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Research Article

Achievable Rates and Scaling Laws for Cognitive Radio Channels

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Cognitive radios have the potential to vastly improve communication over wireless channels. We outline recent information theoretic results on the limits of primary and cognitive user communication in single and multiple cognitive user scenarios. We first examine the achievable rate and capacity regions of single user *cognitive channels*. Results indicate that at medium SNR (0–20 dB), the use of *cognition* improves rates significantly compared to the currently suggested spectral gap-filling methods of secondary spectrum access. We then study another information theoretic measure, the multiplexing gain. This measure captures the number of point-to-point Gaussian channels contained in a cognitive channel as the SNR tends to infinity. Next, we consider a cognitive network with a single primary user and multiple cognitive users. We show that with single-hop transmission, the sum capacity of the cognitive users *scales linearly* with the number of users. We further introduce and analyze the *primary exclusive radius*, inside of which primary receivers are guaranteed a desired outage performance. These results provide guidelines when designing a network with secondary spectrum users.

1. INTRODUCTION

Secondary spectrum usage is of current interest worldwide. Regulatory bodies, including the Federal Communications Commission (FCC) [1] in the US and the European Commission (EC) [2] in Europe, have been licensing entities, such as cellular companies, exclusive rights to portions of the wireless spectrum, and leaving some small *unlicensed* bands, such as the 2.4 GHz Wi-Fi band, for public use. Managing the spectrum this way, however, is nonoptimal. The regulatory bodies have come to realize that, most of the time, large portions of certain licensed frequency bands remain underutilized [3]. To remedy this situation, legislators are easing the way frequency bands are licensed and used. In particular, new regulations would allow for devices which are able to sense and adapt to their spectral environment, such as cognitive radios, to become *secondary* or *cognitive users.* These cognitive users opportunistically employ the spectrum of the *primary users* without excessively harming the latter. Primary users are generally associated with the primary spectral license holder, and thus have a higher priority right to the spectrum.

The intuitive goal behind secondary spectrum licensing is to increase the spectral efficiency of the network, while, depending on the type of licensing, not affecting higher priority users. The exact regulations governing secondary spectrum licensing are still being formulated [4], but it is clear that networks consisting of heterogeneous devices, both in terms of physical capabilities and in the right to the spectrum, are beneficial and emerging.

Of interest in this work is *dynamic spectrum leasing* [4], in which some *secondary* wireless devices opportunistically employ the spectrum granted to the *primary* users. In order to efficiently use the spectrum, we require a device which is able to sense the communication opportunities and take actions based on the sensed information. Cognitive radios are prime candidates.

1.1. Cognitive radios and behavior

Over the past few years, the incorporation of software into radio systems has become increasingly common. This has allowed for faster upgrades, and has given such wireless devices the ability to transmit and receive using a variety of protocols and modulation schemes. This is enabled by reconfigurable software rather than hardware. Mitola [5] took the definition of a software-defined radio one step further, and envisioned a radio which could make decisions as to the network, modulation, and/or coding parameters *based on its surroundings*, and called such a *smart* radio a *cognitive radio*. Such radios could even adapt their transmission strategies to the availability of nearby collaborative nodes, or the regulations dictated by their current location and spectral conditions.

1.2. Outline of this paper

How cognitive radios and their adaptive nature may best be employed in secondary spectrum licensing scenarios is a question being actively pursued from a number of angles. From the fundamental limits of communication at the physical layer to game theoretic analyses at the network level to legal and regulatory issues, this new and exciting field still has many unanswered questions. We outline recent results on one particular subset of cognitive radio research, the fundamental limits of communication. Information theory provides an ideal framework for analyzing this question. The theoretical and ultimately limiting capacity and rate regions achieved in a network with cognitive radios may be used as benchmarks for gauging the efficiency of any practical cognitive radio system.

This paper explores the limits of communication in cognitive channels from three distinct yet related information theoretic angles in its three main sections.

Section 2 looks at the simplest scenario, in which a primary user and a secondary, or cognitive, user wish to communicate over the same channel. We introduce the Gaussian cognitive channel, a two-transmitter two-receiver channel, in which the secondary transmitter knows the message to be transmitted by the primary. This asymmetric message knowledge is what we will term *cognition*, and is precisely what will be exploited to demonstrate better achievable rates than the currently proposed time-sharing schemes for secondary spectrum access. We outline the intuition behind the best-known information theoretic achievable rate regions and compare these regions, at medium SNRs, to channels in which full and no-transmitter cooperation is employed.

Section 3 considers the multiplexing gain of the Gaussian cognitive channel. The multiplexing gain is a different information theoretic measure which captures the number of point-to-point channels contained in a multiple-input multiple-output (MIMO) channel when noise is no longer an impediment, that is, as SNR $\to\infty$. We review recent results on the multiplexing gains of the cognitive as well as the cognitive X-channels.

Section 4 shifts the emphasis from a single-user cognitive channel to a network of cognitive radios. We first explore the scaling laws (as the number of cognitive users approaches infinity) of the sum rate of a network of cognitive devices. We show that with single-hop transmission, provided that each cognitive transmitter and receiver pair is within a bounded distance of each other, a cognitive network can achieve a linear sum-rate scaling. We then examine a primary exclusive radius, which is designed to guarantee an outage performance for the primary user. We provide analytical bounds on this radius, which may help in the design of cognitive networks.

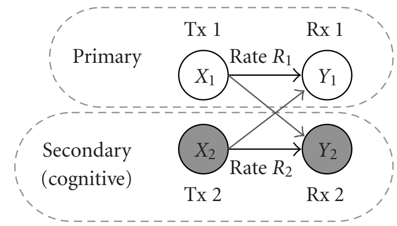

Figure 1: A simple channel in which the primary transmitter Tx 1 wishes to transmit a message to the primary receiver Rx 1, and the secondary (or cognitive) transmitter Tx 2 wishes to transmit a message to its receiver Rx 2. We explore the rates R_1 and R_2 that are achievable in this channel.

2. THE COGNITIVE CHANNEL: RATE REGIONS

We start our discussion by looking at a simple scenario, in which primary and secondary (or cognitive) users share a channel. Consider a primary transmitter and receiver pair (Tx 1 $\to$ Rx 1) which transmits over the same spectrum as a cognitive secondary transmitter and receiver pair (Tx 2 $\to$ Rx 2) as in Figure 1.

