Achievable Rates in Cognitive Radio Channels

Natasha Devroye, Patrick Mitran, *Student Member, IEEE*, and Vahid Tarokh

***Abstract*—*Cognitive radio* promises a low-cost, highly flexible alternative to the classic single-frequency band, single-protocol wireless device. By sensing and adapting to its environment, such a device is able to fill voids in the wireless spectrum and can dramatically increase spectral efficiency. In this paper, the cognitive radio channel is defined as a two-sender, two-receiver interference channel in which sender 2 obtains the encoded message sender 1 plans to transmit. We consider two cases: in the genie-aided cognitive radio channel, sender 2 is noncausally presented the data to be transmitted by sender 1 while in the causal cognitive radio channel, the data is obtained causally. The cognitive radio at sender 2 may then choose to transmit simultaneously over the same channel, as opposed to waiting for an idle channel as is traditional for a cognitive radio. Our main result is the development of an achievable region which combines Gel'fand–Pinkser coding with an achievable region construction for the interference channel. In the additive Gaussian noise case, this resembles dirty-paper coding, a technique used in the computation of the capacity of the Gaussian multiple-input multiple-output (MIMO) broadcast channel. Numerical evaluation of the region in the Gaussian noise case is performed, and compared to an inner bound, the interference channel, and an outer bound, a modified Gaussian MIMO broadcast channel. Results are also extended to the case in which the message is causally obtained.**

***Index Terms*—Cognitive radio channel, dirty-paper coding, Gaussian multiple-input multiple-output (MIMO) broadcast channel, Gel'fand–Pinsker coding, interference channel, wireless communication.**

I. Motivation

RECENTLY, there has been an explosion of interest in cognitive and software radios, as is evidenced by FCC proceedings [5], [6], and papers [15], [19] *Software Defined Radios* (SDR) [14] are wireless communication devices equipped with either a general-purpose processor or programmable silicon as hardware base, and enhanced by a flexible software architecture. They are low cost, can be rapidly upgraded, and may adapt to the environment in real time. Such devices are able to operate in many frequency bands, under multiple transmission protocols, and employ a variety of modulation and coding schemes. Taking this one step further, Mitola [15] coined the term *cognitive radio* for software-defined radios capable of sensing their environment and making decisions instantaneously, without any user intervention. This allows them to rapidly change their modulation schemes and communication protocols so as to better communicate in the sensed environment.

Apart from their low cost and flexibility, another benefit of SDR technology is spectral efficiency. Currently, FCC measurements indicate that at any time roughly 10% of the unlicensed frequency spectrum is actively in use (leaving 90% unused) [7]. If a wireless device such as a cognitive radio is able to sense an idle channel in a particular frequency band (or at a particular time), then it can shift to that frequency band (or time slot) to transmit its own information, potentially leading to a dramatic increase in spectral (or temporal) efficiency.

In current cognitive radio protocol proposals, the device listens to the wireless channel and determines, either in time or frequency, which part of the spectrum is unused [11]. It then adapts its signal to fill this void in the spectrum domain. Thus, a device transmits over a certain time or frequency band only when no other user does. In this paper, the cognitive radio behavior is generalized to allow two users to simultaneously transmit over the same time or frequency. Under our scheme, a cognitive radio will listen to the channel and, if sensed idle, could proceed with the traditional cognitive radio channel model, that is, transmit during the voids. On the other hand, if another sender is sensed, the radio may decide to proceed with simultaneous transmission. The cognitive radio need not wait for an idle channel to start transmission.

Although cognitive radios have spurred great interest and excitement in industry, many of the fundamental theoretical questions on the limits of such technology remain unanswered. In this paper, we propose a general model of a cognitive radio channel and study its theoretic limits. Specifically, we will prove achievability, in the information-theoretic sense, of a certain set of rates at which two senders (cognitive radios, denoted as $\mathcal{S}_1$ and $\mathcal{S}_2$) can transmit simultaneously over a common channel to two independent receivers $\mathcal{R}_1$ and $\mathcal{R}_2$, when $\mathcal{S}_2$ is aware of the message to be sent by $\mathcal{S}_1$. Our methods borrow ideas from Gel'fand and Pinsker's coding for channels with known interference at the transmitter [9], Costa's dirty-paper coding [2], the interference channel [1], the Gaussian multiple-input multiple-output (MIMO) broadcast channel [26], and the achievable region of the interference channel described by Han and Kobayashi [10]. The results are also related, conceptually, to other communication systems in which user cooperation is employed in order to enhance the capacity. These schemes can be traced back to telegraphy, and have recently been considered in the *collaborative communications* of [21], the spatial diversity enhancing schemes obtained through user cooperation described in [24], [25], and many others such as [12], [13], [17], [18].

Manuscript received November 25, 2004; revised October 5, 2005. This work is based upon research supported by the National Science Foundation under the Alan T. Waterman Award, Grant CCR-0139398. The material in this paper was presented in part at the Conference on Information Sciences and Systems, Johns Hopkins University, Baltimore, MD, March 2005.

The authors are with the Division of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138 USA (e-mail:ndevroye@deas.harvard.edu; mitran@deas.harvard.edu; vahid@deas.harvard.edu).

Communicated by M. Médard, Associate Editor for Communications.
Digital Object Identifier 10.1109/TIT.2006.872971

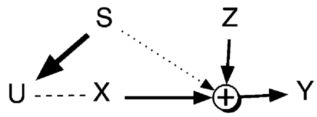


Fig. 1. Dirty-paper coding channel with input X, auxiliary random variable U, interference S known noncausally to the transmitter, additive noise Z, and output Y.

