On The Capacity of the Symmetric Interference Channel with a Cognitive Relay at High SNR

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Motivation

- interference channel (IFC) with one cognitive relay (CR)
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• how does the presence of a CR change the capacity?
General model that contains...

- interference channel

\[ X_1 | h_{11} | + | h_{12} | X_2 \]  
\[ Y_1 | h_{21} | + | h_{22} | Y_2 \]  
\[ Z_1 | W_1 \]  
\[ Z_2 | W_2 \]
General model that contains...

- interference channel
- broadcast channel
General model that contains...

- interference channel
- broadcast channel
- cognitive interference channel
Past work
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- non-causal cognition [Sahin, Erkip, 2007], [Sridharan, Vishwanath, Jafar, Shamai, 2008], [Jiang, Maric, Goldsmith, Cui, 2009], inner + outer bounds for DMC, Gaussian channels
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- outer bounds DMC, 3 bit to capacity for Gaussian no blue interference links [Rini, Tuninetti, Devroye, 2010, 2011]
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- out-of-band cognition/links [Tian, Yener, 2010], [Sahin, Simeone, Erkip, 2010], [Razaghi, Hong, Zhou, Wei, Caire, 2011]

- causal cognition, direct links [Sahin, Erkip, 2007]

- outer bounds DMC, 3 bit to capacity for Gaussian no blue interference links [Rini, Tuninetti, Devroye, 2010, 2011]
• specialize and tighten DMC outer bound of [Rini, Tuninetti, Devroye 2010] to linear deterministic, symmetric IFC-CR (models Gaussian channel at high SNR)
Contributions/outline

• specialize and tighten DMC outer bound of [Rini, Tuninetti, Devroye 2010] to **linear deterministic, symmetric IFC-CR** (models Gaussian channel at high SNR)

• show capacity for symmetric linear deterministic IFC-CR for almost all parameter regimes → insight into **optimal Tx strategies**!
Channel model

Gaussian symmetric IFC-CR:

\[ Y_1 = |h_S|X_1 + |h_C|X_c + h_I X_2 + Z_1 \]
\[ Y_2 = h_I X_1 + |h_C|X_c + |h_S|X_2 + Z_2 \]

\[ N = \max(n_S, n_C, n_I), S = N \times N \text{ shift matrix} \]
Channel model

**Gaussian symmetric IFC-CR:**

\[
Y_1 = |h_S|X_1 + |h_C|X_c + h_I X_2 + Z_1 \\
Y_2 = h_I X_1 + |h_C|X_c + |h_S|X_2 + Z_2
\]

**Linear deterministic sym. IFC-CR:** *(models above at high SNR)*

\[
Y_1 = S^{N-n_s} X_1 + S^{N-n_C} X_c + S^{N-n_I} X_2 \\
Y_2 = S^{N-n_I} X_1 + S^{N-n_C} X_c + S^{N-n_s} X_2
\]

\[N = \max(n_S, n_C, n_I), S = N \times N \text{ shift matrix}\]
Linear deterministic channel model

\[
Y_1 = S^{N-n_s} X_1 + S^{N-n_c} X_c + S^{N-n_I} X_2
\]
\[
Y_2 = S^{N-n_I} X_1 + S^{N-n_c} X_c + S^{N-n_s} X_2
\]

\[
n_S > 0
\]
\[
n_I = \alpha n_S
\]
\[
n_C = \beta n_S
\]
Encoders, decoders and capacity

\[ Y_1 = S^{N-n_s} X_1 + S^{N-n_C} X_C + S^{N-n_I} X_2 \]
\[ Y_2 = S^{N-n_I} X_1 + S^{N-n_C} X_C + S^{N-n_s} X_2 \]

\[ n_S > 0 \]
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Encoders, decoders and capacity

\[ Y_1 = S^{N-n_s} X_1 + S^{N-n_C} X_c + S^{N-n_I} X_2 \]
\[ Y_2 = S^{N-n_I} X_1 + S^{N-n_C} X_c + S^{N-n_s} X_2 \]
\[ n_s > 0 \]
\[ n_I = \alpha n_s \]
\[ n_C = \beta n_s \]