One of the major contributions of information theory is the notion of *channel capacity*. Qualitatively, it is the maximum rate at which information may be sent *reliably* over a channel. When there are multiple information streams being transmitted, we can speak of capacity regions as the maximum set of all rates which can be simultaneously reliably achieved. For example, the capacity region of the channel depicted in Figure 1 is a two-dimensional region, or a set of rates (R_1, R_2), where R_1 is the rate between (Tx 1 $\to$ Rx 1), and R_2 is the rate between (Tx 2 $\to$ Rx 2). For any point (R_1, R_2) inside the capacity region, the rate R_1 on the x-axis corresponds to a rate that can be reliably transmitted at simultaneously, over the same channel, with the rate R_2 on the y-axis. An *achievable rate/region* is an inner bound on the capacity region. Such regions are obtained by suggesting a particular coding (often random coding) scheme and proving that the claimed rates can be reliably achieved, that is, the probability of a decoding error vanishes with increasing block size.

2.1. Cognition: asymmetric message knowledge

What differentiates the cognitive radio channel from a basic two-sender two-receiver interference channel is the asymmetric *message knowledge* at the transmitters, which in turn allows for *asymmetric cooperation* between the transmitters. This message knowledge is possible due to the properties of cognitive radios. If Tx 2 is a cognitive radio and geographically close to the primary Tx 1 (relative to the primary receiver Rx 1), then the wireless channel (Tx 1 $\to$ Tx 2) could be of much higher capacity than the channel (Tx 1 $\to$ Rx 1). Thus in a fraction of the transmission time, Tx 2 could listen

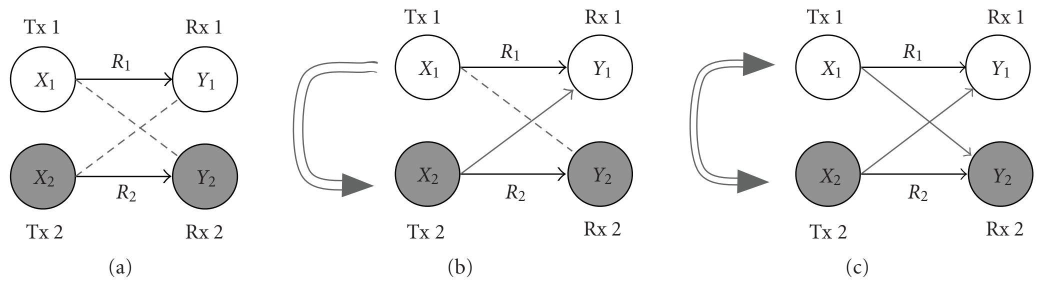

Figure 2: (a) Competitive behavior: the interference channel. The transmitters may not cooperate. (b) Cognitive behavior: the cognitive channel. Asymmetric transmitter cooperation is possible. (c) Cooperative behavior: the two Tx antenna broadcast channel. The transmitters, but not the receivers, may fully and symmetrically cooperate. In these figures, solid lines indicate desired signal paths, while dotted lines indicate undesired (or interfering) signal paths.

to, and obtain, the message transmitted by Tx 1. It could then employ this message knowledge—which translates into exact knowledge of the interference it will encounter—to intelligently attempt to mitigate it.

For the purpose of this paper, we idealize the message knowledge: we suppose that rather than causally obtaining Tx 1 message, Tx 2 is given the message fully prior to transmission. We call this *noncausal* message knowledge. This idealization will provide an upper bound to any real-world scenario, and the solutions to this problem may provide valuable insight to the fundamental techniques that could be employed in such a scenario. The techniques used in obtaining the limits on communication for the channel employing a genie could be extended to provide achievable regions for the case in which Tx 2 obtains Tx 1 message causally. We have suggested causal schemes in [6].

For the purpose of this paper, we also assume that all nodes have full channel-state information at the transmitters as well as the receivers (CSIT and CSIR), meaning that all Txs and Rxs know the channel. This idealization provides an outer bound with respect to what may be achieved in practice. This CSIT may be obtained through various techniques such as, for example, feedback from the receivers or channel reciprocity [7]. One particular challenge in obtaining CSIT in a cognitive setting is obtaining the cross-over channel parameters. That is, if a feedback method is used, the primary Tx and secondary Rx (and likewise the primary Rx and secondary Tx) may need to cooperate to exchange the CSIT.

2.2. The cognitive channel in a classical setting

The key property of a cognitive channel is its asymmetric noncausal message knowledge. This asymmetric transmitter cooperation may be compared to classical information theoretic channels as follows. As shown in Figure 2, there are three possibilities for transmitter cooperation in a two-transmitter (2 Tx) two-receiver (2 Rx) channel. In all of these channels, each receiver decodes independently. Transmitter cooperation in this figure is denoted by a directed double line. These three channels are simple examples of the cognitive decomposition of wireless networks seen in [8], and encompass three possible types of transmitter cooperation or behavior as follows.

(a) *Competitive behavior*: the two transmitters transmit independent messages. There is no cooperation in sending the messages, and thus the two users*compete* for the channel. Such a channel is equivalent to the two-sender two-receiver information theoretic interference channel [9, 10].

(b) *Cognitive behavior*: asymmetric cooperation is possible between the transmitters. This asymmetric cooperation is a result of Tx 2 knowing Tx 1 message, but not vice-versa, and is indicated by the one-way double arrow between Tx 1 and Tx 2. We idealize the concept of message knowledge: whenever the cognitive node Tx 2 is able to hear and decode the message of the primary node Tx 1, we assume it has full a priori knowledge (we use the terms a priori and noncausal interchangeably). We use the term cognitive behavior, or *cognition*, to emphasize the need for Tx 2 to be a *smart* device capable of altering its transmission strategy according to the message of the primary user.

(c) *Cooperative behavior*: the two transmitters know each other's messages (two way double arrows) and can thus fully and symmetrically cooperate in their transmission. The channel pictured in Figure 2(c) may be thought of as a two-antenna sender, two-single-antenna receivers broadcast channel [11].

We are interested in determining the fundamental limits of communication over wireless channels in which transmitters cooperate in an asymmetric fashion. To do so, we approach the problem from an information theoretic perspective, an approach that had thus far been ignored in cognitive radio literature.