A key idea behind achieving high data rates in an environment where two senders share a common channel is interference cancelation or mitigation. When said information is known at the transmitter only, the channel capacity is given by the well-known formula obtained by Gel'fand and Pinsker [9] as

$$C = \max_{p(u,x|s)} [I(U;Y) - I(U;S)]$$

where X is the input to the channel, Y is the output, S is the interference, and U is an auxiliary random variable chosen to make the channel $U \rightarrow Y$ appear causal. The channel model and variables are shown in Fig. 1 for additive interference and noise. We refer to the coding technique used in [9] as Gel'fand–Pinkser coding. In the Gaussian noise and interference case, Costa achieves the capacity of an interference-free channel by assuming the input X to the channel is Gaussian, and then considering an auxiliary variable U of the form $U = X + \alpha S$ for some parameter α whose optimal value is equal to the ratio of the signal power to the signal plus noise power. Since the rate thus obtained is equal to the capacity of an interference-free channel, which provides an upper bound, optimality is achieved by the assumed Gaussian input X. Dirty-paper coding is the term first used by Costa [2] to describe a technique which completely mitigates *a priori* known interference over an input power constrained additive white Gaussian noise (AWGN) channel. We will make use of the coding techniques of Costa [2], Gel'fand and Pinsker [9], as well as Cover and Chiang [3] in our main results in Sections II and IV.

Our methods are also closely related to the interference channel, which is briefly described next. Consider a discrete memoryless *interference channel* [1], with random variables $X_1 \in \mathcal{X}_1$, $X_2 \in \mathcal{X}_2$ as inputs to the channel characterized by the conditional probabilities $p(y_1|x_1,x_2)$, $p(y_2|x_1,x_2)$ with resulting channel output random variables $Y_1 \in \mathcal{Y}_1$, $Y_2 \in \mathcal{Y}_2$. The interference channel corresponds to two independent senders $\mathcal{S}_1, \mathcal{S}_2$, with independent noncooperating receivers $\mathcal{R}_1$, $\mathcal{R}_2$, transmitting over the same channel, and thus interfering with each other.

The additive interference channel is shown in Fig. 2. In this paper, wireless channels are indicated using solid black lines, related variables are connected through dashed lines, while interference is denoted using dotted lines. There, in addition to the additive interference from the other sender, each output is affected by independent additive noise Z_1, Z_2. The parameters a_{12}, a_{21} capture the effects of the interference. The channel outputs are

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} 1 & a_{21} \\ a_{12} & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}. \tag{1}$$

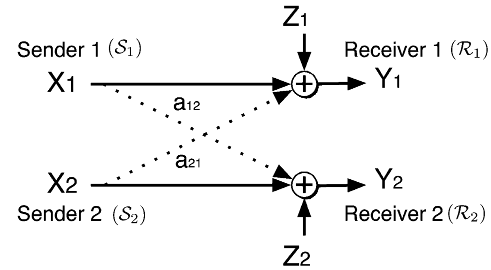


Fig. 2. The additive interference channel with inputs X_1, X_2, outputs Y_1, Y_2, additive noise Z_1, Z_2, and interference coefficients a_{12}, a_{21}.

The interference channel capacity, in the most general case, is still an open problem. In the case of strong interference, as defined in [10], [23], and very-strong interference, as defined in [1], the capacity is known. Achievable regions of the interference channel have been calculated in [10], and recently in [22]. We will make use of the techniques in [10], merged with Gel'fand–Pinsker coding [9] to provide an achievable region for the cognitive radio channel, as defined next.

The main contribution of this paper is to define and prove the achievability of a region of rate pairs for a cognitive radio channel. A cognitive radio channel is defined to be an interference channel in which $\mathcal{S}_2$ has knowledge of the message to be transmitted by $\mathcal{S}_1$. This is either obtained causally, or could possibly be given to the sender noncausally by a "genie." The main theorem and corollary will be proved for the noncausal case, or the genie-aided cognitive radio channel. Once $\mathcal{S}_2$ has $\mathcal{S}_1$'s message, two possible coding techniques become apparent. In the first, $\mathcal{S}_2$ treats the message of $\mathcal{S}_1$ as interference and tries to compensate for it by using a binning technique similar to the one seen in Gel'fand and Pinsker's coding scheme for channels with side information known at the transmitter [9]. This results in an achievable region for the rate pair that enlarges the region in [10], and reduces to that region in the case where no interference mitigation is performed. In the second, $\mathcal{S}_2$ could refrain from transmitting its own information and act as a relay for $\mathcal{S}_1$. In this case, a $2 \rightarrow 1$ multiple-input single-output (MISO) channel between $(\mathcal{S}_1, \mathcal{S}_2) \rightarrow \mathcal{R}_1$ is obtained. The main result argues that time sharing can then achieve the convex hull of the regions obtained using these two coding techniques. Simulations in the Gaussian noise case compare the rate region described in this paper to both the upper bound provided by a 2×2 MIMO Gaussian broadcast channel (with one receiving antenna per receiver) [26], and an additional upper bound provided by an interference-free channel. Simulations also suggest that the larger the power mismatch between the two senders, the better this scheme performs.

The paper is structured as follows: Section II defines the genie-aided cognitive radio channel as an interference channel in which one sender is noncausally given the other sender's message. Section II also proves the main result: achievability of a certain rate region. The employed technique merges the results of Gel'fand and Pinsker [9] on coding for channels with side information known at the transmitter and the achievable region for the interference channel described by Han and Kobayashi [10]. The significance of our result is shown in Section III, where numerical methods are used to compute

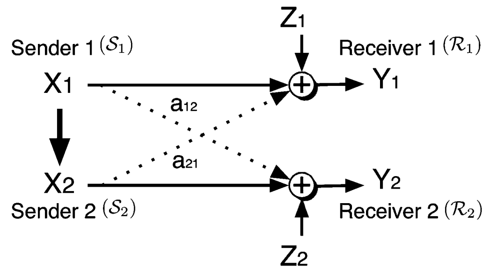


Fig. 3. The additive interference genie-aided cognitive radio channel with inputs X_1, X_2, outputs Y_1, Y_2, additive noise Z_1, Z_2, and interference coefficients a_{12}, a_{21}. S_1's input X_1 is given to S_2 (indicated by the arrow), but not *vice versa*.

