Multi-pair bi-directional relay networks part II: outer bounds and cooperation

Sang Joon Kim, Besma Smida, and Natasha Devroye

***Abstract*—In part II of our investigation of the multi-pair bi-directional relay network - which consists of one base-station, multiple (say m) terminal nodes and one relay, all of which operate in half-duplex modes - we extend and enhance previously derived protocols to exploit over-heard side information through end-user cooperation. That is, each terminal node communicates with the base-station in a bi-directional fashion with the possible help of a relay and may help each other in decoding their messages using a compress-and-forward cooperation strategy. Our contributions in our part II investigation are: 1) the introduction of end-user cooperation in a multi-pair bi-directional relay network which is enabled through the compression of over-heard signals, on top of the network coding and random binning schemes introduced in part I, 2) the derivation of a general achievable rate region for this cooperation strategy, and 3) numerical simulations which highlight the relative gains achieved by network coding, random binning and compress-and-forward-type cooperation between terminal nodes.**

***Index Terms*—bi-directional relaying, capacity region, decode and forward, compress and forward, multi-pair, cooperation**

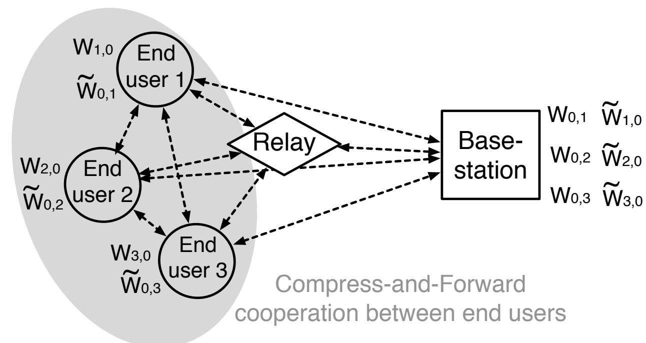

Fig. 1. Our physical channel model consists of multiple independent bi-directional desired communication flows (indicated by arrows) between multiple terminal nodes, a single relay node, and a single base-station. $W_{i,j}$ denotes the message from node i to node j, while $\widetilde{W}_{i,j}$ is the estimate at node j of the message $W_{i,j}$.

I. Introduction

A multi-pair bi-directional relay network which consists of a half-duplex base station (node 0) which wishes to communicate simultaneously in a bi-directional fashion with multiple half-duplex terminal nodes (node 1, $\cdots$, node m) with the help of one half-duplex relay node (node r) is considered. This work builds on part I, in which protocols - or schemes which indicate which nodes transmit when - were derived which made use of Network coding to combine the bi-directional information flows on a flow-by-flow basis, along with Random Binning at the base-station which exploited available over-heard side-information whenever a node was not transmitting. We extend the results in two ways:

- We present modified cut-set-based *outer bounds* on the capacity region of this network.
- We derive achievable rate regions in which not only network coding and random binning are exploited, but *cooperation* between end users is enabled. This is possible in protocols in which certain nodes *over-hear* other nodes' transmissions. In this work we focus on *compress-and-forward*-based cooperation schemes.

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After deriving inner and outer bounds, we evaluate these in an additive white Gaussian noise channel, which demonstrate, at least under Gaussian input distributions, that:

- Cooperation can improve the rate regions over schemes which employ only non-cooperative superposition, Network coding and Random Binning-based schemes. We furthermore illustrate examples of channel conditions under which gains may be expected.
- There are no general inclusion relationships between the derived FMABC, PMABC and TFDBC regions with cooperation.

Results on the classical three-node bi-directional relay channel in which two terminal nodes wish to exchange messages with the help of a single relay [1]–[8] has been extended to scenarios in which a single bi-directional link is aided by multiple relays [9]–[12], and more recently to scenarios where *multiple bi-directional links* share a single, common relay [13]–[16]. In this work we are concerned with a multi-pair bi-directional relay network whose model itself differs from the work in [13]–[16] in three distinct ways:

1) All nodes have direct links with other nodes and traffic is not forced to flow through the relay. This allows us to explore the tension between communicating information directly, through the relay, or through cooperation.