2.3. Achievable rates in Gaussian cognitive channels

In [6, 12], achievable rate regions are derived for the discrete cognitive channel. We refer the interested reader to these works as well as [13, 14] for further results on achievable rate regions for the discrete cognitive channel. Here, we consider the Gaussian cognitive channel for a few central reasons. First, Gaussian noise channels are the most commonly considered continuous alphabet channel and are often used to model noisy channels. Secondly, Gaussian noise channels are computation ally tractable and easy to visualize as they often have the property that the optimal capacity-achieving input distribution is Gaussian as well. The physical Gaussian cognitive channel is described by the relations in (1) as (notice that we have assumed the channel gains between (Tx 1, Rx 1) as well as (Tx 2, Rx 2) are all 1. This can be assumed WLOG by multiplying the entire receive chain at Rx 1 by any (noninfinite)$1/a_{11}^2$, and the receive chain at Rx 2 by (noninfinite)$1/a_{22}^2$ without altering the achievable and/or capacity results),

$$\begin{aligned} Y_1 &= X_1 + a_{21}X_2 + Z_1, \\ Y_2 &= a_{12}X_1 + X_2 + Z_2, \end{aligned} \tag{1}$$

where a_{12} and a_{21} are the crossover (channel) coefficients, $Z_1 \sim \mathcal{N}(0, Q_1)$ and $Z_2 \sim \mathcal{N}(0, Q_2)$ independent additive white Gaussian noise (AWGN) terms, X_1 and X_2 channel inputs with average powers constraints P_1 and P_2, respectively, and Tx 2 given the message encoded by X_1 as well as X_1 itself noncausally.

The key technique used to improve rates in the cognitive channel is *interference mitigation*, or *dirty-paper coding*. This coding technique was first considered by Costa [15], where he showed that in a Gaussian noise channel with noise N of power Q, input X, subject to a power constraint $E[|X|^2] \le P$ and additive interference S of arbitrary power known noncausally to the *transmitter* but not the receiver,

$$Y = X + S + N, \qquad E[|X|]^2 \le P, \qquad N \sim \mathcal{N}(0, Q), \tag{2}$$

has the same capacity as an interference-free channel, or

$$C = \frac{1}{2}\log_2\left(1 + \frac{P}{Q}\right). \tag{3}$$

This remarkable and surprising result has found its application in numerous domains including data storage [16, 17], and watermarking/steganography [18], and most recently, has been shown to be the capacity-achieving technique in Gaussian MIMO broadcast channels [11, 19]. We now apply dirty-paper coding techniques to the Gaussian cognitive channel.

The Gaussian cognitive channel has an interesting and elegant relation to the Gaussian MIMO broadcast channel, which is equivalent to Figure 2(c). In the latter channel, a single transmitter with (possibly) multiple antennas wishes to transmit distinct messages to independent noncooperating receivers, which may also have multiple antennas. The capacity region of the Gaussian MIMO broadcast channel was recently proven to be equal to the region achieved through dirty-paper coding [11], a technique useful whenever a transmitter has noncausal knowledge of interference. We consider a two-transmit-antenna broadcast channel with two independent single-receiver antennas, where the physical channel is described by (1). Let $H_1 = [1\ a_{21}]$ and $H_2 = [a_{12}\ 1]$. Let $X \succeq 0$ denote that the matrix X is positive semidefinite. Then the capacity region of this two-transmit-antenna Gaussian MIMO broadcast channel, under per-antenna power constraints of P_1 and P_2, respectively, may be expressed as the region (4). We note that most of the MIMO broadcast channel literature assumes a *sum power* constraint over the antennas rather than *per-antenna* power constraints as assumed here. However, the framework of [11], which is tailored to the cognitive problem here, is able to elegantly capture both of these constraints.

MIMO BC region = Convex hull of

$$\begin{aligned}
&(R_1, R_2): \\
&\quad R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1 + B_2)H_1^\dagger + Q_1}{H_1(B_2)H_1^\dagger + Q_1}\right) \\
&\quad R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger + Q_2}{Q_2}\right) \\
&\quad\qquad \bigcup \\
&\quad R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1)H_1^\dagger + Q_1}{Q_1}\right) \\
&\quad R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_1 + B_2)H_2^\dagger + Q_2}{H_2(B_1)H_2^\dagger + Q_2}\right) \\
&\quad B_1, B_2 \succeq 0, \\
&\quad B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}, \\
&\quad B_2 = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix}, \\
&\quad B_1 + B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2 \end{bmatrix}, \\
&\quad z^2 \le P_1 P_2.
\end{aligned} \tag{4}$$

The transmit covariance matrix B_k is a positive semidefinite 2×2 whose element $B_k(i, j)$ describes the correlation between the message k at Tx i and Tx j. That is, the encoded signals transmitted on the two transmit antennas are the superposition (sum) of two Gaussian codewords, one corresponding to each message. These codewords are selected from randomly generated Gaussian codebooks which are generated according to $\mathcal{N}(0, B_1)$ for message 1 and $\mathcal{N}(0, B_2)$ for message 2. The constraints on the transmit covariance matrices B_1 and B_2 ensure the matrices are proper covariance matrices (positive semidefinite), and the per-antenna power constraints are met.

We now relate the MIMO broadcast channel region specific to the two-transmit-antenna case to the cognitive channel. Recall that the cognitive channel has the same physical channel model as the MIMO broadcast channel, but the messages are not known at both antennas.

In order to capture this asymmetry, we must restrict the set of transmit covariance matrices to certain forms. Specifically, in the Gaussian cognitive channel, the transmit matrices (B_1, B_2) must lie in the set $\mathcal{B}$, defined as

$$\begin{aligned} \mathcal{B} = \Bigg\{ &(B_1, B_2) \mid B_1,\ B_2 \succeq 0,\ B_1 + B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2 \end{bmatrix}, \\ &B_2 = \begin{pmatrix} 0 & 0 \\ 0 & x \end{pmatrix},\ x \in \mathbb{R}^+ \Bigg\}. \end{aligned} \tag{5}$$

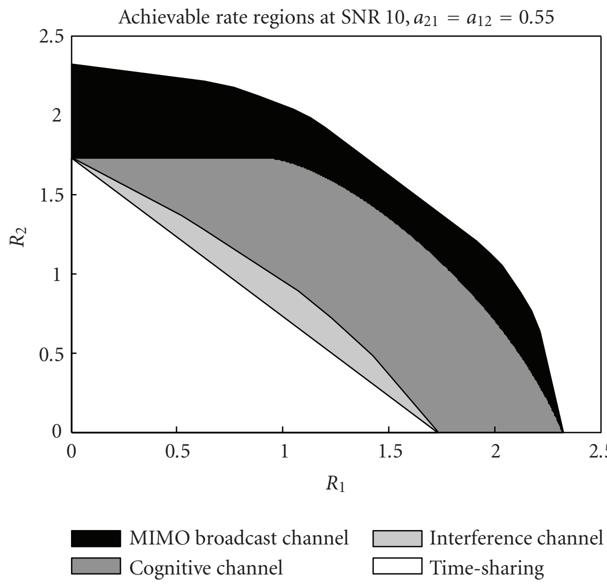

FIGURE 3: Capacity region of the Gaussia 2×1 MIMO two-receiver broadcast channel (outer), cognitive channel (middle), achievable region of the interference channel (second smallest) and time-sharing (innermost) region for Gaussian noise power $Q_1 = Q_2 = 1$, power constraint $P_1 = P_2 = 10$ at the two transmitters, and channel parameter $a_{21} = 0.55, a_{12} = 0.55$.