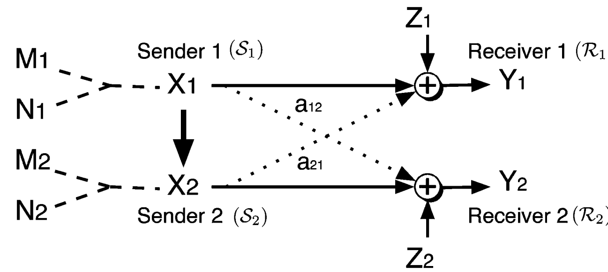


Fig. 4. The modified genie-aided cognitive radio channel with auxiliary random variables M_1, M_2, N_1, N_2, inputs X_1, X_2, additive noise Z_1, Z_2, outputs Y_1, Y_2, and interference coefficients a_{12}, a_{21}.

an achievable region in the AWGN case. Our region extends that of [10] and is compared to an upper bound given by the 2×2 Gaussian MIMO broadcast channel [26] intersected with that of an interference-free channel. Section IV extends the genie-aided cognitive radio channel model of Section II to a more realistic scenario in which all signals are obtained causally. In Section V, we summarize the main contributions of this paper: the definition of a cognitive radio channel, the proof ideas, and the significance of a certain achievable rate region for this channel.

II. Genie-Aided Cognitive Radio Channel Definition

A genie-aided cognitive radio channel C_{COG} is defined to be an interference channel in which $\mathcal{S}_2$ is given, in a noncausal manner (i.e., by a genie), the message x_1^n which $\mathcal{S}_1$ will transmit, as illustrated in Fig. 3. This noncausal constraint will be relaxed in Section IV, and a cognitive radio channel describes the case where the message is causally obtained. $\mathcal{S}_2$ can then exploit the knowledge of $\mathcal{S}_1$'s message, and potentially improve the transmission rate by, for example, using a binning technique (similar to dirty-paper coding). In the following, an achievable rate region for such a cognitive radio channel is constructed in a way which combines the results of Gel'fand and Pinsker [9] on the capacity of channels with known interference, Costa's [2] dirty-paper coding, and the largest known achievable region of the interference channel as described by Han and Kobayashi [10], in which senders are completely independent. Intuitively, the achievable region in [10] should lie entirely within the cognitive achievable region, since in the latter, the senders are permitted to at least partially cooperate. They could choose not to cooperate at all and in that case reduce to the scenario in [10]. An upper bound for our region in the Gaussian case is provided by the 2×2 MIMO broadcast channel whose capacity region, in the Gaussian case, has recently been calculated in [26]. In [26], dirty-paper coding techniques are shown to be optimal for nondegraded vector broadcast channels. Our channel model resembles that of [26], with one important difference. In the scheme of [26], it is presumed that both senders can cooperate in order to precode the transmitted signal. In our scheme, the relation between the two senders is asymmetric. We believe this is a reasonable model for the target application of a cognitive radio channel in which one sender is transmitting and a second sender obtains the first sender's transmission before starting its own. The rate of $\mathcal{S}_2$ is also bounded by the rate achievable in an interference-free channel, with $a_{12} = 0$. For some rate pairs, this interference-free channel provides a tighter bound than the 2×2 MIMO broadcast channel, and *vice versa*.

An (n, K_1, K_2, ϵ) *code* for the genie-aided cognitive radio channel consists of K_1 codewords $x_1^n(i) \in \mathcal{X}_1^n$ for $\mathcal{S}_1$, and $K_1 \cdot K_2$ codewords $x_2^n(i,j) \in \mathcal{X}_2^n$ for $\mathcal{S}_2$, $i \in \{1, 2, \ldots, K_1\}$, $j \in \{1, 2, \ldots, K_2\}$, which together form the *codebook*, revealed to both senders and receivers such that the average error probabilities under some decoding scheme are less than ϵ.

A rate pair (R_1, R_2) is said to be *achievable* for the genie-aided cognitive radio channel if there exists a sequence of $(n, 2^{\lceil nR_1 \rceil}, 2^{\lceil nR_2 \rceil}, \epsilon_n)$ codes such that $\epsilon_n \to 0$ as $n \to \infty$. An *achievable region* is a closed subset of the positive quadrant of $\mathbb{R}^2$ of achievable rate pairs.

The interference channel capacity region, in the most general case, is still an open problem. This is the case for the genie-aided cognitive radio channel as well. In [10], an achievable region of the interference channel is found by first considering a modified interference channel and then establishing a correspondence between the achievable rates of the modified and the original problems. A similar modification is made in the next subsection.

A. The Modified Genie-Aided Cognitive Channel C_{COG}^m

Similar to [10], we introduce a modified genie-aided cognitive radio channel, C_{COG}^m, (m for modified) and demonstrate an achievable region for C_{COG}^m. Then, a relation between an achievable rate for C_{COG}^m and an achievable rate for C_{COG} is used to establish an achievable region for the latter. The modified genie-aided cognitive radio channel C_{COG}^m is defined as in Fig. 4.

Let $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$ be the random-variable inputs to the channel. Let $Y_1 \in \mathcal{Y}_1$ and $Y_2 \in \mathcal{Y}_2$ be the random-variable outputs of the channel. The conditional probabilities of the discrete memoryless C_{COG}^m are the same as those of the discrete memoryless C_{COG} and are fully described by $p(y_1|x_1, x_2)$ and $p(y_2|x_1, x_2)$ for all values $x_1 \in \mathcal{X}_1$, $x_2 \in \mathcal{X}_2$, $y_1 \in \mathcal{Y}_1$, and $y_2 \in \mathcal{Y}_2$.

The modified genie-aided cognitive radio channel introduces two pairs of auxiliary random variables: (M_1, N_1) and (M_2, N_2). The random variables $M_1 \in \mathcal{M}_1$ and $M_2 \in \mathcal{M}_2$ represent, as in [10], the private information to be sent from $\mathcal{S}_1 \to \mathcal{R}_1$ and $\mathcal{S}_2 \to \mathcal{R}_2$ respectively. In contrast, the random variables $N_1 \in \mathcal{N}_1$ and $N_2 \in \mathcal{N}_2$ represent the public information to be sent from $\mathcal{S}_1 \to (\mathcal{R}_1, \mathcal{R}_2)$ and $\mathcal{S}_2 \to (\mathcal{R}_1, \mathcal{R}_2)$,