2) We do not consider independent, interfering bi-directional links, but rather assume that one end of the bi-directional links is a base-station that wishes to communicate in a bi-directional fashion with multiple end users, as motivated by relay-aided satellite and cellular networks which have been proposed to improve coverage, reliability and/or data-rates.

3) To the best of the authors' knowledge, this is the first consideration of *cooperation between end-users* in a multi-pair bi-directional channel. In this work, we consider a Compress-and-Forward-based causal cooperation scheme, as first introduced in the context of a uni-directional relay channel in [17]. Compress and forward (CF) based cooperation is an interesting choice as its operation and performance is in some sense a combination of Amplify-and-Forward based schemes which simply forward the received signals and thus suffer from noise amplification, and Decode-and-Forward based schemes which are able to completely eliminate the noise and re-encode the data, but suffer from reduced rate in order to guarantee proper decoding. Compress-and-Forward schemes operate by compressing the received signal, which effectively eliminates some of the noise, and has been employed in the context of *bi-directional relaying* in [18]–[21] while its usefulness in a more general cooperative communications context is well illustrated in [22]. We have also selected CF-based cooperation / relaying due to the rough intuition that CF outperforms for example DF-based cooperation/relaying when the helping node is close to the final destination, which is expected to be one of the scenarios of practical interest.

II. NOTATION AND DEFINITIONS

We consider a base station (node 0), a set of terminal nodes $\mathcal{B} := \{1, 2, \cdots, \mathsf{m}\}$ and a relay r which aids in the communication between the terminal nodes and the base station. We define $\mathcal{M} := \mathcal{B} \cup \{0\} = \{0, 1, 2, \cdots, \mathsf{m}\}$. We use $R_{i,j}$ to denote the rate of communication from node i to node j, i.e. the message between node i and node j, $W_{i,j}$, lies in the set $\mathcal{S}_{i,j} := \{0, \ldots, \lfloor 2^{nR_{i,j}} \rfloor - 1\}$. Similarly, $R_{S,T}$ is the vector of rates from set S to set T where $S, T \subseteq \mathcal{M}$ at which the messages $W_{S,T} := \{W_{i,j} | i \in S, j \in T, \ S, T \subseteq \mathcal{M}\}$ may be reliably communicated. We assume that each end user communicates with the base station bi-directionally and that no information is directly exchanged between end users: i.e. every pair of terminal nodes $\mathsf{0}$ and $i \in [1, m]$ wish to exchange independent messages while $R_{i,j}$ is undefined for all $i, j \in \mathcal{B}$.

Communication takes place over a number of channel uses, n and rates are achieved in the classical asymptotic sense as $n \to \infty$ [3]. Node i has input alphabet $\mathcal{X}_i^* = \mathcal{X}_i \cup \{\varnothing\}$ and channel output alphabet $\mathcal{Y}_i^* = \mathcal{Y}_i \cup \{\varnothing\}$, which are related through a discrete memoryless channel[1]. Lower case letters x_i denote instances of the upper case X_i which lie in the calligraphic alphabets $\mathcal{X}_i^*$. Boldface $\mathbf{x}_i$ represents a vector indexed by time at node i. Finally, it is convenient to denote by $\mathbf{x}_S := \{\mathbf{x}_i | i \in S\}$, a set of vectors indexed by time, and $\bigotimes$ as the cartesian product, i.e., $\bigotimes_{i=1}^{3} \mathcal{X}_i = \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3$.

During phase ℓ we use $X_i^{(\ell)}$ to denote the input distribution and $Y_i^{(\ell)}$ to denote the distribution of the received signal of node i, and we use the dummy symbol $\varnothing$ to denote that there is no input or no output at a particular node during a particular phase. It is also convenient to define $X_S^{(\ell)} := \{X_i^{(\ell)} | i \in S\}$, a set of input distributions during phase ℓ.