The covariance matrix corresponding to message 1, B_1, may have nonzero elements at all locations. This is because message 1 is known by both transmitters, and thus message 1 may be encoded and placed onto both antennas. In contrast, B_2 may only have a nonzero element $B_2(2,2)$ as transmit antenna 2 is the only one that knows message 2, and thus power related to message 2 can only be placed at that antenna. An achievable rate region for the Gaussian cognitive channel may then be expressed as (6). It is of interest to note that this region is exactly that of [20], and furthermore, corresponds to the complete capacity region when $a_{21} \le 1$, as shown in [20],

Cognitive region = Convex hull of

$$
\begin{aligned}
&(R_1, R_2) : \\
&\quad R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^{\dagger}+Q_1}{H_1(B_2)H_1^{\dagger}+Q_1}\right) \\
&\quad R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^{\dagger}+Q_2}{Q_2}\right) \\
&\quad B_1, B_2 \succeq 0, \\
&\quad B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}, \\
&\quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & c_{22} \end{bmatrix}, \\
&\quad B_1 + B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2 \end{bmatrix}, \\
&\quad z^2 \le P_1 P_2.
\end{aligned}
\tag{6}
$$

We evaluate the bounds by varying the power parameters and compare four regions related to the cognitive channel in Figure 3. We illustrate the regions when the transmitters have identical powers ($P_1 = P_2 = 10$) and identical receiver noise powers ($Q_1 = Q_2 = 1$). The crossover coefficients in the interference channel are $a_{12} = a_{21} = 0.55$, while the direct coefficients are 1. The four regions, from smallest to largest, illustrated in Figure 3 correspond to the following.

(a) The time-sharing region displays the result of pure time sharing of the wireless channel between Tx 1 and Tx 2. Points in this region are obtained by letting Tx 1 transmit for a fraction of the time, during which Tx 2 refrains, and vice versa.
(b) The interference channel region corresponds to the best-known achievable region [21] of the classical information theoretic interference channel. In this region, both senders encode independently, and there is no a priori message knowledge by either transmitter of the other's message.
(c) The cognitive channel region is described by (6). We see that both users—not only the incumbent Tx 2 which has the extra message knowledge—benefit from using this scheme. This is expected: if Tx 2 allocated power to mitigate interference from Tx 1, it boosts R_2 rates, while allocating power to amplifying Tx 1 message boosts R_1 rates, and so gracefully, combining the two will yield benefits to both users.
(d) The capacity region of the two-transmit-antenna Gaussian broadcast channel [11], subject to individual-transmit-antenna power constraints P_1 and P_2, respectively, is described by (4). The multiple antenna broadcast channel region is an outer bound of any achievable rate region for the cognitive channel: the only difference between the two is the symmetry of the cooperation. In the cognitive channel, Tx 2 knows Tx 1 message, but not vice versa. In the MIMO broadcast channel, both transmitters know each others' messages.

From Figure 3, we see that both users–not only the incumbent Tx 2 which has the extra message knowledge–benefit from behaving in a cognitive, rather than simple time-sharing, manner. Time sharing would be the maximal theoretically achievable region in spectral gap-filling models for cognitive channels. That is, under the assumption that an incumbent cognitive was to perfectly sense the gaps in the spectrum and fill them by transmitting at the capacity of the point-to-point channel between (Tx 2, Rx 2), the best rate region one can hope to achieve is the time-sharing rate region.

The largest region is naturally the one in which the two transmitters fully cooperate. However, such a scheme is also unreasonable in a secondary spectrum licensing scenario in which a primary user should be able to continue transmitting in the same fashion regardless of whether a secondary cognitive user is present or not. The cognitive channel, with asymmetric transmitter cooperation shifts the burden of cooperation to the opportunistic secondary user of the channel.

3. THE MULTIPLEXING GAINS OF COGNITIVE CHANNELS

The previous section showed that when two interfering point-to-point links act in a*cognitive* fashion, or employ asymmetric noncausal side information, interference may be at least partially mitigated, allowing for higher spectral efficiency. It is thus possible for the cognitive secondary user to communicate at a nonzero rate while the primary user suffers no loss in rate. At medium SNR levels (Figure 3 operates at a receiver SNR of 10), there is a definitive advantage to cognitive transmission. One immediate question that arises is how cognitive transmission performs in the high SNR regime, when noise is no longer an impediment. For Gaussian noise channels, the *multiplexing gain* is defined as the limit of the ratio of the maximal achieved sum rate, $R(\text{SNR})$ to the $\log(\text{SNR})$ as the SNR tends to infinity (note that the usual factor 1/2 is omitted in any rate expressions, but rather, the number of times the sum rate looks like $\log(\text{SNR})$ is the multiplexing gain. Also, the SNR on all links is assumed to grow at the same rate). That is,

$$\text{multiplexing gain} := \lim_{\text{SNR}\to\infty} \frac{R(\text{SNR})}{\log(\text{SNR})}. \tag{7}$$

Since a Gaussian noise point-to-point channel has channel capacity

$$C = \frac{1}{2}\log_2(1+\text{SNR}), \tag{8}$$

as the $\text{SNR}\to\infty$, the capacity of a single point-to-point channel scales as $\log_2(\text{SNR})$.