- \( W_1 \in \{1, 2, \ldots, 2^n R_1\} \), \( W_2 \in \{1, 2, \ldots, 2^n R_2\} \)
Encoders, decoders and capacity

\[ Y_1 = S^{N-n_s} X_1 + S^{N-n_c} X_c + S^{N-n_I} X_2 \]
\[ Y_2 = S^{N-n_I} X_1 + S^{N-n_c} X_c + S^{N-n_s} X_2 \]

- \( W_1 \in \{1, 2, \ldots, 2^{nR_1}\} \), \( W_2 \in \{1, 2, \ldots, 2^{nR_2}\} \)
- encoding functions \( X_1^n(W_1), X_2^n(W_2), X_c^n(W_1, W_2) \)

\( n_S > 0 \)
\( n_I = \alpha n_S \)
\( n_C = \beta n_S \)
Encoders, decoders and capacity

$W_1 \to X_1 \xrightarrow{n_s} Y_1 \to \tilde{W}_1$

$W_2 \to X_2 \xrightarrow{n_s} Y_2 \to \tilde{W}_2$

$Y_1 = S^{N-n_S}X_1 + S^{N-n_C}X_c + S^{N-n_I}X_2$

$Y_2 = S^{N-n_I}X_1 + S^{N-n_C}X_c + S^{N-n_s}X_2$

$n_S > 0$

$n_I = \alpha n_S$

$n_C = \beta n_S$

- $W_1 \in \{1, 2, \ldots, 2^{nR_1}\}$, $W_2 \in \{1, 2, \ldots, 2^{nR_2}\}$
- encoding functions $X_1^n(W_1)$, $X_2^n(W_2)$, $X_c^n(W_1, W_2)$
- decoding functions $\tilde{W}_1(Y_1^n)$, $\tilde{W}_2(Y_2^n)$
Encoders, decoders and capacity

- $W_1 \in \{1, 2, \ldots, 2^{nR_1}\}$, $W_2 \in \{1, 2, \ldots, 2^{nR_2}\}$
- encoding functions $X_1^n(W_1)$, $X_2^n(W_2)$, $X_c^n(W_1, W_2)$
- decoding functions $\hat{W}_1(Y_1^n)$, $\hat{W}_2(Y_2^n)$
- seek rates $(R_1, R_2)$ for which $\exists$ en/decoding functions such that $\Pr[(W_1, W_2) \neq (\hat{W}_1, \hat{W}_2)] \to 0$ as $n \to \infty$

\[
Y_1 = S^{N-n_s}X_1 + S^{N-n_C}X_c + S^{N-n_I}X_2
\]
\[
Y_2 = S^{N-n_I}X_1 + S^{N-n_C}X_c + S^{N-n_s}X_2
\]

$n_S > 0$

$n_I = \alpha n_S$

$n_C = \beta n_S$
Encoders, decoders and capacity

\[ Y_1 = S^{N-n_S} X_1 + S^{N-n_C} X_c + S^{N-n_I} X_2 \]
\[ Y_2 = S^{N-n_I} X_1 + S^{N-n_C} X_c + S^{N-n_S} X_2 \]

- \( W_1 \in \{1, 2, \ldots, 2^{nR_1}\} \), \( W_2 \in \{1, 2, \ldots, 2^{nR_2}\} \)
- encoding functions \( X_1^n(W_1), X_2^n(W_2), X_c^n(W_1, W_2) \)
- decoding functions \( \widehat{W}_1(Y_1^n), \widehat{W}_2(Y_2^n) \)
- seek rates \( (R_1, R_2) \) for which \( \exists \) en/decoding functions such that \( \Pr[(W_1, W_2) \neq (\widehat{W}_1, \widehat{W}_2)] \to 0 \) as \( n \to \infty \)

**Maximal rates = capacity**
• we TIGHTEN outer bound of [Rini, Tuninetti, Devroye, ITW Dublin 2010] for general memoryless IFC-CR (exploits Sato’s argument for BC) + linear det. IFC-CR