TABLE I
DEFINITIONS OF VARIABLES

Variable	Description
$\mathcal{S}_1$, $\mathcal{S}_2$	Sender 1, Sender 2
$\mathcal{R}_1$, $\mathcal{R}_2$	Receiver 1, Receiver 2
M_1	Private information to be sent from $\mathcal{S}_1 \to \mathcal{R}_1$
N_1	Public information to be sent from $\mathcal{S}_1 \to (\mathcal{R}_1, \mathcal{R}_2)$
M_2	Private information to be sent from $\mathcal{S}_2 \to \mathcal{R}_2$
N_2	Public information to be sent from $\mathcal{S}_2 \to (\mathcal{R}_1, \mathcal{R}_2)$
X_1, X_2	Encodings transmitted by $\mathcal{S}_1$, $\mathcal{S}_2$ respectively
Y_1, Y_2	Received signals at $\mathcal{R}_1$, $\mathcal{R}_2$ respectively
a_{12}, a_{21}	Interference coefficients
R_{11}	Rate achieved between $\mathcal{S}_1 \to \mathcal{R}_1$
R_{12}	Rate achieved between $\mathcal{S}_1 \to (\mathcal{R}_1, \mathcal{R}_2)$
R_{21}	Rate achieved between $\mathcal{S}_2 \to (\mathcal{R}_1, \mathcal{R}_2)$
R_{22}	Rate achieved between $\mathcal{S}_2 \to \mathcal{R}_2$

respectively. The function of these M_1, N_1, M_2, N_2 is as in [10]: to decompose or define *explicitly* the information to be transmitted between various input and output pairs.

In this work, M_2 and N_2 also serve a dual purpose: these auxiliary random variables are analogous to the auxiliary random variables of Gel'fand and Pinsker [9] or Cover and Chiang [3]. They serve as fictitious inputs to the channel, so that after $\mathcal{S}_2$ is informed of the encoded message of $\mathcal{S}_1$ noncausally, the channel still behaves like a discrete memoryless channel (DMC) from $(M_1, N_1, M_2, N_2) \to (Y_1, Y_2)$. As in [3], [9], there is a penalty in using this approach which will be reflected by a reduction in achievable rates (compared to the fictitious DMC from (M_1, N_1, M_2, N_2) to (Y_1, Y_2)) for the links which use noncausal information. In Gel'fand and Pinsker, use of side information S to transmit the auxiliary random variable U reduces the achievable rate by $I(U;S)$. In our case, using the side information M_1, N_1 (which we will obtain from X_1) to help transmit the auxiliary random variables M_2 and N_2 will reduce their rates by $I(M_2; M_1, N_1)$ and $I(N_2; M_1, N_1)$, respectively. For clarity, Table I describes all variables employed.

Similar to the definition of a *code* in the genie-aided cognitive radio channel case, define an $(n, K_{11}, K_{12}, K_{21}, K_{22}, \epsilon)$ code for the modified genie-aided cognitive radio channel as a set of $K_{11} \cdot K_{12}$ codewords $x_1^n(i,j) \in \mathcal{X}_1^n$, for $\mathcal{S}_1$ and $K_{11} \cdot K_{12} \cdot K_{21} \cdot K_{22}$ codewords $x_2^n(i,j,k,l) \in \mathcal{X}_2^n$, $i \in \{1, 2, \ldots, K_{11}\}$, $j \in \{1, 2, \ldots, K_{12}\}$, $k \in \{1, 2, \ldots, K_{21}\}$, $l \in \{1, 2, \ldots, K_{22}\}$, such that the average probability of decoding error is less than ϵ. Call a quadruple $(R_{11}, R_{12}, R_{21}, R_{22})$ achievable if there exists a sequence of

$$(n, 2^{\lceil nR_{11} \rceil}, 2^{\lceil nR_{12} \rceil}, 2^{\lceil nR_{21} \rceil}, 2^{\lceil nR_{22} \rceil}, \epsilon_n)$$

codes such that $\epsilon_n \to 0$ as $n \to \infty$. An achievable region for the modified genie-aided cognitive radio channel is the closure of a subset of the positive region of $\mathbb{R}^4$ of achievable rate quadruples.

As mentioned in [10], the introduction of a time-sharing random variable W is thought to strictly extend the achievable region obtained using a convex hull operation. Thus, let $W \in \mathcal{W}$ be a time-sharing random variable whose n-sequences $w^n \triangleq (w^{(1)}, w^{(2)}, \ldots, w^{(n)})$ are generated independently of the messages, according to $\prod_{t=1}^{n} p(w^{(t)})$. The n-sequence w^n is given to both senders and both receivers. The paper's main theorem and corollary are stated next.

Theorem 1: Let

$$Z \triangleq (Y_1, Y_2, X_1, X_2, M_1, N_1, M_2, N_2, W)$$

and let $\mathcal{P}$ be the set of distributions on Z that can be decomposed into the form

$$\begin{aligned} &p(w)p(m_1|w)p(n_1|w)p(x_1|m_1,n_1,w) \\ &\quad \times p(m_2|m_1,n_1,w)p(n_2|m_1,n_1,w) \\ &\quad \times p(x_2|m_2,n_2,w)p(y_1|x_1,x_2)p(y_2|x_1,x_2). \end{aligned} \tag{2}$$

For any $Z \in \mathcal{P}$, let $S(Z)$ be the set of all quadruples $(R_{11}, R_{12}, R_{21}, R_{22})$ of nonnegative real numbers such that there exist nonnegative real (L_{21}, L_{22}) satisfying

