To adapt or not in two-way interference channels?

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Why is the two-way problem so hard?

Adaptation!

One-way: no adaptation possible

Two-way: no adaptation = "restricted"

Two-way: full adaptation

Two-way:
\[
M_{12} = (y_{2,1}, y_{2,2}, \ldots, y_{2,i-1})
\]
Capacity known for certain examples where

\[ X_1(M_{12}) \rightarrow Y_2 \rightarrow X_2(M_{21}) \]

\[ \hat{M}_{21} \rightarrow Y_1 \rightarrow \hat{M}_{12} \]

equal to

\[ X_1(M_{12}) \rightarrow Y_2 \rightarrow \hat{M}_{12} \]

in parallel with

\[ \hat{M}_{21} \rightarrow Y_1 \rightarrow X_2(M_{21}) \]

Wednesday, July 25, 2012
When is capacity **known**

- Parallel two-way channel
- Binary modulo 2 adder channel
- Two-way restricted channel
- Two-way "push-to-talk" channel
- Two-way Gaussian noise channel

When adaptation is useless (does not increase capacity)!

When is capacity **unknown**

- General discrete memoryless channel
- Binary multiplier channel

When adaptation is useful?
What about two-way interfering networks?
Two-way capacity

≡

One-way ➔ One-way

(as in two-way binary adder, two-way Gaussian channels)
Results: Capacity achieved without adaptation for:

- Node 1
- Node 3
- Node 2

(a) Two-way MAC/BC

- Binary adder channel
- Linear deterministic channel (models Gaussian noise channel at high SNR)

(b) Two-way Z channel

- Binary adder channel
- Linear deterministic channel

(c) Two-way interference channel

- Binary adder channel (always)
- Linear deterministic channel (only with restricted interaction!)
- Constant gap certain Gaussian channels (with/without restricted interaction!)

[Cheng, Devroye, Allerton 2011]
[Cheng, Devroye, ISIT 2012]
[Cheng, Devroye, submitted IT Trans., 2012]
Two-way interference channel related work

One-way IC

1 → 2
2 → 3
3 → 4

[El Gamal, Costa 1982]

[Bresler, Tse 2008]
[Etkin, Tse, Wang 2008]

One-way IC with rate-limited FB

[El Gamal, Costa 1982]

[Bresler, Tse 2008]
[Etkin, Tse, Wang 2008]

One-way IC with interfering FB

[El Gamal, Costa 1982]

[Bresler, Tse 2008]
[Etkin, Tse, Wang 2008]

One-way IC with generalized FB

[El Gamal, Costa 1982]

[Bresler, Tse 2008]
[Etkin, Tse, Wang 2008]

One-way IC with FB

[El Gamal, Costa 1982]

[Bresler, Tse 2008]
[Etkin, Tse, Wang 2008]

Two-way IC

1 → 2
2 → 3
3 → 4

[Cheng, Devroye sub. Trans IT, 2012. On Arxiv]
Two-way Modulo 2 Adder IC

\[ Y_1 = X_1 \oplus X_2 \oplus X_4 \]
\[ Y_2 = X_1 \oplus X_2 \oplus X_3 \]
\[ Y_3 = X_2 \oplus X_3 \oplus X_4 \]
\[ Y_4 = X_1 \oplus X_3 \oplus X_4. \]

**Theorem 2:** The capacity region of the two-way modulo 2 adder interference channel is the set of non-negative rate tuples \((R_{12}, R_{21}, R_{34}, R_{43})\) such that

\[ R_{12} + R_{34} \leq 1 \quad (1) \]
\[ R_{21} + R_{43} \leq 1. \quad (2) \]

*Adaptation useless (can not increase capacity)!*

Achievable via time-sharing, cut-set outer bound or directly via alternative proof (see arxiv)
Two-way linear deterministic IC

If partial adaptation:
Then capacity:

\[
N = \max(n_{11}, n_{22}, n_{33}, n_{44}, n_{12}, n_{21}, n_{32}, n_{23}, n_{14}, n_{41}, n_{34}, n_{43})
\]

\[
Y_1 = S^{N-n_{11}} X_1 + S^{N-n_{21}} X_2 + S^{N-n_{41}} X_4
\]

\[
Y_2 = S^{N-n_{12}} X_1 + S^{N-n_{22}} X_2 + S^{N-n_{32}} X_3
\]

\[
Y_3 = S^{N-n_{23}} X_2 + S^{N-n_{33}} X_3 + S^{N-n_{43}} X_4
\]

\[
Y_4 = S^{N-n_{14}} X_1 + S^{N-n_{34}} X_3 + S^{N-n_{44}} X_4
\]

\[
X_{1,i} = f_1(M_{12}), \quad X_{2,i} = f_2(M_{21}, Y_2^{i-1})
\]

\[
X_{3,i} = f_3(M_{34}), \quad X_{4,i} = f_4(M_{43}, Y_4^{i-1})
\]
Partial adaptation key lemma:

Lemma: Under partial adaptation conditions, for some deterministic functions $f_5$ and $f_6$,

\[
X_{2,i} = f_5(M_{12}, M_{21}, M_{34}) \perp M_{43}, \forall i \\
X_{4,i} = f_6(M_{43}, M_{34}, M_{12}) \perp M_{21}, \forall i
\]

where $\perp$ denotes independence.

Central to many of the partial adaptation converses!
Two-way linear deterministic IC

\[ N = \max(n_{11}, n_{22}, n_{33}, n_{44}, n_{12}, n_{21}, n_{23}, n_{23}, n_{14}, n_{41}, n_{34}, n_{43}) \]

\[ Y_1 = S^{N-n_{11}} X_1 + S^{N-n_{21}} X_2 + S^{N-n_{41}} X_4 \]

\[ Y_2 = S^{N-n_{12}} X_1 + S^{N-n_{22}} X_2 + S^{N-n_{32}} X_3 \]

\[ Y_3 = S^{N-n_{23}} X_2 + S^{N-n_{33}} X_3 + S^{N-n_{43}} X_4 \]

\[ Y_4 = S^{N-n_{14}} X_1 + S^{N-n_{34}} X_3 + S^{N-n_{44}} X_4 \]

If FULL adaptation:

\[ X_{1,i} = f_1(M_{12}, Y_1^{i-1}) \]

\[ X_{2,i} = f_1(M_{21}, Y_2^{i-1}) \]

\[ X_{3,i} = f_1(M_{34}, Y_3^{i-1}) \]

\[ X_{4,i} = f_1(M_{43}, Y_4^{i-1}) \]

If partial adaptation in this bound, instead, we consider full deterministic symmetric interference channel when \( m \) other.

Remark 8:

\[ \alpha_m \]

Then we still have the outer bounds:

- If (A) IC in \( \rightarrow \) direction:

  \[ R_{21} \leq n_{21}, R_{43} \leq n_{43}, R_{12} \leq \max(n_{12}, n_{43}), R_{34} \leq \max(n_{34}, n_{21}) \]

  \[ R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{34} - n_{32}]^+ + \max([n_{34} - n_{32}]^+, n_{14}) \]

  \[ 2R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{12} - n_{32}]^+ + \max([n_{12} - n_{32}]^+, n_{14}) \]

  \[ R_{12} + 2R_{34} \leq \max(n_{34}, n_{14}) + [n_{34} - n_{32}]^+ + \max([n_{12} - n_{32}]^+, n_{32}) \]

- If (B) IC in \( \leftarrow \) direction:

  \[ R_{21} \leq n_{21}, R_{43} \leq n_{43}, R_{21} \leq \max(n_{21}, n_{34}), R_{43} \leq \max(n_{43}, n_{12}) \]

  \[ R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{43} - n_{41}]^+ + \max([n_{43} - n_{41}]^+, n_{23}) \]

  \[ 2R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{21} - n_{23}]^+ + \max([n_{43} - n_{41}]^+, n_{23}) \]

  \[ R_{21} + 2R_{43} \leq \max(n_{43}, n_{23}) + [n_{43} - n_{41}]^+ + \max([n_{21} - n_{23}]^+, n_{41}) \]
**One** open problem (least ambitious):

Existing outer bounds under FULL adaptation:

- \( R_{12} \leq n_{12}, \quad R_{34} \leq n_{34} \)
- \( R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{34} - n_{32}]^+ \)
- \( R_{12} + R_{34} \leq \max(n_{34}, n_{14}) + [n_{12} - n_{14}]^+ \)
- \( R_{12} + R_{34} \leq \max([n_{12} - n_{34}]^+, n_{32}) + \max([n_{34} - n_{32}]^+, n_{14}) \)
- \( 2R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{12} - n_{14}]^+ + \max([n_{34} - n_{32}]^+, n_{14}) \)
- \( R_{12} + 2R_{34} \leq \max(n_{34}, n_{14}) + [n_{34} - n_{32}]^+ + \max([n_{12} - n_{34}]^+, n_{32}) \)

