494, Nov. 2010.

Related work

Multi-pair Two-way lattice coding (successive decoding)

- **Encoding**

$$x_i = (c_i - u_i) \bmod \Lambda_i$$

- **Decoding**

$$y_r = \sum_{i=1}^{2M} x_i + z_r$$

$$y_{r1} = (\alpha_1 y_r + (u_1 + u_2)) \bmod \Lambda_1$$

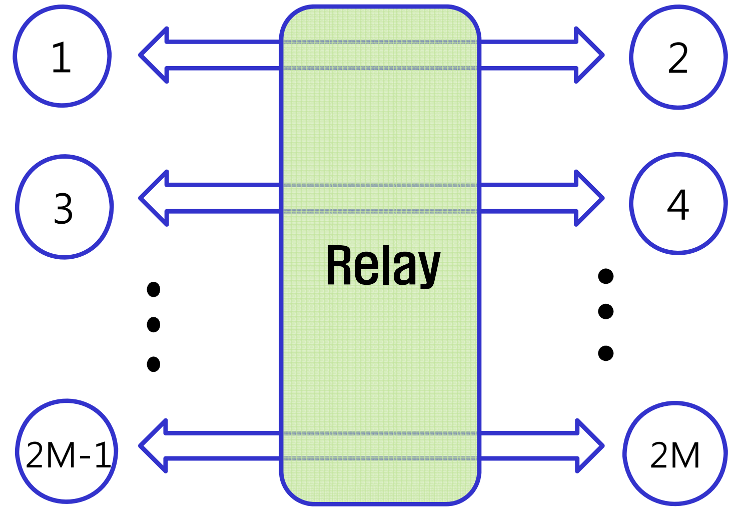
$$= (c_1^{sum} + z_{r1}) \bmod \Lambda_1$$

$$c_1^{sum} = Q_C(y_{r1})$$

$$x_1 + x_2 = (c_1^{sum} - (u_1 + u_2)) \bmod \Lambda_1$$

$$y_{r2} = (\alpha_1 (y_r - (x_1 + x_2)) + (u_3 + u_4)) \bmod \Lambda_1$$

Successive decoding



- **Limitation**

$$x_1 + x_2 = (c_1 - u_1) \bmod \Lambda_1 + (c_2 - u_2) \bmod \Lambda_2$$

$$\neq (c_1 + c_2 - u_1 - u_2) \bmod \Lambda_1$$

$$= (c_1^{sum} - (u_1 + u_2)) \bmod \Lambda_1$$

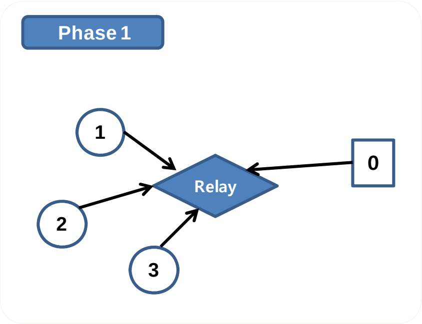
** cannot decode x_1+x_2 from c_1^{sum} if the channel is not symmetric

** D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, "The multi-way relay channel," in Proc. IEEE Int. Symp. Inform. Theory, Seoul, Jul. 2009, pp.339–343.

Coding scheme

FMABC protocol

$$y_r = x_{01} + x_{02} + x_{03} + x_{10} + x_{20} + x_{30} + z_r$$



Relay decodes :

$$\tilde{t}_1 = \tilde{c}_1$$
$$\tilde{t}_2 = \tilde{c}_1 + \tilde{c}_2$$
$$\tilde{t}_3 = \tilde{c}_1 + \tilde{c}_2 + \tilde{c}_3$$

reconstruction

$$\tilde{c}_1 = \tilde{t}_1$$
$$\tilde{c}_2 = \tilde{t}_2 - \tilde{t}_1$$
$$\tilde{c}_3 = \tilde{t}_3 - \tilde{t}_2$$