The multiplexing gain is thus a measure of how well a MIMO channel is able to avoid self interference. This is particularly relevant in studying cooperative communication in distributed systems where multiple Txs and Rxs wish to share the same medium. It may be thought of as the number of *parallel* point-to-point channels captured by the MIMO channel. As such, the multiplexing gain of various multiple-input multiple-output systems has been recently studied in the literature [22]. For the single user point-to-point MIMO channel with M_T transmit and N_R receive antennas, the maximum multiplexing gain is known to be $\min(M_T, N_R)$ [23, 24]. For the two user MIMO multiple-access channel (MAC) with N_R receive antennas and M_{T_1}, M_{T_2} transmit antennas at the two transmitters, the maximal multiplexing gain is $\min(M_{T_1} + M_{T_2}, N_R)$. Its dual [25], the two user MIMO broadcast channel (BC) with M_T transmit antennas and N_{R_1}, N_{R_2} receive antennas at the two transmitters, respectively, the maximum multiplexing gain is $\min(M_T, N_{R_1} + N_{R_2})$. These results, as outlined in [22], demonstrate that when joint signal processing is available at either the transmit or receive sides (as is the case in the MAC and BC channels), then the multiplexing gain is significant. However, when joint processing is not possible neither at the transmit nor receive sides, as is the case for the interference channel, then the multiplexing gain is severely limited. Results for the maximal multiplexing gain when cooperation is permitted at the transmitter or receiver side through noisy communication channels can be found in [26, 27].

In the cognitive radio channel, a form of partial joint processing is possible at the transmitter. It is thus unclear whether this channel will behave more like the MAC and BC channels, or whether it will suffer from interference at high SNR as in the interference channel. In [28], it was shown that the multiplexing gain of the cognitive channel is one. That is, only one stream of information may be sent by the primary and/or secondary transmitters. Thus, just like the interference channel, the cognitive radio channel, at high SNR, is fundamentally interference limited.

4. SCALING LAWS OF COGNITIVE NETWORKS

The previous two sections consider an achievable rate region and the multiplexing gain of a single cognitive user channel. In this section, we outline recent results on *cognitive networks*, in which multiple secondary users (cognitive radios) as well as primary users must share the same spectrum [29, 30]. Naturally, cognitive users should only be granted spectrum access if the induced performance degradation (if any at all) on the primary users is acceptable. Specifically, the interference from the cognitive users to the primary users must be such that an outage performance may be guaranteed for the primary user. With the additional complexity of multiple users in a network setting, in contrast to the previous two sections, here we assume that the cognitive users have no knowledge of the primary user messages. In other words, we assume all devices encode and decode their messages independently.

In a network of primary and secondary devices, there are numerous interesting questions to be pursued. We focus on two fundamental questions: what is the minimum distance from a primary user at which secondary users can start transmitting to guarantee a primary outage performance, and, how does the total throughput achieved by these cognitive users scale with the number of users?

The scaling law question is closely related to results on ad-hoc network. Initiated by the work of Gupta and Kumar [31], this area of research has been actively pursued under a variety of wireless channel models and communication protocol assumptions [32–41]. These papers usually assume n pairs of ad-hoc devices are randomly located on a plane. Each transmitter has a single, randomly selected receiver. The setting can be either an extended network, in which the node density stays constant and the area increases with n, or a dense network, in which the network area is fixed and the node density increases with n. The scaling of the network throughput as $n\to\infty$ then depends on the node distribution and on the signal processing capability. Results in the literature can be roughly divided into two groups. When nodes in the ad-hoc network use only the simple decode-and-forward scheme without further cooperation, then the per user network capacity decreases as $1/\sqrt{n}$ as $n\to\infty$ [31, 32, 35]. This decreasing capacity can be viewed as a consequence of the unmitigated interference experienced. In contrast, when nodes are able to cooperate, using more sophisticated signal processing, the per user capacity approaches a constant [41].

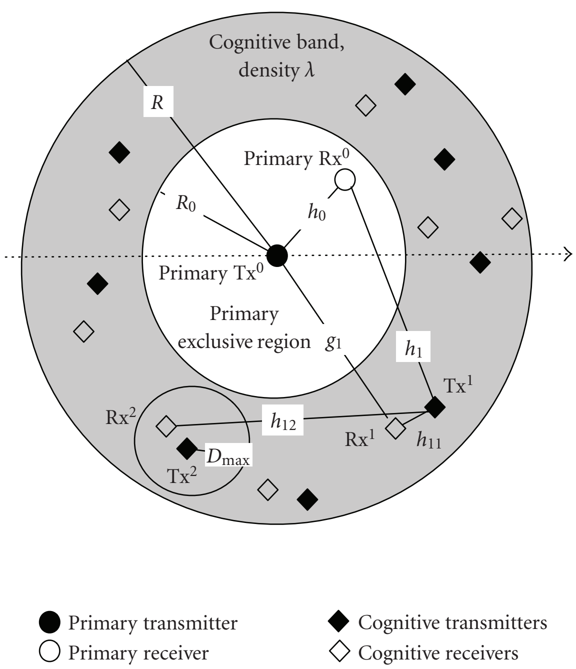

FIGURE 4: A cognitive network: a single primary transmitter Tx^0 is placed at the origin and wishes to transmit to its primary receiver Rx^0 in the circle of radius R_0 (the *primary exclusive region*). The n cognitive nodes are randomly placed with uniform density λ in the shaded *cognitive band*. The cognitive transmitter Tx^i wishes to transmit to a single cognitive receiver Rx^i which lies within a distance $< D_{max}$ away. The cognitive transmissions must satisfy a primary outage constraint.

In this work, we study a cognitive network of the interference-limited type, in which nodes simply treat other signals as noise. Because of the opportunistic nature of the cognitive users, we consider a network and communication model different from the previously mentioned ad-hoc networks. We assume that each cognitive transmitter communicates with a receiver within a *bounded distance* D_{max}, using *single-hop* transmission. Different from multihop communication in ad-hoc networks, single-hop communication appears suitable for cognitive devices which are mostly short range. Our results, however, are not limited to short-range communication. There can be other cognitive devices (transmitters and receivers) in between a Tx-Rx pair. This is different from the local scenarios of ad-hoc networks, in which every node is talking to its neighbor. In practice, we may preset a D_{max} based on a large network and use the same value for all networks of smaller sizes. (If we allow the cognitive devices to scale its power according to the distance to the primary user, then D_{max} may scale with the network size by a feasible exponent.) Furthermore, we assume that any interfering transmitter must be at a nonzero distance away from the interfered receiver.

We find that with *single-hop* transmission, the network capacity scales *linearly* ($O(n)$) in the number of cognitive users. Equivalently, in the limit as the number of cognitive users tends to infinity, the *per-user capacity* remains constant. Our results thus indicate that an initial approach to building a scalable cognitive network should involve limiting cognitive transmissions to a single hop. This scheme appears reasonable for secondary spectrum usage, which is opportunistic in nature.

In the following sections, we summarize our results for the network case with multiple cognitive users and a single primary user, assuming constant transmit power for both types of users. These results have been extended to networks with multiple primary users and to the scenario in which the cognitive transmitters can scale their power according to their distance to the primary user. Due to space limitation, however, we refer the readers to [30] for details on these extensions.