(A) IC in \( \rightarrow \) direction

- \( R_{21} \leq n_{21}, \quad R_{43} \leq n_{43} \)
- \( R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{43} - n_{41}]^+ \)
- \( R_{21} + R_{43} \leq \max(n_{43}, n_{23}) + [n_{21} - n_{23}]^+ \)
- \( R_{21} + R_{43} \leq \max([n_{21} - n_{43}]^+, n_{41}) + \max([n_{43} - n_{41}]^+, n_{23}) \)
- \( 2R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{21} - n_{23}]^+ + \max([n_{43} - n_{41}]^+, n_{23}) \)
- \( R_{21} + 2R_{43} \leq \max(n_{43}, n_{23}) + [n_{43} - n_{41}]^+ + \max([n_{21} - n_{43}]^+, n_{41}) \)

(B) IC in \( \leftarrow \) direction

What is the general capacity region of linear deterministic two-way IC?

Key technical issues:

- Fully adaptive version of ETW-type outer bounds not clear (tried!)
- What type of adaptive scheme achieves capacity?
- So many channel gains in asymmetric case: how to make sense?
Full adaptation example converse:

\[ n(R_{12} + R_{34} - \epsilon) \]

\[ \leq I(M_{12}; Y_2^n| M_{21}, M_{43}) + I(M_{34}; Y_4^n| M_{12}, M_{21}, M_{43}) \]

\[ \leq I(M_{12}; Y_2^n| M_{21}, M_{43}) + I(M_{34}; Y_2^n| M_{21}, M_{12}, M_{43}) + H(Y_4^n| M_{21}, M_{12}, M_{43}, Y_2^n) \]

\[ = I(M_{12}; Y_2^n| M_{21}, M_{43}) + I(M_{34}; Y_2^n| M_{21}, M_{12}, M_{43}) \]

\[ + \sum_{i=1}^{n} [H(S^{N-n_{34}} X_{3,i}| M_{21}, M_{12}, M_{43}, Y_4^{i-1}, X_4^i, Y_2^n, X_2^n, X_1^i)] \]

\[ \leq \sum_{i=1}^{n} [H(Y_2,i| Y_2^{i-1}, M_{21}, X_2^i) - H(Y_2,i| Y_2^{i-1}, M_{12}, M_{21}, M_{43}) + H(Y_2,i| Y_2^{i-1}, M_{12}, M_{21}, M_{43}) \]

\[ + H(S^{N-n_{34}} X_{3,i}| M_{21}, M_{12}, M_{43}, Y_4^{i-1}, X_4^i, S^{N-n_{12}} X_1^n + S^{N-n_{22}} X_2^n + S^{N-n_{32}} X_3^n, X_2^n, X_1^n)] \]

\[ \leq \sum_{i=1}^{n} [H(S^{N-n_{12}} X_1,i + S^{N-n_{32}} X_3,i) + H(S^{N-n_{34}} X_{3,i}| S^{N-n_{32}} X_{3,i})] \]

\[ \leq n(\max(n_{12}, n_{32}) + [n_{34} - n_{32}]^+) \]

Given \( M_{12}, X_2^n, X_4^i \), we can construct \( X_1^i \).
Remark on partial adaptation:

\[ X_{1,i} = f_1(M_{12}), \quad X_{2,i} = f_2(M_{21}, Y_{i-1}^2) \]
\[ X_{3,i} = f_3(M_{34}), \quad X_{4,i} = f_4(M_{43}, Y_{i-1}^4) \]

Blocks routing at node 1,3

Adaptation useful in general!
Routing via interference!

However, it is sometimes useless!
Symmetric two-way linear deterministic IC

Notice symmetry!

Define: $C_{sys}(\alpha) := \frac{R_{12} + R_{34}}{2} = \frac{R_{21} + R_{43}}{2}$, $\alpha := \frac{n_I}{n_D}$
Let’s compare $C_{sym}$ for 4 models:

**One-way IC**
- 1
- 2
- 3
- 4
  - 2 messages
  - [Bresler, Tse 2008]
  - [Etkin, Tse, Wang 2008]

**One-way IC with FB**
- 1
- 2
- 3
- 4
  - 2 messages
  - [Suh, Tse 2011]

**One-way IC with rate-limited FB**
- 1
- 2
- 3
- 4
  - 2 messages
  - [Vahid, Suh, Avestimehr 2012]

**Two-way IC**
- 1
- 2
- 3
- 4
  - 4 messages
  - [Cheng, Devroye 2012]
One-way IC with perfect feedback

One-way IC = Two-way IC with partial adaptation

One-way IC with rate-limited feedback

Two-way IC with full adaptation

Fig. 3. $C_{\text{sym}}$ for various linear deterministic ICs as a function of $\alpha := \frac{q}{p}$.