**$c_i = c_{0i} + c_{i0}$: pair-wise lattice sum

Coding scheme

FMABC protocol

- **Encoding**

$$x_{0i} = (c_{0i} - u_{0i}) \bmod \Lambda_{0i}$$
$$x_0 = x_{01} + x_{02} + \cdots + x_{0m}$$
$$x_{i0} = (c_{i0} - u_{i0}) \bmod \Lambda_{i0}$$

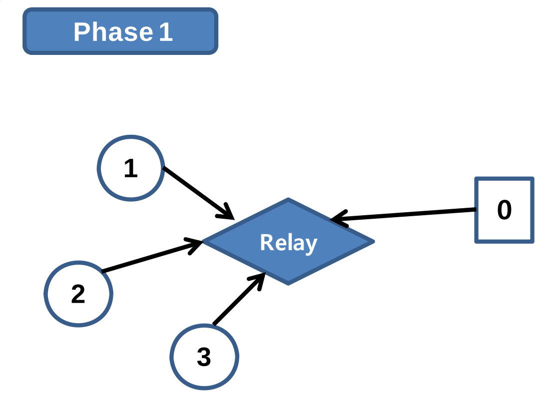
- **Decoding**

$$y_r = \sum_{i=1}^{M} (x_{0i} + x_{i0}) + z_r$$

for $1 \le i \le l$

$$y_{r,i} = (\alpha_i y_r + \sum_{j=1}^{M} g_{ji}(u_{0j} + u_{j0})) \bmod \Lambda_{max}$$
$$= (t_i + z_{r,i}) \bmod \Lambda_{max}$$
$$t_i = Q_C(y_{r,i})$$

$$c = tH \bmod \Lambda_{max}$$



- **Two step decoding: Linear combination**

$$t = cG \bmod \Lambda_{max} \qquad G \in \{0,1\}^{m \times l}, H \in Z^{l \times m}$$
$$c = tH \bmod \Lambda_{max} \qquad GH = I$$

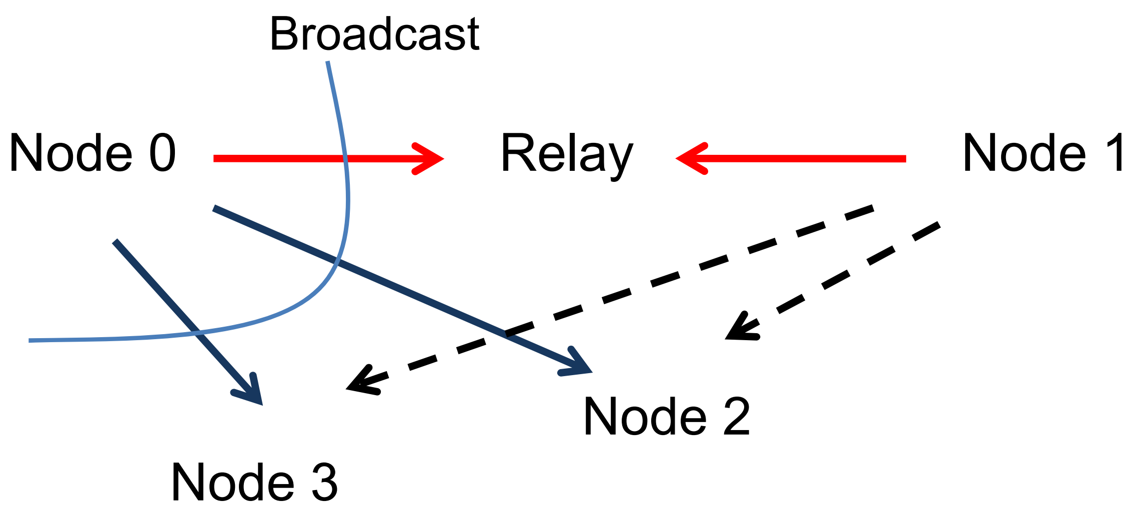
** decode "t" first by linear combination of lattice sum
→ extract "c" from G and H