[1]Extensions to Gaussian noise channels will be addressed in Section VI.

III. COOPERATION PROTOCOLS

A. Temporal protocols

In part I we considered three main multi-pair bi-directional relaying protocols: 1) the *Full Multiple Access Broadcast* (FMABC) protocol in which all terminal nodes transmit for the whole duration of the multiple-access phase and all listen during the relay *broadcasting* period/phase, 2) the *Partial Multiple Access Broadcast* (PMABC) protocol, where node $\mathsf{0}$ uses the whole duration and the terminal nodes $1, \cdots, \mathsf{m}$ transmit sequentially during the multiple-access phase, all terminal nodes listen during the relay broadcasting period, and 3) the *Full Time Division Broadcast* (FTDBC) protocol, where the nodes transmit in the order $\mathsf{0}, 1, \cdots, \mathsf{m}, \mathsf{r}$. Illustrations of these protocols may be found in part I as well as [23] online.

B. Cooperation between terminal nodes

"Over-heard" transmissions received at a terminal node when it is not transmitting may be used to allow them to cooperate in decoding the messages $W_{0,i}$ for $i \in \mathcal{B}$. Cooperation is enabled through a compress and forward strategy in which each terminal node in $\mathcal{B}$ compresses the signals received during the relay broadcast period using an auxiliary message set, which it then transmits during the next multiple access period. If other nodes can decode this auxiliary message, they are able to obtain the compressed received signals which in turn may be used to decode messages from the relay. We note that not all nodes need to cooperate or compress the received signals - our results allow for any subset of the terminal nodes in $\mathcal{B}$ to cooperate.

To concretely illustrate how our cooperation strategy operates, we describe cooperation for the PMABC protocol; the FTDBC protocol can be similarly constructed. We apply *sliding window* and *Compress and Forward* schemes when node i $(\in \mathcal{B})$ is transmitting: first, we divide the total time duration into $K+1$ slots; each slot consists of $m+1$ phases. Every message $w_{i,j}$ is also divided into K blocks as $\{w_{i,j|(1)}, \cdots, w_{i,j|(K)}\}$, and node i transmits $\{w_{i,j|(k)}\}$ during slot k and phase i (for PBMAC). After relay r broadcasts $\mathbf{x}_\mathsf{r}$ during slot k and phase $m+1$, node i compress $\mathbf{y}_i$ to $\hat{\mathbf{y}}_i$ with auxiliary message set $\{w_{\{i\},\mathcal{B}}\}$. Then node i broadcasts $\mathbf{x}_i(w_{i,\mathsf{0}|(k+1)}, w_{\{i\},\mathcal{B}|(k)})$ during slot $k+1$ and phase i except for the first and last slots. During the first and last slot, i sends $\mathbf{x}_i(w_{i,\mathsf{0}|(1)}, 1)$ and $\mathbf{x}_i(1, w_{\{i\},\mathcal{B}|(K)})$, respectively.

In general, joint typicality is non-transitive. However, by using strong joint-typicality, and the fact that for the distributions of interest $x \to y \to \hat{y}$, we will be able to argue joint typicality between $\mathbf{x}$ and $\hat{\mathbf{y}}$ by the *Markov lemma* (Lemma 4.1 in [24]). If node j $(\in \mathcal{B}\ , j \neq i)$ can decode $\tilde{w}_{\{i\},\mathcal{B}|(k)}$ at the end of slot $k+1$ and phase i, node j can use the sequence $\hat{\mathbf{y}}_i^{(m+1)}(w_{\{i\},\mathcal{B}|(k)})$ for decoding $\tilde{w}_{\mathsf{0},j|(k)}$. Let $\mathcal{J}_j$ be the set of nodes whose message can be decoded by node j, i.e., $\mathcal{J}_j = \{i | \tilde{w}_{\{i\},\mathcal{B}|(k)} = w_{\{i\},\mathcal{B}|(k)} \ , \forall k \in [1, K+1]\}$. Then node j uses the jointly typical sequences $(\mathbf{x}_\mathsf{r}^{(m+1)}(w_\mathsf{r}), \mathbf{y}_j^{(m+1)}, \hat{\mathbf{y}}_{\mathcal{J}_j}^{(m+1)}(w_{\mathcal{J}_j,\mathcal{B}|(k)}))$ to decode $\tilde{w}_{\mathsf{0},j|(k)}$. Fig. 2 illustrates an example of the cooperative PMABC protocol with $m = 2$ terminal nodes (and hence four messages).