$$R_{11} \le I(M_1; X_1|N_1, W) \tag{3}$$
$$R_{12} \le I(N_1; X_1|M_1, W) \tag{4}$$
$$R_{11} + R_{12} \le I(M_1, N_1; X_1|W) \tag{5}$$
$$R_{21} \le L_{21} - I(N_2; M_1, N_1|W) \tag{6}$$
$$R_{22} \le L_{22} - I(M_2; M_1, N_1|W) \tag{7}$$
$$R_{11} \le I(Y_1, N_1, N_2; M_1|W) \tag{8}$$
$$R_{12} \le I(Y_1, M_1, N_2; N_1|W) \tag{9}$$
$$L_{21} \le I(Y_1, M_1, N_1; N_2|W) \tag{10}$$
$$R_{11} + R_{12} \le I(Y_1, N_2; M_1, N_1|W) \tag{11}$$
$$R_{11} + L_{21} \le I(Y_1, N_1; M_1, N_2|W) \tag{12}$$
$$R_{12} + L_{21} \le I(Y_1, M_1; N_1, N_2|W) \tag{13}$$
$$R_{11} + R_{12} + L_{21} \le I(Y_1; M_1, N_1, N_2|W) \tag{14}$$
$$L_{22} \le I(Y_2, N_1, N_2; M_2|W) \tag{15}$$
$$R_{12} \le I(Y_2, N_2, M_2; N_1|W) \tag{16}$$
$$L_{21} \le I(Y_2, N_1, M_2; N_2|W) \tag{17}$$
$$L_{22} + L_{21} \le I(Y_2, N_1; M_2, N_2|W) \tag{18}$$
$$L_{22} + R_{12} \le I(Y_2, N_2; M_2, N_1|W) \tag{19}$$
$$R_{12} + L_{21} \le I(Y_2, M_2; N_1, N_2|W) \tag{20}$$
$$L_{22} + R_{21} + L_{12} \le I(Y_2; M_2, N_1, N_2|W). \tag{21}$$

Let S be the closure of $\cup_{Z \in \mathcal{P}} S(Z)$. Then any element of S is achievable for the modified genie-aided cognitive radio channel C_{COG}^{m}. □

Proof: It is sufficient to show the achievability of the interior elements of $S(Z)$ for each $Z \in \mathcal{P}$. So, fix

$$Z = (Y_1, Y_2, X_1, X_2, M_1, N_1, M_2, N_2, W)$$

and take any $(R_{11}, R_{12}, R_{21}, R_{22})$ and (L_{21}, L_{22}) satisfying the constraints of the theorem. The standard notation and notions of strong ϵ-typicality, strong joint typicality, and strongly typical sets of [4] will be used.

Codebook generation: Let some distribution on Z of the form (2) be given. For any $\epsilon > 0$ it is sufficient to prove that there exists a large enough block length n to ensure that the probability of error is less than ϵ. To generate the codebook, first let $w^n \triangleq (w^{(1)}, w^{(2)}, \ldots, w^{(n)})$ be a sequence in $\mathcal{W}^n$ chosen randomly according to $\prod_{t=1}^{n} p(w^{(t)})$ and known to $\mathcal{S}_1, \mathcal{S}_2, \mathcal{R}_1$, and $\mathcal{R}_2$. Next, note that

$$p(m_2|w) = \sum_{m_1,n_1 \in \mathcal{M}_1,\mathcal{N}_1} p(m_2|m_1,n_1,w)p(m_1|w)p(n_1|w)$$

and

$$p(n_2|w) = \sum_{m_1,n_1 \in \mathcal{M}_1,\mathcal{N}_1} p(n_2|m_1,n_1,w)p(m_1|w)p(n_1|w).$$

We will generate the codebook according to the distribution

$$\begin{array}{c} p(w)p(m_1|w)p(n_1|w)p(x_1|m_1,n_1,w) \\ p(m_2|w)p(n_2|w)\, p(x_2|m_2,n_2,w) \\ p(y_1|x_1,x_2)p(y_2|x_1,x_2). \end{array} \tag{22}$$

To do so,

1) generate $2^{\lceil n(R_{11}-10\epsilon)\rceil}$ n-sequences $m_1(i)$ i.i.d. according to $\prod_{t=1}^{n} p(m_1^{(t)}|w^{(t)})$;
2) generate $2^{\lceil n(R_{12}-10\epsilon)\rceil}$ n-sequences $n_1(j)$ i.i.d. according to $\prod_{t=1}^{n} p(n_1^{(t)}|w^{(t)})$;
3) generate $2^{\lceil n(L_{21}-10\epsilon)\rceil}$ n-sequences $n_2(l)$ i.i.d. according to $\prod_{t=1}^{n} p(n_2^{(t)}|w^{(t)}) \rightarrow$ throw into $2^{\lceil n(R_{21}-10\epsilon)\rceil}$ bins uniformly;
4) generate $2^{\lceil n(L_{22}-10\epsilon)\rceil}$ n-sequences $m_2(k)$ i.i.d. according to $\prod_{t=1}^{n} p(m_2^{(t)}|w^{(t)}) \rightarrow$ throw into $2^{\lceil n(R_{22}-10\epsilon)\rceil}$ bins uniformly.

We note that the binning scheme used here resembles that of Gel'fand and Pinsker [9]. Define the message index spaces

$$\mathcal{S}_{11} \triangleq \{1,2,\ldots,2^{\lceil n(R_{11}-10\epsilon)\rceil}\}$$
$$\mathcal{S}_{12} \triangleq \{1,2,\ldots,2^{\lceil n(R_{12}-10\epsilon)\rceil}\}$$
$$\mathcal{S}_{21} \triangleq \{1,2,\ldots,2^{\lceil n(R_{21}-10\epsilon)\rceil}\}$$

and

$$\mathcal{S}_{22} \triangleq \{1,2,\ldots,2^{\lceil n(R_{22}-10\epsilon)\rceil}\}.$$

The aim is to send a four dimensional message

$$s \triangleq (s_{11},s_{12},s_{21},s_{22}) \in \mathcal{S} \triangleq \mathcal{S}_{11} \times \mathcal{S}_{12} \times \mathcal{S}_{21} \times \mathcal{S}_{22}$$

whose first two components are message indices and whose last two components are bin indices. Note that if such a message can be sent with arbitrarily small probability of error, then the rates achieved will be $(R_{11}, R_{12}, R_{21}, R_{22})$ for the respective sender $\rightarrow$ receiver pairs $\mathcal{S}_1 \rightarrow \mathcal{R}_1$, $\mathcal{S}_1 \rightarrow (\mathcal{R}_1,\mathcal{R}_2)$, $\mathcal{S}_2 \rightarrow (\mathcal{R}_1,\mathcal{R}_2)$, and $\mathcal{S}_2 \rightarrow \mathcal{R}_2$.

Recall that the messages actually sent over the genie-aided cognitive radio channel are elements of $\mathcal{X}_1^n$, $\mathcal{X}_2^n$. The message indices are mapped into the signal space $\mathcal{X}_1^n$ as follows.