4.1. Problem formulation

Our problem formulation may be summarized as follows. We consider a single primary user at the center of a network wishing to communicate with a primary receiver located within the primary exclusive region of radius R_0. In the same plane outside this radius, we throw n cognitive transmitters, each of which wishes to transmit to its own cognitive receiver within a fixed distance away. We then obtain lower and upper bounds on the total sum rate of the n cognitive users as $n \to \infty$, and establish the scaling law. Next, we proceed to examine the outage constraint on the primary user rate in terms of cognitive node placement. We analyze the *exclusive region* radius R_0 around the primary transmitter, in which the primary user has the *exclusive* right to transmit and no cognitive users may do so.

4.1.1. Network model

We introduce our network model in Figure 4. We assume that all users transmitters and receivers are distributed on a plane. Let Tx^0 and Rx^0 denote the primary transmitter and receiver, while Tx^i and Rx^i are pairs of secondary transmitters and receivers, respectively, $i = 1, 2, \dots, n$. The primary transmitter is located at the center of the primary exclusive region with radius R_0, and the primary receiver can be located anywhere within this exclusive region. This model is based on the premises that the primary receiver location may not be known to the cognitive users, which is typical in, for example, broadcast scenarios. All the cognitive transmitters and receivers, on the other hand, are distributed in a ring outside this exclusive region with an outer radius R. We assume that the cognitive transmitters are located randomly and uniformly in the ring. Each cognitive receiver, however, is within a D_{max} distance from its transmitter. We also assume that any interfering cognitive transmitter must be at least a distance ϵ away from the interfered receiver, for some $\epsilon > 0$. This practical constraint simply ensures that the interfering transmitters and receivers are not located at the same point. Furthermore, the cognitive user density is constant at λ users-per-unit area. The outer radius R therefore grows as the number of cognitive users increases. The notation is summarized in Table 1.

Table 1: Variable names and definitions.

Definitions	Variable names
Primary transmitter and receiver	Tx^0, Rx^0
Cognitive user ith transmitter and receiver	Tx^i, Rx^i
Primary exclusive region radius	R_0
Outer radius for cognitive transmission	R
Channel from Tx^0 to Rx^0	h_0
Channel from Tx^0 to Rx^i	g_i
Channel from Tx^i to Rx^0	h_i
Channel from Tx^i to Rx^j	h_{ij}
Number of cognitive users	n
Maximum cognitive Tx^i-Rx^i distance	$D_{\max}$
Minimum cognitive Tx^i-Rx^k distance ($i \neq k$)	ϵ
Cognitive user density	λ

4.1.2. Signal and interference characteristics

The received signal at Rx^0 is denoted by y_0, while that at Rx^i is denoted by y_i. These relate to the signals x_0 transmitted by the primary Tx^0 and x_i by the cognitive Tx^i as

$$\begin{aligned} y_0 &= h_0 x_0 + \sum_{i=1}^{n} h_i x_i + n_0, \\ y_i &= h_{ii} x_i + g_i x_0 + \sum_{j \neq i} h_{ji} x_j + n_i. \end{aligned} \tag{9}$$

We assume that each user has no knowledge of each other's signal, and hence treats other signals as noise. By the law of large numbers, the total interference can then be approximated as Gaussian. Thus all users optimal signals are zero-mean Gaussian (optimal input distribution for a Gaussian noise channel [42]) and independent. While treating other signals as noise is not necessarily capacity optimal, it provides us with a simple, easy to implement lower bound on the achievable rates. These rates may be improved later by using more sophisticated encoding and decoding schemes.

4.1.3. Channel model

We consider a path-loss only model for the wireless channel. Given a distance d between the transmitter and the receiver, the channel is therefore given as

$$h = \frac{A}{d^{\alpha/2}}, \tag{10}$$

where A is a frequency-dependent constant and α is the power path loss. We consider $\alpha > 2$, which is typical in practical scenarios.

4.2. Cognitive network throughput and primary exclusive region

We are interested in two measures: the sum rate of all cognitive users and the optimal radius of the primary exclusive region. Assume that each cognitive user transmits with the same power P, and the primary user transmits with power P_0. Denote I_i ($i = 0, \ldots, n$) as the total interference power from the cognitive transmitters to user i, then

$$\begin{aligned} I_0 &= \sum_{i=1}^{n} P \left| h_i \right|^2, \\ I_i &= \sum_{j \neq i} P \left| h_{ji} \right|^2. \end{aligned} \tag{11}$$

With Gaussian signaling, the rate of each cognitive user can thus be written as

$$C_i = \log \left(1 + \frac{P \left| h_{ii} \right|^2}{P_0 \left| g_i \right|^2 + \sigma_n^2 + I_i} \right), \quad i = 1, \ldots, n, \tag{12}$$

where σ_n^2 is the thermal noise power. The sum rate of the cognitive network is then simply

$$C_n = \sum_{i=1}^{n} C_i. \tag{13}$$

The radius R_0 of the primary exclusive region is determined by the outage constraint on the primary user given as

$$\Pr \left[\log \left(1 + \frac{P_0 \left| h_0 \right|^2}{\sigma_{n0}^2 + I_0} \right) \leq C_0 \right] \leq \beta, \tag{14}$$

where C_0 and β are prechosen constants, and σ_{n0}^2 is the thermal noise power at the primary receiver.

We assume the channel gains depend only on the distance between transmitters and receivers as in (10), and do not suffer from fading or shadowing. Thus, all randomness is a result of the random distribution of the cognitive nodes in the cognitive band of Figure 4.

4.3. The scaling law of a cognitive network

We now study the scaling law of the sum capacity as the number of cognitive users n increases to infinity. Since the single primary transmitter has fixed power P_0 and minimum distance R_0 from any cognitive receiver, its interference has no impact on asymptotic rate analysis and can be treated as an additive noise term. In [30], lower and upper bounds on the network sum capacity were computed, and are outlined next.

A lower bound on the network sum capacity can be derived by upper bounding the interference to a cognitive receiver. An interference upper bound is obtained by, first, filling the primary exclusive region with cognitive users. Next, consider a uniform network of n cognitive users. The worst case interference then is to the user with the receiver at the center of the network. Let R_c be the radius of the circle centered at the considered receiver that covers all other cognitive transmitters. With constant user density (λ users per-unit area), then R_c^2 grows linearly with n. Furthermore, any interfering cognitive transmitter must be at least a distance ϵ away from the interfered receiver for some $\epsilon > 0$.