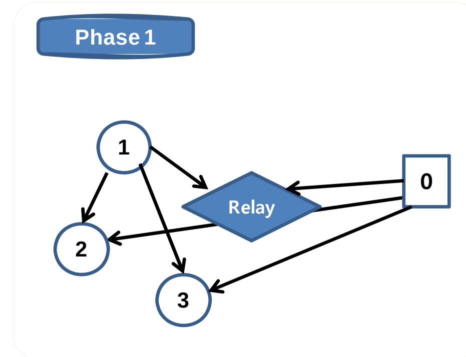
- **Example of G,H**

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lattice code

PMABC protocol





At phase i

- Lattice encoding for the i^{th} message pair
- Broadcasting $\{W_{0,i}\}$'s
- Using direct link side information

Achievable rate region

FMABC protocol

Theorem 2: An achievable rate region of the half-duplex bi-directional Gaussian relay channel under the FMABC protocol is the convex hull of the set of all points $(R_{0,b}, R_{b,0})$ for all $b \in \mathcal{B}$ satisfying

$$R_{0,i} < \Delta_1 \min_{j \in g(i)} \left\{ \left[\log_2 \left(\frac{|h_{0,r}|^2 P_{0i}}{\sum_{k=1}^{m} g_{jk}(|h_{0,r}|^2 P_{0k} + |h_{k,r}|^2 P_k)} + \frac{|h_{0,r}|^2 P_{0i}}{\sum_{k=1}^{m} (1-g_{jk})(|h_{0,r}|^2 P_{0k} + |h_{k,r}|^2 P_k) + 1} \right) \right]^+ \right\}$$

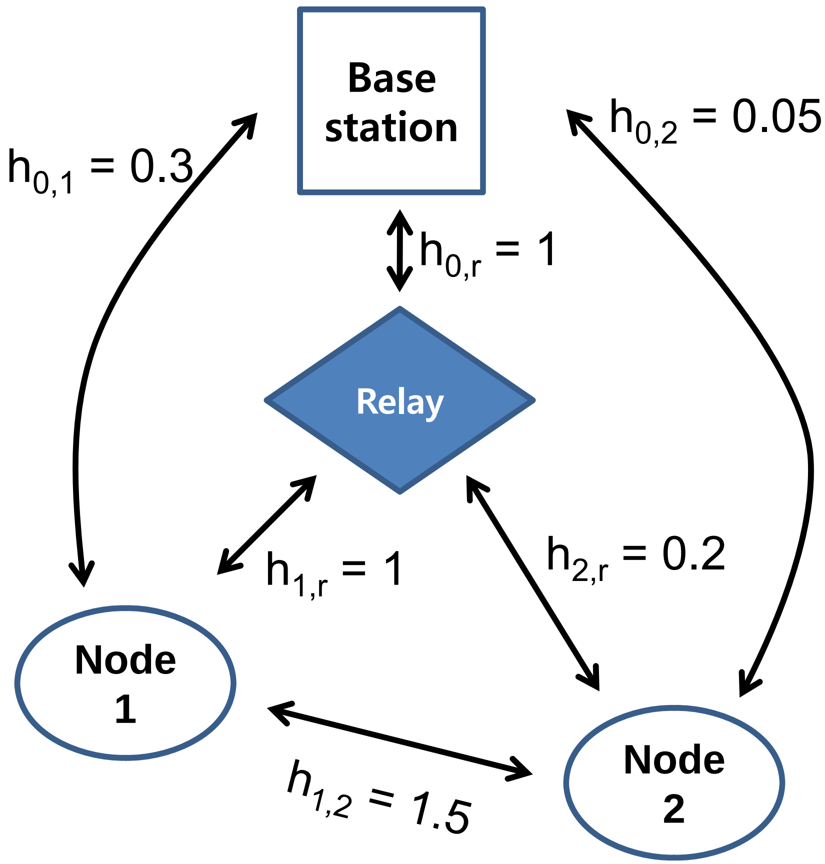
$$R_{i,0} < \Delta_1 \min_{j \in g(i)} \left\{ \left[\log_2 \left(\frac{|h_{i,r}|^2 P_i}{\sum_{k=1}^{m} g_{jk}(|h_{0,r}|^2 P_{0k} + |h_{k,r}|^2 P_k)} + \frac{|h_{i,r}|^2 P_i}{\sum_{k=1}^{m} (1-g_{jk})(|h_{0,r}|^2 P_{0k} + |h_{k,r}|^2 P_k) + 1} \right) \right]^+ \right\}$$