With the parameterization \( n_I = \alpha n_S, n_C = \beta n_S \), the outer bound in [RTD, ITW 2010] becomes

\[
\begin{align*}
R_1 & \leq \max\{1, \beta\}, \\
R_2 & \leq \max\{1, \beta\}, \\
R_1 + R_2 & \leq [1 - \max\{\alpha, \beta\}]^+ + \beta + \max\{1, \alpha\} \\
R_1 + R_2 & \leq \max\{1, \beta\} \text{ apply for } \alpha = 1 \text{ only} \\
R_1 + R_2 & \leq 2 \max\{1 - \alpha, \alpha, \beta\} + \text{MLP} \\
2R_1 + R_2 & \leq \max\{1, \beta, \alpha\} + \max\{1 - \alpha, \alpha, \beta\} + \max\{1 - \alpha, \beta\} + \text{MLP} \\
R_1 + 2R_2 & \leq \max\{1, \beta, \alpha\} + \max\{1 - \alpha, \alpha, \beta\} + \max\{1 - \alpha, \beta\} + \text{MLP}
\end{align*}
\]

with MLP := 2 \min\{\alpha, \beta\}.
Outer bounds - linear deterministic IFC-CR

• we TIGHTEN outer bound of [Rini, Tuninetti, Devroye, ITW Dublin 2010] for general memoryless IFC-CR (exploits Sato’s argument for BC) + linear det. IFC-CR

With the parameterization $n_I = \alpha n_S$, $n_C = \beta n_S$, the outer bound in [RTD, ITW 2010] becomes

\[
\begin{align*}
R_1 &\leq \max\{1, \beta\}, \\
R_2 &\leq \max\{1, \beta\}, \\
R_1 + R_2 &\leq [1 - \max\{1 - \alpha, \beta\}]\max\{1, \alpha\} \\
R_1 + R_2 &< \max\{1 - \alpha, \alpha, \beta\} + \text{MLP} \quad \text{for } \alpha = 1 \text{ only} \\
R_2 &\leq \max\{1, \beta, \alpha\} + \max\{1 - \alpha, \alpha, \beta\} + \max\{1 - \alpha, \beta\} + \text{MLP} \\
R_1 + 2R_2 &\leq \max\{1, \beta, \alpha\} + \max\{1 - \alpha, \alpha, \beta\} + \max\{1 - \alpha, \beta\} + \text{MLP}
\end{align*}
\]

with MLP := $2 \min\{\alpha, \beta\}$. Which max are tight determine regions!
Sub-regions

\[
\begin{align*}
\beta \\
\| & \| \| \\
I & II & III \\
1 & 1 & 1 \\
V & VI & IV \\
1 & 1 & 1 \\
\end{align*}
\]

\[
\begin{align*}
\alpha & \alpha \\
1 & 1 & 1 \\
I & II & III \\
1 & 1 & 1 \\
\end{align*}
\]

\[
\begin{align*}
W_1 & \rightarrow X_1 & n_S & \rightarrow Y_1 & \rightarrow \tilde{W}_1 \\
& \downarrow & n_l & \downarrow & \\
& \downarrow & n_c & \downarrow & \\
W_2 & \rightarrow X_2 & n_S & \rightarrow Y_2 & \rightarrow \tilde{W}_2 \\
& \downarrow & n_c & \downarrow & \\
& \downarrow & n_l & \downarrow & \\
X_c & & & & \\
\end{align*}
\]

\[
\begin{align*}
n_S & > 0 \\
n_I & = \alpha n_S \\
n_C & = \beta n_S
\end{align*}
\]
Three generic cognitive achievability strategies

- bit cancellation
Three generic cognitive achievability strategies

- **bit cancellation**

- **bit sharing**
Three generic cognitive achievability strategies

• bit cancellation

• bit sharing

• bit cleaning
\[ \alpha = 1 \]

Outer bound reduces to

\[
R_1 + R_2 \leq \max\{1, \beta\}
\]

- Achieved via time-sharing between nodes (and CR fully helps non-silent node)
Regimes I and II: $\alpha > \max\{1, \beta\}$

(a) Optimal Strategy for Regime I for $\alpha > 2$. 