This tells us that allowing partial adaptation is useless – i.e. may as well not adapt. Interestingly, the same holds true even for full adaptation for $\alpha > \frac{2}{3}$. This was also concluded for the linear deterministic one-way interference channel with interfering feedback links in [20]; what is interesting is that we can just as well squeeze in extra information messages in the feedback link – in the two-way interference channel model – rather than use the backwards links for feedback. The symmetric sum-capacity for the fully adaptive two-way IC remains open for $\alpha < \frac{2}{3}$; it is solved for partial adaptation.

Recently, the work in [23] has considered a one-way interference channel with interfering feedback links – again forming an interference channel – a generalization of some of the deterministic interference channels with feedback considered in [20] where the feedback link spends fraction $\lambda$ of its time sending feedback and uses the remaining $(1-\lambda)$ for other things – such as for example sending independent backwards messages – though adaptation as in (1–) is not considered. This is quite different from our model which integrates sending feedback and messages over all links and does not force this separation.

While the symmetric sum-capacity for this one-way interference channel with interfering feedback links is obtained in [23] in our notation for $\alpha \geq 1$, it is a function of this parameter $\lambda$ and is thus not plotted here.
So far all

Two-way capacity

≡

One-way ➔
One-way ➙

for deterministic models

What about noisy channels?
Question: does a non-adaptive scheme (Han + Kobayashi) ever perform close to the adaptive outer bounds?

where $g_{jk}$ are the complex channel gains, $E[|X_j|^2] \leq 1$, i.i.d. $Z_j \sim \mathcal{CN}(0, 1)$ and $\text{SNR}_{jk} = |g_{jk}|^2$, $\text{INR}_{jk} = |g_{jk}|^2$
**Theorem: Outer bound: full adaptation.** For the Gaussian two-way symmetric IC under full adaptation, any achievable symmetric rate $R_{sym} = \frac{R_{12} + R_{34}}{2} = \frac{R_{21} + R_{43}}{2}$, achievable by each user, satisfies,

\[
R_{sym} \leq \frac{1}{2} \log \left( 1 + \text{SNR} + \text{INR} + 2 \sqrt{\text{SNR} \times \text{INR}} \right) + \frac{1}{2} \log \left( 1 + \frac{\text{SNR}}{1 + \text{INR}} \right) \quad (1)
\]

same as perfect output feedback sum-rate, though arrived at differently!

**Open problem:** more computable Gaussian full-adaptation bounds?
\[ R_{sym} = \frac{R_{12} + R_{34}}{2} \leq \frac{1}{2} \log \left( 1 + \text{SNR} + \text{INR} + 2\sqrt{\text{SNR} \times \text{INR}} \right) + \frac{1}{2} \log \left( 1 + \frac{\text{SNR}}{1 + \text{INR}} \right) \]

\[ n(R_{12} + R_{34} - \epsilon) \]
\[ \leq I(M_{12}; Y^2_2 | M_{21}, M_{43}, Z^n_1) + I(M_{34}; Y^n_4 Y^n_2 M_{12}, M_{21}, M_{43}, Z^n_1) \]
\[ \leq I(M_{12}; Y^2_2 | M_{21}, M_{43}, Z^n_1) + I(M_{34}; Y^n_2 | M_{21}, M_{12}, M_{43}, Z^n_1) + H(Y^n_4 | M_{21}, M_{12}, M_{43}, Y^n_2, Z^n_1) - H(Z^n_4) \]
\[ \overset{(a)}{=} I(M_{12}; Y^2_2 | M_{21}, M_{43}, Z^n_1) + I(M_{34}; Y^n_2 | M_{21}, M_{12}, M_{43}, Z^n_1) \]
\[ + \sum_{i=1}^{n} \left[ H(g_{34}X^i_3 + Z^i_4 | M_{21}, M_{12}, M_{43}, Y^{i-1}_4 X^i_4 Y^n_2 X^n_2 Z^n_1 X^n_1) - H(Z^n_4) \right] \]
\[ \overset{(b)}{\leq} \sum_{i=1}^{n} \left[ H(Y^i_2 | Y^{i-1}_2 M_{21}, X^i_2) - H(Y^i_2 | Y^{i-1}_2 M_{12}, M_{21}, M_{43}, Z^n_1) \right] + H(Y^i_2 | Y^{i-1}_2 M_{12}, M_{21}, M_{43}, Z^n_1) - H(Z^i_2) + H(g_{34}X^i_3 + Z^i_4 | X^i_4, g_{32}X^i_3 + Z^i_2, X^i_1, X^n_2) - H(Z^i_4) \]
\[ \overset{(c)}{\leq} \sum_{i=1}^{n} H(g_{12}X^i_1 + g_{32}X^i_3 + Z^i_2 | X^i_2) - H(Z^i_2) + H(g_{34}X^i_3 + Z^i_4 | X^i_4, g_{32}X^i_3 + Z^i_2) - H(Z^i_4) \]