It can then be shown that the average worst-case interference, caused by $n = \lambda\pi(R_c^2 - \epsilon^2)$ cognitive users, is given by

$$I_{\text{avg},n} = \frac{2\pi\lambda P}{(\alpha - 1)} \left(\frac{1}{\epsilon^{\alpha-2}} - \frac{1}{R_c^{\alpha-2}} \right). \tag{15}$$

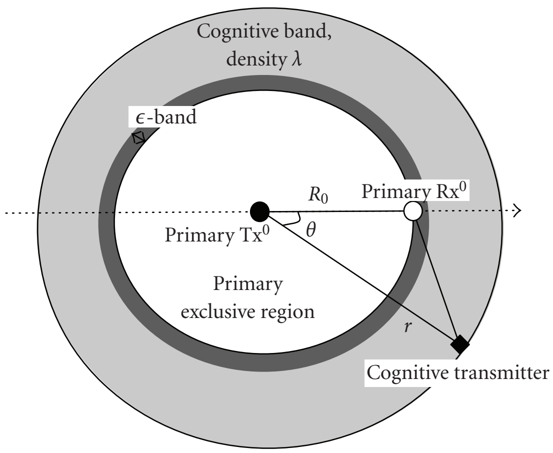

FIGURE 5: Worst-case interference to a primary receiver: the receiver is on the boundary of the primary exclusive region of radius R_0. We seek to find R_0 to satisfy the outage constraint on the primary user.

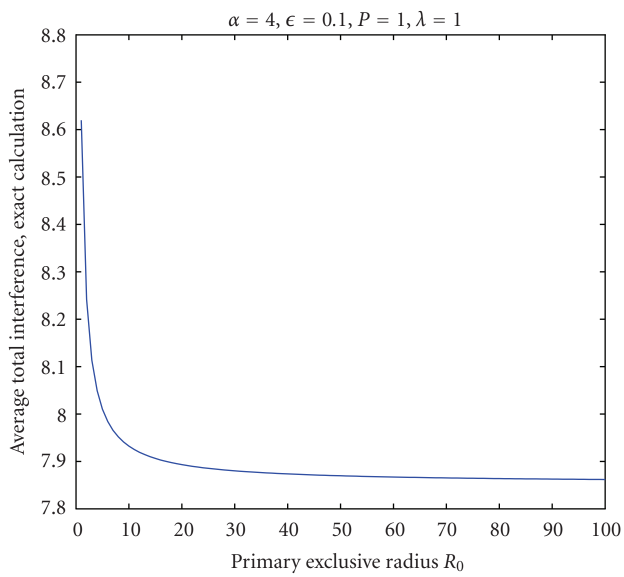

FIGURE 6: The average interference at the primary receiver as a function of the primary exclusive radius R_0, when $R \to \infty$.

As $n \to \infty$, provided that $\alpha > 2$, this average interference to the cognitive receiver at the center approaches a constant as

$$I_{\text{avg},n} \xrightarrow{n\to\infty} \frac{2\pi\lambda P}{(\alpha-1)\epsilon^{\alpha-2}} \triangleq I_\infty. \quad (16)$$

This may be used to show that the expected capacity of each user is lower bounded by a constant as $n \to \infty$ [30],

$$E[C_i] \geq \log\left(1 + \frac{P_{r,\min}}{\sigma^2_{0,\max} + I_\infty}\right) \triangleq \overline{C}_1, \quad (17)$$

where $P_{r,\min} = P/D^\alpha_{\max}$ and $\sigma^2_{0,\max} = \sigma^2_n + P_0/R^\alpha_0$. Thus the average per-user rate of a cognitive network remains at least a constant as the number of users increases.

For the upper bound, we can simply remove the interference from all other cognitive users. Assuming that the capacity of a single cognitive user under noise alone is bounded by a constant, then the total network capacity grows at most linearly with the number of users.

From these lower and upper bounds, we conclude that the sum capacity of the cognitive network grows linearly in the number of users

$$E[C_n] = nK\overline{C}_1 \quad (18)$$

for some constant K, where $\overline{C}_1$ defined in (17) is the achievable average rate of a single cognitive user under constant noise and interference power.

4.4. *The primary exclusive region*

To study the primary exclusive region, we consider the worst case when the primary receiver is at the edge of this region, on the circle of radius R_0, as shown in Figure 5. The outage constraint must also hold in this (worst) case, and we find a bound on R_0 that will ensure this.

Since each receiver has a protected radius ϵ, and assuming that the cognitive users are not aware of the location of the primary receiver, then all cognitive transmitters must be placed minimally at a radius $R_0+\epsilon$. In other words, they cannot be placed in the guard band of width ϵ in Figure 5.

Consider interference at the worst-case primary receiver from a cognitive transmitter at radius r and angle θ. The distance $d(r,\theta)$ (the distance depends on r and θ) between this interfering transmitter and the primary receiver satisfies

$$d(r,\theta)^2 = R_0^2 + r^2 - 2R_0 r\cos\theta. \quad (19)$$

For uniformly distributed cognitive users, θ is uniform in $[0, 2\pi]$, and r has the density $f_r(r) = 2r/(R^2 - (R_0+\epsilon)^2)$.

The expected interference, plus noise power experienced by the primary receiver Rx^0 from all $n = \lambda\pi(R^2 - (R_0+\epsilon)^2)$ cognitive users, is then given as

$$E[I_0] = \int_{R_0+\epsilon}^{R}\int_0^{2\pi} \frac{2rPdrd\theta}{2\pi\left(R_0^2 + r^2 - 2R_0 r\cos\theta\right)^{\alpha/2}}. \quad (20)$$

When $\alpha/2$ is an integer, we may evaluate the integral for the exact interference using complex contour integration techniques. As an example for $\alpha = 4$, the expected interference is given by

$$E[I_0] = \lambda\pi P\left[-\frac{R^2}{\left(R^2 - R_0^2\right)^2} + \frac{\left(R_0+\epsilon\right)^2}{\epsilon^2\left(2R_0+\epsilon\right)^2}\right]. \quad (21)$$

In Figure 6, we plot this expected interference versus the radius R_0. As R_0 increases, the interference decreases to a constant level. For any $\alpha > 2$, bounds on the expected interference may be obtained [30].