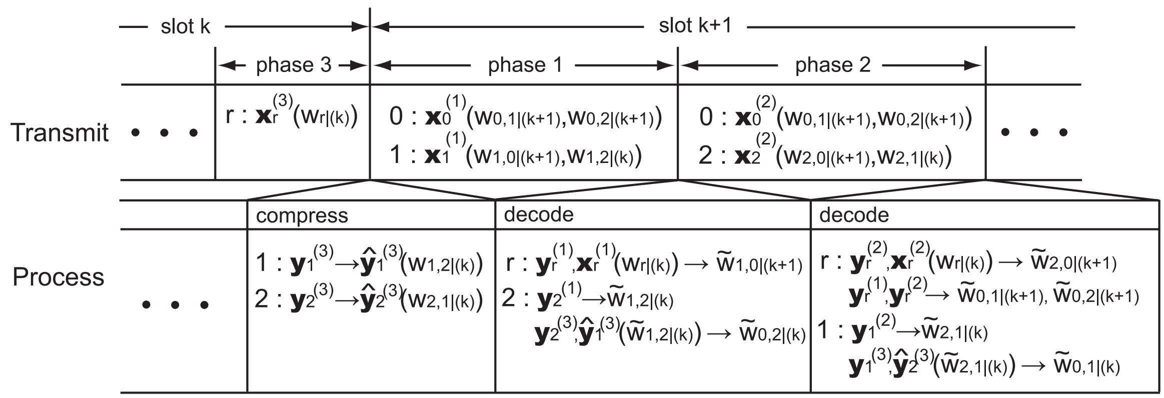

Fig. 2. An example of the PMABC protocol with terminal-node cooperation with two end-users $m = 2$ and one base-station.

IV. Achievable rate regions

We now present achievable rate regions for the PMABC-NRC and FTDBC-NRC protocols, where the 'N' stands for Network coding, 'R' stands for Random binning, 'C' stands for Cooperation. Due to space limitations all proofs are omitted but are available in [23] online.

A. PMABC-NRC Protocol

We allow the terminal nodes to *Cooperate* with each other in resolving the messages $w_{0,i}, \forall i \in \mathcal{B}$ - in addition to the flow-by-flow Network coding at the relay and the Random Binning at the base-station. In the following theorem, the U_i variables are the auxiliary random variables similar to those seen in Marton's BC-channel region [25] and its extension [23], while V_{0i} are auxiliary random variables used for binning the message $W_{0,i}$ at the base-station node 0 for node i. The V_{i1} are the auxiliary random variables for transmitting new information from node i to node 0. The V_{i2} are the auxiliary random variables for transmitting cooperative information from node i to other terminal nodes in the set $\mathcal{I}_i$, defined as the set of nodes which can decode $\hat{\mathbf{y}}_i^{(m+1)}(w_{\{i\},\mathcal{B}})$ at the end of transmission of node i. In a similar vein, $\mathcal{J}_i$ as the set of nodes whose quantized channel output is used at node i and so $\mathcal{I}_i = \{j | i \in \mathcal{J}_j \ , \ \forall j\}$. Note that we will derive achievable rate regions over all "cooperation" sets $\mathcal{I}_i$ and $\mathcal{J}_i$ and that the question of which sets to select is left for future work.