$R_{X_1}$  

$C = A_2 \oplus B_2$  

$R_{X_2}$
Regimes I and II: $\alpha > \max\{1, \beta\}$

user 1 wants A

(a) Optimal Strategy for Regime I for $\alpha > 2$. 
Regimes I and II: $\alpha > \max\{1, \beta\}$

\[
\begin{align*}
R_{x_1} & \quad C = A_2 \oplus B_2 \\
R_{x_2} &
\end{align*}
\]

(a) Optimal Strategy for Regime I for $\alpha > 2$.

user 1 wants \textbf{A}  
user 2 wants \textbf{B}
Regimes I and II: $\alpha > \max\{1, \beta\}$

**High interference:** bit-cancellation!

User 1 wants A
User 2 wants B

(a) Optimal Strategy for Regime I for $\alpha > 2$. 

$R_{X_1}$ \( C = A_2 \oplus B_2 \) \( R_{X_2} \)
Regimes III and IV: $\beta > \max\{1, \alpha\}$

**High cognitive:**
- bit-sharing +
- bit-cancellation +
- bit-cleaning!

- $C_1 = A_1 \oplus B_1$
- $C_2 = A_2 \oplus B_2$

(b) Optimal Strategy for Regime III.
Regimes III and IV: \( \beta > \max\{1, \alpha\} \)

<table>
<thead>
<tr>
<th>cognitive</th>
<th>direct</th>
<th>interference</th>
</tr>
</thead>
</table>

- **high cognitive:**
  - bit-sharing +
  - bit-cancellation+
  - bit-cleaning!

\[
\begin{align*}
\beta - \alpha & < 0 \\
\beta - \alpha + 1 & \geq 0
\end{align*}
\]

\[
\begin{align*}
C_1 = A_1 \oplus B_1 \\
C_2 = A_2 \oplus B_2
\end{align*}
\]

\[
\begin{align*}
A_1 & = 0 \\
A_2 & = \beta - \alpha + 1 \\
A_3 & = \beta - \alpha
\end{align*}
\]

\[
\begin{align*}
B_1 & = 0 \\
B_2 & = \alpha
\end{align*}
\]

\[
\begin{align*}
X_1 & = 0 \\
X_C & = \beta \\
X_2 & = 1
\end{align*}
\]

(b) Optimal Strategy for Regime III.
Regimes III and IV: $\beta > \max\{1, \alpha\}$

high cognitive: bit-sharing + bit-cancellation + bit-cleaning!

(b) Optimal Strategy for Regime III.
Regimes III and IV: \( \beta > \max\{1, \alpha\} \)

High cognitive: bit-sharing + bit-cancellation + bit-cleaning!

(b) Optimal Strategy for Regime III.
Regime V: $0 < \alpha < \beta < 1$

(high direct: \hspace{1cm} bit-cancellation\hspace{1cm} + \hspace{1cm} bit-cleaning!)

(c) Optimal Strategy for Regime V.
Regime V: $0 < \alpha < \beta < 1$

(c) Optimal Strategy for Regime V.

(high direct: bit-cancellation + bit-cleaning!)
Regime V: $0 < \alpha < \beta < 1$

(c) Optimal Strategy for Regime V.

high direct: bit-cancellation+
bit-cleaning!
Regime VI.1: $0 < \beta < \alpha < \frac{1}{2}$

(a) Optimal Strategy for Regime VI.I.

**bit-cancellation**

**bit-cleaning!**
Regime VI.1: \( 0 < \beta < \alpha < \frac{1}{2} \)

(a) Optimal Strategy for Regime VI.1.
Regime VI.1: $0 < \beta < \alpha < \frac{1}{2}$

(a) Optimal Strategy for Regime VI.1.
Missing in regimes VI.2 VI.3, VI.4?

- related to most complex part of “W” curve for IC

- we believe outer bound is problem: too loose bounding does not capture correlation between signals, or may need another $2R_1 + R_2$ - type bound

- mention some possible strategies in paper

- what is Gaussian version of 3 transmission strategies?
Conclusion

• focus on role of cognitive cooperation in an interference channel

• depending on strength of cognition, different behaviors perform well
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• missing last small piece...
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• how to translate results back to Gaussian?