Given the system parameters P_0, β, and C_0, one can combine (21) with the primary outage constraint (14) to design the exclusive region radius R_0 and the band ϵ so as to meet the desired outage constraint [30]. Specifically, for $\alpha = 4$, the outage constraint results in

$$\frac{\left(R_0+\epsilon\right)^2}{\epsilon^2\left(2R_0+\epsilon\right)^2} \leq \frac{\beta}{\lambda\pi P}\left(\frac{P_0/R_0^4}{2^{C_0}-1} - \sigma^2\right). \quad (22)$$

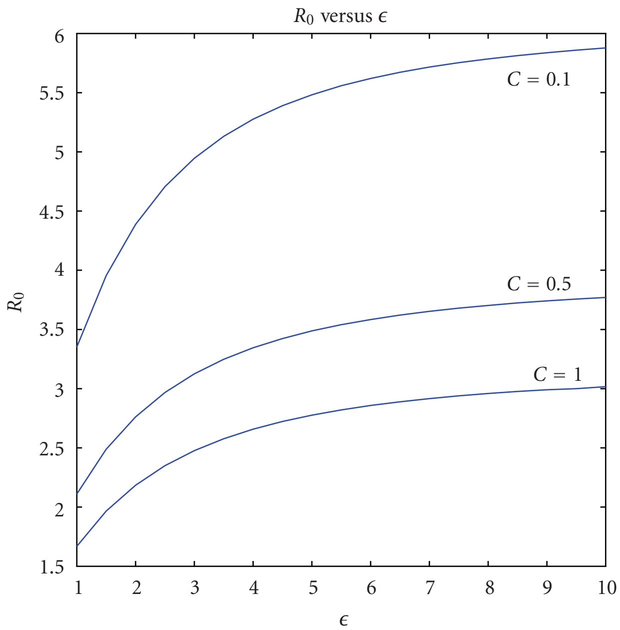

Figure 7: The relation between the exclusive region radius R_0 and the guard band ϵ according to (22) for $\lambda = 1, P = 1, P_0 = 100, \sigma^2 = 1$, $\beta = 0.1$, and $\alpha = 4$.

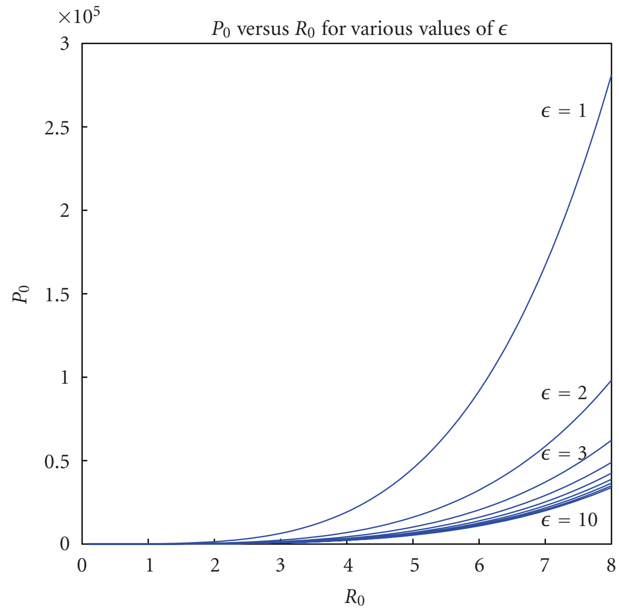

Figure 8: The relation between the BS power P_0 and the exclusive region radius R_0 according to (22) for $\lambda = 1, P = 1, \sigma^2 = 1, \beta = 0.1, C_0 = 3$ and $\alpha = 4$.

In Figure 7, we plot the relation between the exclusive region radius R_0 and the guard-band width ϵ for various values of the outage capacity C_0, while fixing all other parameters according to (22). The plots show that R_0 increases with ϵ, which is intuitive. Furthermore, as C_0 increases, R_0 decreases for the same ϵ. Alternatively, we can fix the guard band ϵ and the secondary user power P and seek the relation between the primary power P_0 and the exclusive radius R_0 that can support the outage capacity C_0, as in Figure 8. The fourth-order increase in power (in relation to the radius R_0) here is in line with the path loss $\alpha = 4$. Interestingly, a small increase in the gap band ϵ can lead to a large reduction in the required primary transmit power P_0 to reach a receiver at a given radius R_0 while satisfying the given outage constraint.

5. CONCLUSION

As the deployment of cognitive radios and networks draws near, fundamental limits of possible communication may offer system designers both guidance as well as benchmarks against which to measure cognitive network performance. In this paper, we outlined three different fundamental limits of communication possible in cognitive channels and networks. These illustrated three different and noteworthy aspects of cognitive system design.

In Section 2, we explore the simplest of cognitive channels: a channel in which one primary Tx-Rx link and one cognitive Tx-Rx link share spectral resources. Currently, secondary spectrum usage proposals involve sharing the channel in time or frequency, that is, the secondary cognitive user will listen for spectral gaps (in either time or frequency) and will proceed to fill in these gaps. We showed that this is not optimal in terms of primary and secondary user rates. Rather, we showed that if the secondary user obtains the message of the primary user, *both* users rates may be significantly improved. Thus encouraging primary users to make their messages publicly known ahead of time, or encouraging secondary user protocols to learn the primary users message may improve the overall spectral efficiency of cognitive systems.

In Section 3, we explore the multiplexing gains of cognitive radio systems. We showed that as SNR $\rightarrow\infty$, the cognitive channel achieves a multiplexing gain of one, just like the interference channel. The fully cooperative channel, on the other hand, achieves a multiplexing gain of two, meaning that, roughly speaking, two parallel streams of information may be sent between the 2 Txs and the 2 Rxs. This result suggests that cognition, or asymmetric transmitter cooperation, while achieving better rates than, for example, a time-sharing scheme, is valuable at all SNR, as the SNR $\rightarrow\infty$, the incentive to share messages two ways, or to encourage full transmitter cooperation becomes stronger. We also note that practical SNRs do not fall into the high SNR regime, and thus these results are primarily of theoretical interest.

Finally, in Section 4, we consider a cognitive network which consists of a single primary user and multiple cognitive users. We show that when cognitive links are of bounded distance (which does not grow as the network radius grows), then *single-hop* transmissions achieve a linear sum-rate scaling as the number of cognitive users grows. This result suggests that in designing cognitive networks, cognitive links should not scale with the network size as in arbitrary ad-hoc networks [31]. Single-hop communication, which is suitable for cognitive devices of opportunistic nature, should then be deployed. Furthermore, we analyze the impact the cognitive network has on the primary user in terms of an outage constraint. We illustrate how the outage constraint may be used to jointly design the primary exclusive radius R_0 and the guard band ϵ, thus providing the primary user a cognitive transmission-free zone.

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