Theorem 1: An achievable rate region of the half-duplex bi-directional relay channel under the PMABC-NRC protocol is the closure of the set of all points $(R_{0,b}, R_{b,0})$ under given sets $\mathcal{I}_b$ and $\mathcal{J}_b$ for all $b \in \mathcal{B}$ satisfying (1) – (4) subject to $\Delta_{m+1} I(Y_i^{(m+1)}; \hat{Y}_i^{(m+1)} | Y_j^{(m+1)}) < \Delta_i I(V_{i2}^{(i)}; Y_j^{(i)} | Q)$ for all $j \in \mathcal{I}_i$. where $\mathcal{I}_i^{\min} = \text{argmin}_{j \in \mathcal{I}_i} \{\Delta_i I(V_{i2}^{(i)}; Y_j^{(i)} | Q) - \Delta_{m+1} I(Y_i^{(m+1)}; \hat{Y}_i^{(m+1)} | Y_j^{(m+1)})\}$ for all $i \in \mathcal{B}$ over all joint distributions of the form (5), $S(i) := \{j | j < i, j \in S\}$, and $V_{0i}, V_{i1}, V_{i2}, U_i$'s are the auxiliary random variables and $V_{0T} := \{V_{0s} | s \in T\}$ with $|\mathcal{Q}| \leq 2^{m+1} + m^2 + m + 2$ over the alphabet $\bigotimes_{i=0}^m \mathcal{X}_i \times \bigotimes_{j=1}^m (\mathcal{V}_{0j} \times \mathcal{V}_{j1} \times \mathcal{V}_{j2} \times \mathcal{U}_j) \times \mathcal{X}_{\mathsf{r}} \times \mathcal{Q}$.

We note that (1) result from the multiple access period, while (2) and (3) result from the relay broadcast period. Also, (4) and the condition relating Y_i and $\hat{Y}_i$ enable CF-based cooperation between terminal nodes.

B. FTDBC-NRC protocol

In the following theorem the auxiliary random variables $U_i, V_{0i}, V_{i1}, V_{i2}$ serve the same purpose as in Theorem 1.

Theorem 2: An achievable rate region of the half-duplex bi-directional relay channel under the FTDBC-NRC protocol is the closure of the set of all points $(R_{0,b}, R_{b,0})$ under given sets $\mathcal{I}_b$ and $\mathcal{J}_b$ for all $b \in \mathcal{B}$ satisfying (6) – (9) subject to $\Delta_{m+2} I(Y_i^{(m+2)}; \hat{Y}_i^{(m+2)} | Y_j^{(m+2)}) < \Delta_{i+1} I(V_{i2}^{(i+1)}; Y_j^{(i+1)})$ for all $j \in \mathcal{I}_i$ where $\mathcal{I}_i^{\min} = \text{argmin}_{j \in \mathcal{I}_i} \{\Delta_{i+1} I(V_{i2}^{(i+1)}; Y_j^{(i+1)}) - \Delta_{m+2} I(Y_i^{(m+2)} ; \hat{Y}_i^{(m+2)} | Y_j^{(m+2)})\}$ for all $i \in \mathcal{B}$ and $S \subseteq \mathcal{B}$ over all joint distributions as in (10) where $V_{0j}, V_{j1}, V_{j2}, U_j$'s are the auxiliary random variables and $V_{0T} := \{V_{0s} | s \in T\}$ over the alphabet $\bigotimes_{i=0}^m \mathcal{X}_i \times \bigotimes_{j=1}^m (\mathcal{V}_{0j} \times \mathcal{V}_{j1} \times \mathcal{V}_{j2} \times \mathcal{U}_j) \times \mathcal{X}_{\mathsf{r}}$.

Here (6) results from the relay decoding messages during the multiple access period, while (7) and (8) result from the relay broadcast period - where we see that the base-station and terminal nodes combine information from the relay and from direct links. Also, (4) and the condition relating Y_i and $\hat{Y}_i$ enable CF-based cooperation between terminal nodes.

V. Outer Bounds

The FMABC, PMABC and FTDBC outer bounds are obtained by applying the cut-set bound lemma tailored to half-duplex multi-phase protocols first derived in [3] to the different protocols, where the "cuts" will look different depending on what nodes are permitted to transmit during each phase. The PMABC outer bound is included as an example of the type of outer bounds obtained, the FMABC and FTDBC outer bounds and proofs are omitted for brevity and may be found in [23].