(a): \( X^i_1 \) is constructed from \( (M_{12}, X^n_2, X^i_4, Z^n_1) \).
Theorem: Outer bound: partial adaptation. For the Gaussian two-way IC under partial adaptation, in addition to the bounds in full-adaptation Theorem, we may also conclude that any achievable rates \( (R_{12}, R_{21}, R_{34}, R_{43}) \), and \( R_{sym\rightarrow} = \frac{R_{12} + R_{34}}{2} \) and \( R_{sym\leftarrow} = \frac{R_{21} + R_{43}}{2} \) must satisfy,

\[
\begin{align*}
R_{12} &\leq \log(1 + \text{SNR}_{12}) \quad (1) \\
R_{21} &\leq \log(1 + \text{SNR}_{21}) \quad (2) \\
R_{34} &\leq \log(1 + \text{SNR}_{34}) \quad (3) \\
R_{43} &\leq \log(1 + \text{SNR}_{43}) \quad (4)
\end{align*}
\]

same as Etkin, Tse, Wang

\[
R_{sym\rightarrow} \leq \log \left( 1 + \text{INR} + \text{SNR} - \frac{\text{INR} \times \text{SNR}}{1 + \text{INR}} \right) \quad (5)
\]

\[
R_{sym\leftarrow} \leq \begin{cases} 
\log \left( 1 + \frac{\text{SNR}}{\text{INR}} \right), & \text{if } \text{SNR} \leq \text{INR}^3 \\
\log \left( 1 + \frac{\sqrt{\text{SNR}} + \sqrt{\text{INR}}}{1 + \text{INR}} \right), & \text{if } \text{SNR} > \text{INR}^3
\end{cases} \quad (6)
\]

If partial adaptation:

\[
\begin{align*}
X_{1,i} &= f_1(M_{12}), & X_{2,i} &= f_2(M_{21}, Y_i^{-1}) \\
X_{3,i} &= f_3(M_{34}), & X_{4,i} &= f_4(M_{43}, Y_i^{-1})
\end{align*}
\]
Partial adaptation key lemma:

Lemma: partial adaptation Gaussian two-way IC. Under partial adaptation, for some deterministic functions $f_5$ and $f_6$,

$$X_{2,i} = f_5(M_{12}, M_{21}, M_{34}, Z_{2}^{i-1}) \perp M_{43}, \ \forall i$$

$$X_{4,i} = f_6(M_{43}, M_{34}, M_{12}, Z_{4}^{i-1}) \perp M_{21}, \ \forall i$$

where $\perp$ denotes independence.

Central to many of the converses!
\[ R_{\text{sym}} \leq \log \left( 1 + \text{INR} + \text{SNR} - \frac{\text{INR} \times \text{SNR}}{1 + \text{INR}} \right) \]

**Proof (details available in arxiv):**

1) Inspired by the outer bounds for the linear deterministic model.

2) Start with Fano’s inequality and provide side information.

3) Use the definition of partial adaptation.