1) To send s_{11} and s_{12}, look up the sequences $m_1^n(s_{11})$ and $n_1^n(s_{12})$.
2) Generate x_1^n i.i.d. according to

$$\prod_{t=1}^{n} p(x_1^{(t)}|m_1^n(s_{11}), n_1^n(s_{12}), w^n)$$

and send x_1^n.

A genie now presents $\mathcal{S}_2$ with x_1^n. From x_1^n, $\mathcal{S}_2$ can recover m_1^n and n_1^n using a joint typicality-based decoder. The MAC (3)–(5) ensure that this is indeed possible. Then, to send its own messages (s_{21}, s_{22}), $\mathcal{S}_2$ proceeds as follows.

1) To send s_{21} and s_{22}, look in bin s_{21} and s_{22} for sequences n_2^n and m_2^n such that $(n_2^n, m_1^n, n_1^n, w^n)$ and $(m_2^n, m_1^n, n_1^n, w^n)$ are *jointly typical* tuples, respectively, according to the joint distribution in (2).
2) Generate x_2^n i.i.d. according to

$$\prod_{t=1}^{n} p(x_2^{(t)}|m_2^{(t)}, n_2^{(t)}, w^{(t)})$$

and send this x_2^n.

Decoding: $\mathcal{R}_1$ and $\mathcal{R}_2$ decode independently, based on strong joint typicality. The inputs x_1^n, x_2^n to the genie-aided cognitive radio channel are received at $\mathcal{R}_1$, $\mathcal{R}_2$ as y_1^n, y_2^n according to the conditional distributions

$$p^n(y_1^n|x_1^n,x_2^n) = \prod_{t=1}^{n} p(y_1^{(t)}|x_1^{(t)},x_2^{(t)})$$

and

$$p^n(y_2^n|x_1^n,x_2^n) = \prod_{t=1}^{n} p(y_2^{(t)}|x_1^{(t)},x_2^{(t)}).$$

$\mathcal{R}_1$ attempts to recover (s_{11}, s_{12}, s_{21}) ($\mathcal{R}_2$ attempts to recover (s_{12}, s_{21}, s_{22})) based on y_1^n (y_2^n resp.) and w^n. Thus, the decoders at $\mathcal{R}_1$ ($\mathcal{R}_2$ resp.) are functions

$$\psi_1 : \mathcal{Y}_1^n \times \mathcal{W}^n \rightarrow \mathcal{S}_{11} \times \mathcal{S}_{12} \times \mathcal{S}_{21},$$
$$\psi_1(y_1^n, w^n) = (\psi_1^{11}(y_1^n, w^n), \psi_1^{12}(y_1^n, w^n), \psi_1^{21}(y_1^n, w^n))$$
$$\psi_2 : \mathcal{Y}_2^n \times \mathcal{W}^n \rightarrow \mathcal{S}_{12} \times \mathcal{S}_{21} \times \mathcal{S}_{22},$$
$$\psi_2(y_2^n, w^n) = (\psi_2^{12}(y_2^n, w^n), \psi_2^{21}(y_2^n, w^n), \psi_2^{22}(y_2^n, w^n)).$$

When $\mathcal{R}_1$ ($\mathcal{R}_2$ resp.) receives the n-sequence y_1^n (y_2^n resp.) and w^n, it looks at the set of all input sequences m_1^n, n_1^n, n_2^n (m_2^n, n_2^n, n_1^n resp.) that are ϵ_d-*jointly typical* (for some $\epsilon_d > 0$), according to the distribution (22) with the received y_1^n (y_2^n resp.) and w^n. Thus, $\mathcal{R}_1$ ($\mathcal{R}_2$) forms the set, for the given $w^n \in \mathcal{W}^n$

$$S_1(y_1^n, w^n) \triangleq \{(m_1^n, n_1^n, n_2^n) : (y_1^n, m_1^n, n_1^n, n_2^n, w^n) \in A_{\epsilon_d}^n(Y_1, M_1, N_1, N_2|W)\}$$
$$S_2(y_2^n, w^n) \triangleq \{(m_2^n, n_1^n, n_2^n) : (y_2^n, m_2^n, n_1^n, n_2^n, w^n) \in A_{\epsilon_d}^n(Y_2, M_2, N_1, N_2|W)\}.$$

Since $\mathcal{R}_1$ and $\mathcal{R}_2$ are decoding message and bin indices, let $B(m_1^n)$ and $B(n_1^n)$ be the message indices in $\mathcal{S}_{11}$ and $\mathcal{S}_{12}$, respectively, while $B(m_2^n)$ and $B(n_2^n)$ are the bin indices of the n-sequences in $\mathcal{S}_{22}$ and $\mathcal{S}_{21}$, respectively. Then the decoding function $\psi_1(\cdot,\cdot)$ is as follows.

If all $(m_1^n, \cdot, \cdot) \in S_1(y_1^n, w^n)$ have the same message index, then we let $\psi_1^{11}(y_1^n, w^n) = B(m_1^n)$.

If all $(\cdot, n_1^n, \cdot) \in S_1(y_1^n, w^n)$ have the same message index, then we let $\psi_1^{12}(y_1^n, w^n) = B(n_1^n)$.

If all $(\cdot, \cdot, n_2^n) \in S_1(y_1^n, w^n)$ have the same bin index, then we let $\psi_1^{21}(y_1^n, w^n) = B(n_2^n)$.

Otherwise, an error is declared. $\psi_2(\cdot)$ is defined analogously.

We defer the probability of error analysis to the Appendix. The analysis shows that if $(R_{11}, R_{12}, R_{21}, R_{22})$ and (L_{21}, L_{22}) are as in the statement of the theorem, then reliable communication is possible.