Theorem 3: (Outer bound) The capacity region of the half-duplex bi-directional relay channel under the PMABC protocol is outer bounded by the set of all points $(R_{0,b}, R_{b,0})$ for all $b \in \mathcal{B}$ satisfying

$$R_{\{0\},\mathcal{B}} \leq \sum_{i=1}^m \Delta_i I(X_0^{(i)}; Y_{\mathsf{r}}^{(i)} | X_i^{(i)}, Q) \tag{11}$$

$$R_{\mathcal{B},\{0\}} \leq \Delta_{m+1} I(X_{\mathsf{r}}^{(m+1)}; Y_0^{(m+1)}) \tag{12}$$

$$R_{\{0\},S} < \sum_{j=1}^{m} \Delta_j I(V_{0S}^{(j)}; Y_{\mathsf{r}}^{(j)}, V_{0\bar{S}}^{(j)} | V_{j1}^{(j)}, Q), \qquad R_{i,0} < \Delta_i I(V_{i1}^{(i)}; Y_{\mathsf{r}}^{(i)} | Q) \tag{1}$$

$$R_{\{0\},S} < \sum_{i\in S}\sum_{j\in\mathcal{J}_i} \Delta_j I(V_{0i}^{(j)}; Y_i^{(j)} | V_{j2}^{(j)}, Q) - \Delta_j I(V_{0i}^{(j)}; V_{0S(i)}^{(j)} | Q) + \sum_{j\notin\mathcal{J}_i} \Delta_j I(V_{0i}^{(j)}; Y_i^{(j)} | Q) - \Delta_j I(V_{0i}^{(j)}; V_{0S(i)}^{(j)} | Q)$$
$$+ \Delta_{m+1} I(U_i^{(m+1)}; Y_i^{(m+1)}, \hat{Y}_{\mathcal{J}_i}^{(m+1)}) - \Delta_{m+1} I(U_i^{(m+1)}; U_{S(i)}^{(m+1)}) \tag{2}$$

$$R_{S,\{0\}} < \Delta_{m+1} I(U_S^{(m+1)}; Y_0^{(m+1)}, U_{\bar{S}}^{(m+1)}) \tag{3}$$

$$R_{i,0} < \Delta_i I(V_{i1}^{(i)}; Y_{\mathsf{r}}^{(i)} | Q) + \Delta_i I(V_{i2}^{(i)}; Y_{\mathcal{I}_i^{\min}}^{(i)} | Q) - \Delta_i I(V_{i1}^{(i)}; V_{i2}^{(i)} | Q) - \Delta_{m+1} I(Y_i^{(m+1)}; \hat{Y}_i^{(m+1)} | Y_{\mathcal{I}_i^{\min}}^{(m+1)}) \tag{4}$$

$$p(q)\cdot \prod_{i=1}^{m} p^{(i)}(v_{01},\cdots,v_{0,\mathsf{m}},x_0|q) p^{(i)}(v_{i1},v_{i2},x_i|q)\cdot p^{(m+1)}(u_1,\cdots,u_m,x_{\mathsf{r}})\cdot p^{(m+1)}(y_{\mathcal{B}}|x_{\mathsf{r}})\cdot \prod_{i=1}^{m} p^{(m+1)}(\hat{y}_i|y_i). \tag{5}$$

$$R_{\{0\},S} < \Delta_1 I(V_{0S}^{(1)}; Y_{\mathsf{r}}^{(1)}, V_{0\bar{S}}^{(1)}), \qquad R_{i,0} < \Delta_{i+1} I(V_{i1}^{(i+1)}; Y_{\mathsf{r}}^{(i+1)}) \tag{6}$$