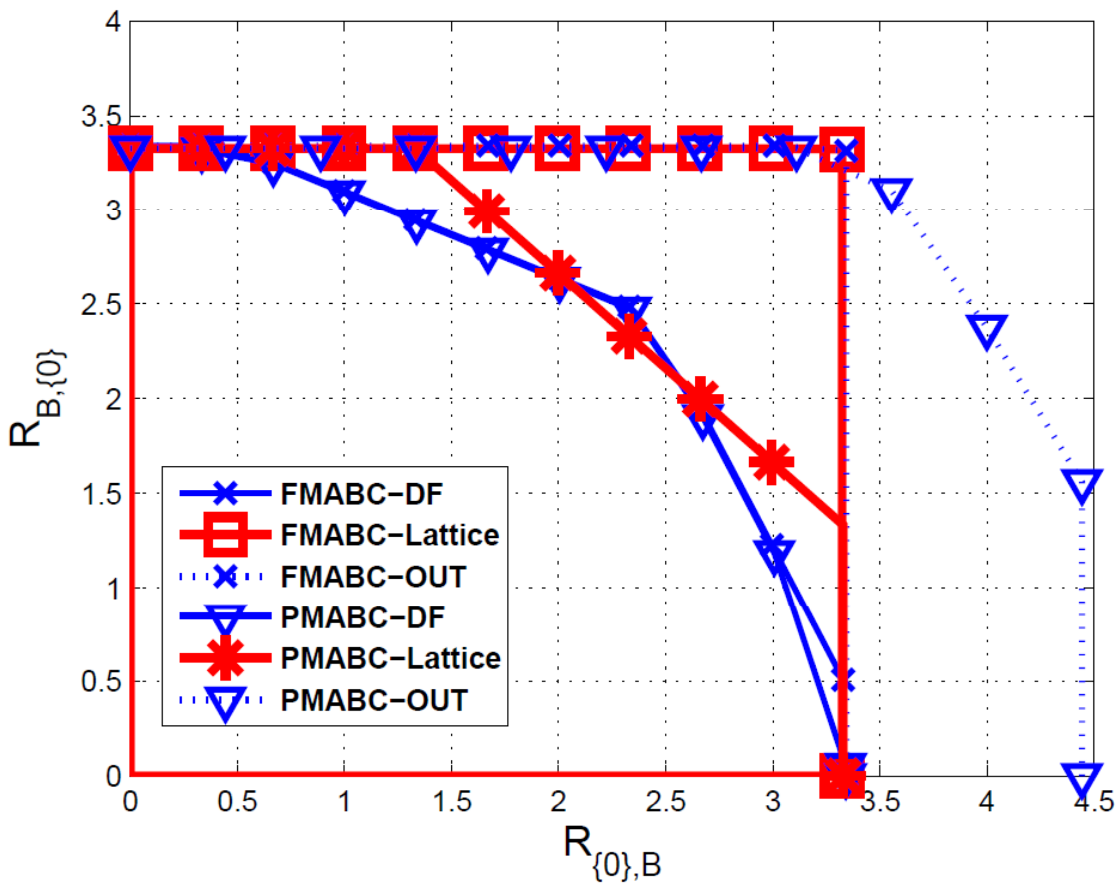
$$R_{\{0\},S} < \sum_{i \in S} \Delta_2 C_B(U_{ri}, Y_i) - \Delta_2 C_B(U_{ri}, U_{rS(i)})$$