4) For evaluation, algebra with correlation coefficients.

\[ n(R_{\text{sym}} - \epsilon) \leq I(M_{12}, Y_{12}; g_{14}X_4^g + Z_4^g, M_{24}, M_{34}) + I(M_{34}, Y_{14}^g, g_{23}X_3^g + Z_3^g, M_{43}, M_{43}) \]

\[ \leq H(Y_{12}^g, g_{14}X_4^g + Z_4^g, M_{14}, M_{24}) + H(g_{14}X_4^g + Z_4^g; M_{14}, M_{24}) + H(Y_{14}^g, g_{23}X_3^g + Z_3^g, M_{34}, M_{43}) + H(g_{23}X_3^g + Z_3^g; M_{34}, M_{43}) + H(Y_{14}^g, g_{23}X_3^g + Z_3^g; M_{34}, M_{43}) \]

\[ \leq H(Y_{12}^g, g_{14}X_4^g + Z_4^g, M_{14}, M_{24}) + \sum_{i=1}^n [H(g_{14}X_4^g + Z_4^g, M_{14}) - H(Y_{12}^g, g_{14}X_4^g + Z_4^g, M_{14}, M_{24})] \]

\[ + H(Y_{14}^g, g_{23}X_3^g + Z_3^g, M_{34}, M_{43}) + \sum_{i=1}^n [H(g_{23}X_3^g + Z_3^g, M_{34}, M_{43}) - H(Y_{14}^g, g_{23}X_3^g + Z_3^g, M_{34}, M_{43})] \]

\[ + H(g_{14}X_4^g + Z_4^g; M_{14}, M_{24}) + \sum_{i=1}^n [H(g_{14}X_4^g + Z_4^g, M_{14}) - H(Y_{12}^g, g_{14}X_4^g + Z_4^g, M_{14}, M_{24})] \]

\[ + H(g_{23}X_3^g + Z_3^g; M_{34}, M_{43}) + \sum_{i=1}^n [H(g_{23}X_3^g + Z_3^g, M_{34}, M_{43}) - H(Y_{14}^g, g_{23}X_3^g + Z_3^g, M_{34}, M_{43})] \]

\[ \leq \sum_{i=1}^n [H(g_{14}X_4^g + Z_4^g, M_{14}) - H(g_{14}X_4^g + Z_4^g, M_{14}, M_{24})] + H(g_{14}X_4^g + Z_4^g, M_{14}) - H(g_{14}X_4^g + Z_4^g, M_{14}, M_{24}) \]

\[ + H(g_{14}X_4^g + Z_4^g, M_{14}) - H(g_{14}X_4^g + Z_4^g, M_{14}, M_{24}) \]
\[ R_{sym}^- \leq \begin{cases} 
\log \left(1 + \frac{\text{SNR}}{\text{INR}} + \frac{\text{SNR}}{\text{INR}}\right), & \text{if } \text{SNR} \leq \text{INR}^3 \\
\log \left(1 + \frac{\sqrt{\text{SNR}} + \sqrt{\text{INR}}}{1 + \text{INR}}\right), & \text{if } \text{SNR} > \text{INR}^3
\end{cases} \]

Proof (details available in arxiv):

Differences compared to the previous outer bound:

1) Use the lemma.

2) Evaluation: The correlation coefficient between \( X_2 \) and \( X_4 \) is not 0.

\[ |\lambda_{24}| = \min(1, \frac{\sqrt{\text{SNR}} \times \text{INR}}{\text{INR}^2}) \]
Symmetric H+K scheme of Etkin, Tse, Wang achieves the following gaps:

<table>
<thead>
<tr>
<th>Two-way Interference</th>
<th>Constant Gaps per user per direction, in bits (to outer bound)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Strong</td>
<td>0 (partial)</td>
</tr>
<tr>
<td>Strong</td>
<td>1 (full)</td>
</tr>
<tr>
<td>Weak</td>
<td></td>
</tr>
<tr>
<td>INR &lt; 1</td>
<td></td>
</tr>
<tr>
<td>HK1 is active</td>
<td>1.5 (full)</td>
</tr>
<tr>
<td>HK2 is active</td>
<td></td>
</tr>
<tr>
<td>→ direction</td>
<td>1 (partial)</td>
</tr>
<tr>
<td>← direction</td>
<td></td>
</tr>
<tr>
<td>SNR ≤ INR$^3$</td>
<td>1 (partial)</td>
</tr>
<tr>
<td>SNR &gt; INR$^3$</td>
<td>2 (partial)</td>
</tr>
</tbody>
</table>

**TABLE I**

**CONSTANT GAPS BETWEEN NON-ADAPTIVE SYMMETRIC HAN AND KOBAYASHI SCHEMES IN EACH DIRECTION AND PARTIALLY OR FULLY ADAPTIVE OUTER BOUNDS FOR THE TWO-WAY GAUSSIAN IC.**
Conclusion

Adaptation appears to be useless when:

(a) Can cancel “self-interference”
(b) No coherent gains to be had
(c) No “routing” (interference is good!) possible

In general, when in adaptation useless in two-way networks?