$$R_{\{0\},S} < \sum_{i\in S} \Delta_1 I(V_{0i}^{(1)}; Y_i^{(1)}) - \Delta_1 I(V_{0i}^{(1)}; V_{0S(i)}^{(1)}) + \Delta_{m+2} I(U_i^{(m+2)}; Y_i^{(m+2)}, \hat{Y}_{\mathcal{J}_i}^{(m+2)}) - \Delta_{m+2} I(U_i^{(m+2)}; U_{S(i)}^{(m+2)}) \tag{7}$$

$$R_{S,\{0\}} < \sum_{i\in S} \Delta_{i+1} I(V_{i1}^{(i+1)}; Y_0^{(i+1)}) + \Delta_{m+2} I(U_S^{(m+2)}; Y_0^{(m+2)}, U_{\bar{S}}^{(m+2)}) \tag{8}$$

$$R_{i,0} < \Delta_{i+1} I(V_{i1}^{(i+1)}; Y_{\mathsf{r}}^{(i+1)}) + \Delta_{i+1} I(V_{i2}^{(i+1)}; Y_{\mathcal{I}_i^{\min}}^{(i+1)}) - \Delta_{i+1} I(V_{i1}^{(i+1)}; V_{i2}^{(i+1)}) - \Delta_{m+2} I(Y_i^{(m+2)}; \hat{Y}_i^{(m+2)} | Y_{\mathcal{I}_i^{\min}}^{(m+2)}) \tag{9}$$

$$p^{(1)}(v_{01},\cdots,v_{0\mathsf{m}},x_0) \prod_{j=1}^{m} p^{(j+1)}(v_{j1},v_{j2},x_j) p^{(m+2)}(u_1,\cdots,u_m,x_{\mathsf{r}}) p^{(m+2)}(y_{\mathcal{B}}|x_{\mathsf{r}})\cdot \prod_{k=1}^{m} p^{(m+2)}(\hat{y}_k|y_k) \tag{10}$$

$$R_{S,\{0\}} \le \sum_{i\in S} \Delta_i I(X_i^{(i)}; Y_{\mathsf{r}}^{(i)}, Y_{\bar{S}}^{(i)} | X_0^{(i)}, Q) \tag{13}$$

$$R_{\{0\},S} \le \sum_{i\in \bar{S}} \Delta_i I(X_0^{(i)}, X_i^{(i)}; Y_S^{(i)} | Q) \tag{14}$$
$$+ \sum_{i\in S} \Delta_i I(X_0^{(i)}; Y_{S\setminus\{i\}}^{(i)} | X_i^{(i)}, Q) + \Delta_{m+1} I(X_{\mathsf{r}}^{(m+1)}; Y_S^{(m+1)})$$

for all choices of the joint distribution $p(q)\prod_{i=1}^{m} p^{(i)}(x_0|q)p^{(i)}(x_i|q)p^{(m+1)}(x_{\mathsf{r}})$ with $|\mathcal{Q}| \le 2^{m+1}-1$ over the restricted alphabet $\bigotimes_{i=0}^{m} \mathcal{X}_i \times \mathcal{X}_{\mathsf{r}}$ for all possible $S \subseteq \mathcal{B}$.

VI. Numerical Analysis

We assume an additive white Gaussian noise (AWGN) channel model, assume Gaussian input distributions for the achievability schemes, and evaluate the mutual information terms in order to obtain insight into the usage of terminal-node cooperation. The resulting fully worked out rate regions under specific Gaussian input assumptions are in [23].

A. Channel model

The corresponding mathematical channel model is, for each channel use k :

$$\mathbf{Y}[k] = \mathbf{H}\mathbf{X}[k] + \mathbf{Z}[k]$$

where $\mathbf{Y}[k]$, $\mathbf{X}[k]$ and $\mathbf{Z}[k]$ are independent, of unit power, additive, white Gaussian, complex and circularly symmetric, and $\mathbf{H} \in \mathbb{C}^{(m+2)\times(m+2)}$ relate the vector channel inputs and output, which are placed in the order $0,1,2,\cdots m,\mathsf{r}$. In phase ℓ, if node i is in transmission mode $X_i[k]$ follows the input distribution $X_i^{(\ell)} \sim \mathcal{CN}(0,P_i)$. Otherwise, $X_i[k] = \varnothing$, which means that the input symbol does not exist in the above mathematical channel model. $[\mathbf{H}]_{i,j} = h_{i,j}$ is the effective channel gain between transmitter i and receiver j which are assumed to be fully known to all nodes, i.e., full CSI.