$$R_{S,\{0\}} < \Delta_2 C_B(U_{rS}, Y_0 \cup U_{r\bar{S}})$$

where $g(i) := \{j | g_{ij} = 1\}$ for $i \in \mathcal{B}$, $S \subseteq \mathcal{B}$ over all possible combination matrices $\mathbf{G}$. □

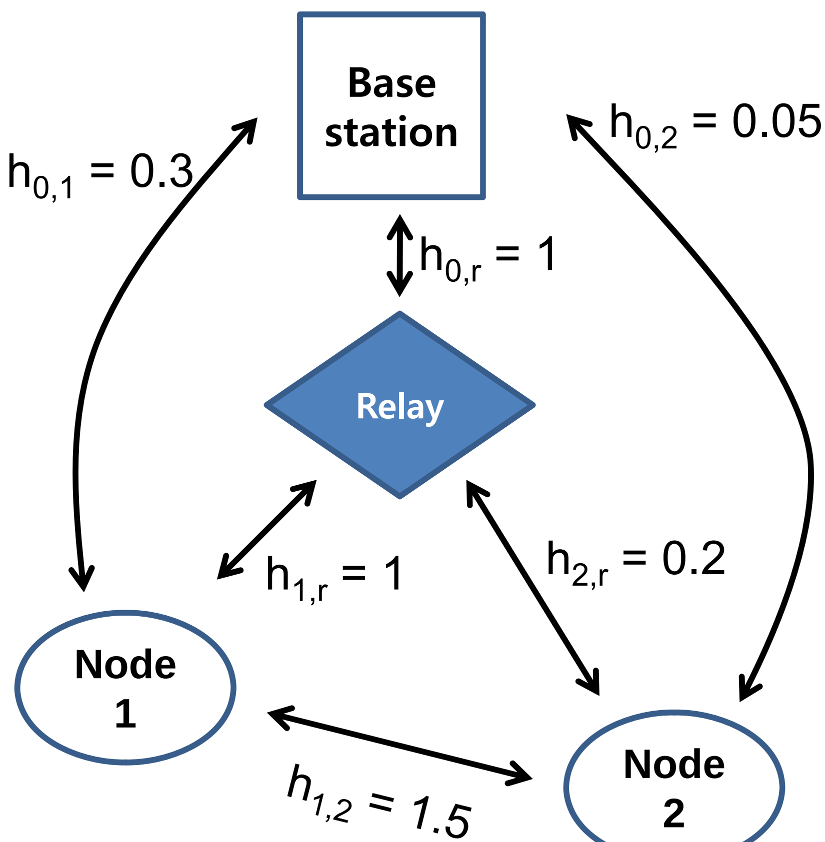
Achievable rate region

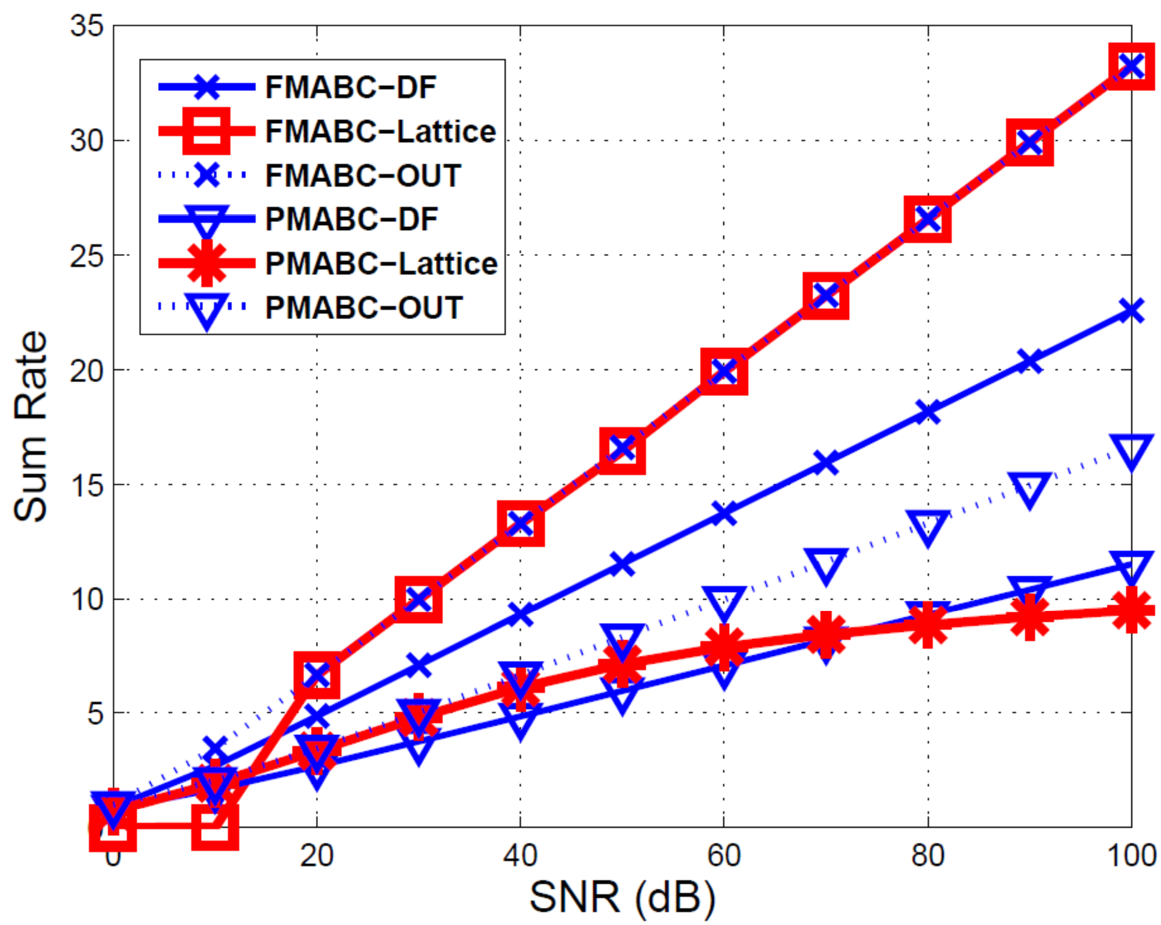




** $P_0 = P_1 = P_2 = P_r$ = 20 dB, B = {1,2}

Sum rate comparison





Constant gap

Theorem : In the multiple access division of the FMABC protocol, if $|h_{1,r}|=|h_{2,r}|=\cdots=|h_{m,r}|$ and $P_1=P_2=\cdots=P_m, \forall S\subseteq\{1,2,\cdots,m\}$

$$R^{out}_{\{0\},S}-R^{in}_{\{0\},S}\le|S|$$

$$R^{out}_{S,\{0\}}-R^{in}_{S,\{0\}}\le|S|$$

Where $R^{in}_{S,T}$ *is the* maximum rate of the lattice code achievable rate region from set S to set T and $R^{out}_{S,T}$ *is the minimum sum-rate of the cut-set* outer bound from set S to set T.

Conclusion

• Derive achievable rate regions of two temporal protocols using lattice sum decode and forward.

• In the FMABC protocol, the proposed scheme outperforms the DF scheme and very tight to cut-set outer bound.

• In the PMABC protocol, there exists performance trade off between DF and the proposed schemes.

• In the FMABC protocol, a constant gap between cut-set and the lattice inner bound.