B. Rate region comparisons with $m = 2$

We use the following channel gain matrix for $m = 2$ case:

$$\mathbf{H} = \begin{bmatrix} 0 & 0.3 & 0.05 & 1 \\ 0.3 & 0 & 1.5 & 1 \\ 0.05 & 1.5 & 0 & 0.2 \\ 1 & 1 & 0.2 & 0 \end{bmatrix} \tag{15}$$

To show the cooperation coding gain, we plot the achievable rate region of the different protocols with and without cooperation. In Fig. 3, we fixed the data rates $(R_{0,1}, R_{2,0})$ to the rate pair ((0.19,0.01) and plot rate regions in the $(R_{1,0}, R_{0,2})$ domain. We do this to highlight the cooperation gain, which comes from re-allocating node 1's transmission resources (i.e. relative power) to the two information flows; $1 \to \mathsf{r}$ $(R_{1,0})$ and $1 \to 2$ $(R_{0,2})$. As expected -NRC protocols achieve much better performance than -NR protocols, which were outlined in part I of this work. Notably, the cooperation protocols improve $R_{0,2}$ without any degradation of $R_{1,0}$ in the FTDBC protocol. In contrast, the maximum $R_{1,0}$ of the PMABC-NRC protocol is less than that of its PMABC-NR only protocol. We explain this by the fact that in our achievable rate region, we used a simplified and sub-optimal (successive decoding like) receiver in the PMABC-NRC protocol instead of using a fully general joint-decoder (as is done in the simpler PMABC-NR protocol), which limits the $R_{1,0}$. If we were to enhance the PMABC-NRC scheme by using the general joint decoder, the

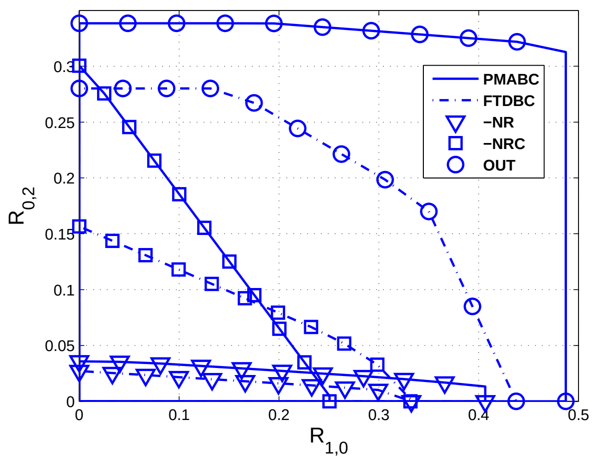

Fig. 3. Comparison with $P_0 = P_1 = P_2 = P_r = 0$ dB, and $R_{0,1} = 0.19$, $R_{2,0} = 0.01$.

maximum $R_{1,0}$ would be reached and the overall performance would improve - a technically challenging task left for future work. We furthermore expect the gains of cooperation to increase if many more terminal nodes are able to exploit node 1's cooperative broadcasting; however these situations with current regions are too complex to be evaluated numerically.

VII. Conclusion

In this paper, we presented cut-set based outer bounds for multi-pair bi-directional relay networks, and derived inner bounds using compress-and-forward-based cooperation between terminal nodes. As expected, cooperation gains may significantly improve upon the already fairly sophisticated network coding and random-binning based schemes presented in part I of this work. As we have evaluated the obtained regions in Gaussian noise, in the future we intend to pursue finite-gap capacity results in the flavor of [26] for this channel by careful choice of powers, compression and phase-duration parameters, as well as seek simplified metrics and/or sub-schemes for quantifying the gains seen in using cooperation for more than 2 terminal nodes.

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