Another important rate pair for the genie-aided cognitive radio channel is achievable: that in which $\mathcal{S}_2$ transmits no

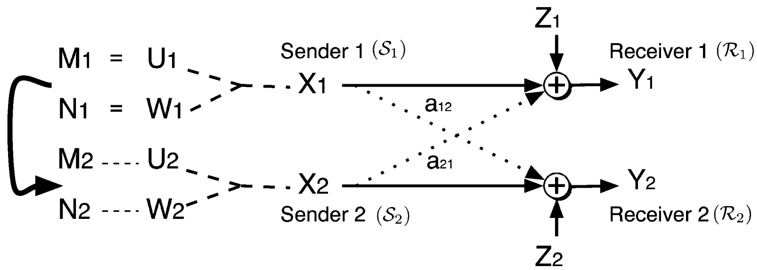


Fig. 5. The modified Gaussian genie-aided cognitive radio channel with inputs X_1, X_2, auxiliary random variables $U_1, W_1, U_2, W_2, M_1, N_1, M_2, N_2$, outputs Y_1, Y_2, additive Gaussian noise Z_1, Z_2, and interference coefficients a_{12}, a_{21}.

information of its own to $\mathcal{R}_2$, and simply aids $\mathcal{S}_1$ in sending its to $\mathcal{R}_1$. When this is the case, the rate pair $(R_1^*, 0)$ is achievable, where R_1^* is the capacity of the vector channel $(\mathcal{S}_1, \mathcal{S}_2) \to \mathcal{R}_1$. Note, however, that the analogous rate pair $(0, R_1^*)$ is not achievable, since that would involve $\mathcal{S}_1$ aiding $\mathcal{S}_2$ in sending $\mathcal{S}_2$'s message. This cannot happen under our assumptions; $\mathcal{S}_2$ knows $\mathcal{S}_1$'s message, but not *vice versa*. The overall achievable region of this paper is given by the following corollary.

Corollary 2: Let $\mathcal{C}_0$ be the set of all points $(R_{11}+R_{12}, R_{21}+R_{22})$ where $(R_{11}, R_{12}, R_{21}, R_{22})$ is an achievable rate tuple of Theorem 1. Consider the vector channel $(\mathcal{S}_1, \mathcal{S}_2) \to \mathcal{R}_1$ described by the conditional probability density $p(y_1|x_1, x_2)$ for all $y_1 \in \mathcal{Y}_1$, $x_1 \in \mathcal{X}_1$, $x_2 \in \mathcal{X}_2$, and define

$$R_1^* \triangleq \max_{p(x_1,x_2)} I(X_1, X_2; Y_1). \tag{23}$$

Then the convex hull of the region $\mathcal{C}_0$ with the point $(R_1^*, 0)$ is achievable for the genie-aided cognitive radio channel.

Proof: Direct application of Lemma 2.1 in [10] to the modified genie-aided cognitive radio channel demonstrates that if the rate quadruple $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable for the modified genie-aided cognitive radio channel, then the rate pair $(R_{11}+R_{12}, R_{21}+R_{22})$ is achievable for the genie-aided cognitive radio channel. The point $(R_1^*, 0)$ is also achievable. The convex hull is then achievable by standard time-sharing arguments.

Next, an achievable region is demonstrated in the Gaussian case.

III. The Gaussian Cognitive Radio Channel

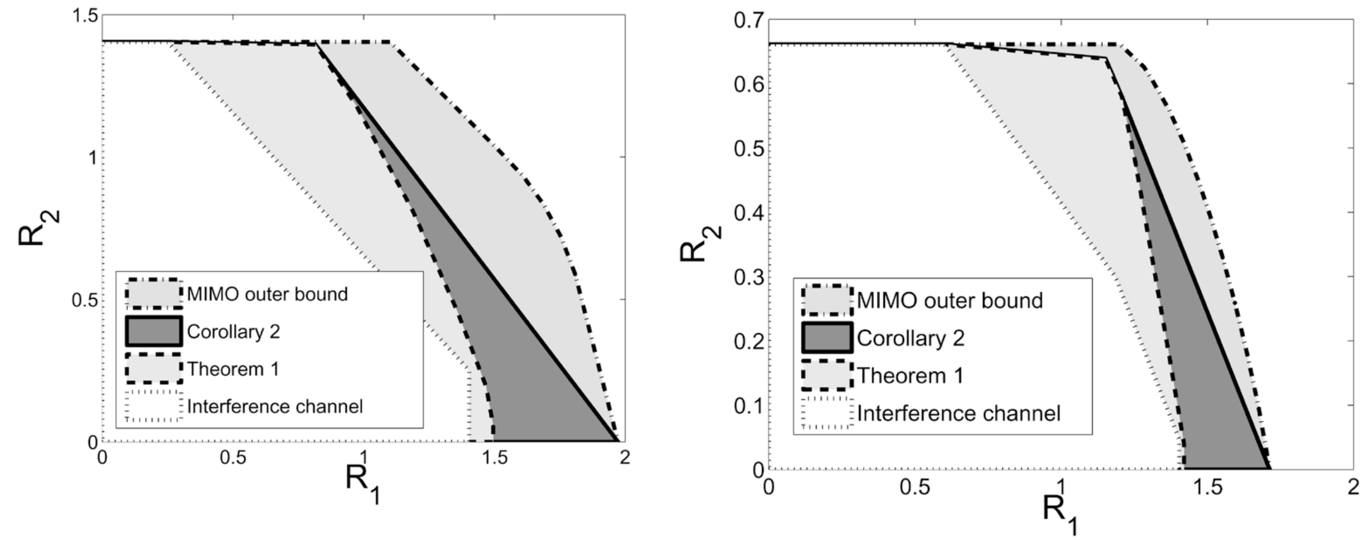
Consider the genie-aided cognitive radio channel, depicted in Fig. 5, with independent additive noise $Z_1 \sim \mathcal{N}(0, Q_1)$ and $Z_2 \sim \mathcal{N}(0, Q_2)$. We assume the two transmitters are power limited to P_1 and P_2, respectively. In order to determine an achievable region for the modified Gaussian genie-aided cognitive radio channel, specific forms of the random variables described in Theorem 1 are assumed. As in [2], [8], [10], Theorem 1 and Corollary 2 can readily be extended to memoryless channels with discrete time and continuous alphabets by finely quantizing the input, output, and interference variables (Gaussian in this case). Let W, the time-sharing random variable, be constant. Consider the case where, for certain $\alpha, \beta \in \mathbb{R}$ and $\lambda, \overline{\lambda}, \gamma, \overline{\gamma} \in [0,1]$, with $\lambda + \overline{\lambda} = 1$, $\gamma + \overline{\gamma} = 1$, and additional independent auxiliary random variables U_1, W_1, U_2, W_2 as in Fig. 5, the following hold:

$$\begin{aligned}
U_1 &= M_1, \text{ distributed according to } \mathcal{N}(0, \lambda P_1)\\
W_1 &= N_1, \text{ distributed according to } \mathcal{N}(0, \overline{\lambda} P_1)\\
X_1 &= U_1 + W_1 = M_1 + N_1,\\
&\quad \text{distributed according to } \mathcal{N}(0, P_1)\\
M_2 &= U_2 + \alpha X_1 = U_2 + \alpha(M_1 + N_1),\\
&\quad \text{where } U_2 \text{ is distributed according to } \mathcal{N}(0, \gamma P_2)\\
N_2 &= W_2 + \beta X_1 = W_2 + \beta(M_1 + N_1),\\
&\quad \text{where } W_2 \text{ is distributed according to } \mathcal{N}(0, \overline{\gamma} P_2)\\
X_2 &= U_2 + W_2, \text{ distributed according to } \mathcal{N}(0, P_2).
\end{aligned}$$

In this model, the received signals are given by

$$\begin{aligned}
Y_1 &= X_1 + a_{21} X_2 + Z_1\\
&= U_1 + W_1 + a_{21}(U_2 + W_2) + Z_1 \qquad (24)\\
Y_2 &= a_{12} X_1 + X_2 + Z_2\\
&= a_{12}(U_1 + W_1) + U_2 + W_2 + Z_2. \qquad (25)
\end{aligned}$$

Note that the MAC channel between $(M_1, N_1) \to X_1$ is a noiseless additive channel with Gaussian inputs. Thus, the bounds in (3)–(5) do not impose any constraints on R_{11} and R_{12}. Notice that although U_1, W_1, U_2, W_2 are mutually independent, M_1, N_1, M_2, N_2 are not necessarily so. Bounds on the rates $R_{11}, R_{12}, R_{21}, R_{22}$ can be calculated as functions of the free parameters $\alpha, \beta, \lambda, \gamma$, as well as $a_{12}, a_{21}, Q_1, Q_2, P_1, P_2$. First, we calculate the covariance matrix between all variables as shown in (26) at the bottom of the following page, where $\Theta \triangleq (Y_1\ Y_2\ M_1\ N_1\ M_2\ N_2\ X_1)$. The values for λ and γ are repeatedly randomly selected from the interval $[0,1]$. The values of α and β are also repeatedly generated according to $\mathcal{N}(0,1)$. There exist bounds on the admissible values of α and β in order to keep all upper bounds on the rates R_{11}, R_{12}, R_{21} and R_{22} positive. However, these are not explicitly considered, and whenever α, β values cause any bound to be negative, those particular values of α and β are rejected. For each 4-tuple $\lambda, \gamma, \alpha, \beta$, the above covariance matrix (26) yields all the information necessary to calculate the 14 remaining bounds (8)–(21) on (R_{11}, R_{12}, R_{21}, R_{22}) of Theorem 1. Each mutual information bound can be expanded in terms of entropies, which can then be evaluated by taking the determinant of appropriate submatrices of (26). The achievable regions thus obtained for the Gaussian genie-aided cognitive radio channel are plotted in Fig. 6. The innermost region corresponds to the achievable



(a) (b)

Fig. 6. The innermost (dotted) polyhedron is the achievable region of [10]. The next to smallest (dashed) is the achievable region for the genie-aided cognitive radio channel in Theorem 1. The second to largest (solid) region is the achievable region of Corollary 2. The largest (dot-dashed) region is the intersection of the capacity region of the 2×2 MIMO broadcast channel with the outer bound on R_2 of an interference-free Gaussian channel of capacity $1/2\log(1+P_2/Q_2)$. In (a) $Q_1 = Q_2 = 1, a_{12} = a_{21} = 0.55, P_1 = P_2 = 6$, in (b) $Q_1 = Q_2 = 1, a_{12} = a_{21} = 0.55, P_1 = 6, P_2 = 1.5$. Note that since $\mathcal{S}_2$ knows $\mathcal{S}_1$'s message, it could aid $\mathcal{S}_1$ in sending it and boost R_1 above the interference-free channel case of $a_{21} = 0$, up to the vector channel rate of R_1^*.

region of [10], and may be obtained by setting $\alpha = \beta = 0$. As expected, because of the extra information at the encoder and the partial use of a binning (dirty-paper coding) technique, our achievable region in Theorem 1 (the second to smallest region in Fig. 6) extends that of [10]. Our overall achievable region, that of Corollary 2, further extends that of Theorem 1, as seen by the second largest region in Fig. 6. 4-tuples were created until the regions extended negligibly. An upper bound on our achievable rate region is provided by the 2×2 Gaussian MIMO broadcast channel, whose capacity was recently computed in [26]. The largest region in Fig. 6 is the intersection of the 2×2 Gaussian MIMO broadcast channel capacity region with the bound on $\mathcal{S}_2$'s rate $R_2 \le \frac{1}{2}\log(1+P_2/Q_2)$ provided by the interference-free channel (in which $a_{12} = 0$). The Gaussian MIMO broadcast channel capacity region is computed using a covariance matrix constraint on the inputs $X = (X_1, X_2)^T$ of the form $E[XX^T] \preceq S$, where, in order to mimic the individual power constraints P_1 and P_2 on the two users for the MIMO case, the input covariance matrix S was constrained to have diagonal elements P_1 and P_2, and is of the form

$$S = \begin{pmatrix} P_1 & c \\ c & P_2 \end{pmatrix}$$

for some $-\sqrt{P_1P_2} \le c \le \sqrt{P_1P_2}$ (which ensures S is positive semidefinite). For each such S, and all positive semidefinite matrices B and D, where $B + D \preceq S$, both rate pairs

$$R_1 \le \frac{1}{2}\log\left(\frac{H_1BH_1^T + Q_1}{Q_1}\right)$$
$$R_2 \le \frac{1}{2}\log\left(\frac{H_2(B+D)H_2^T + Q_2}{H_2BH_2^T + Q_2}\right)$$

and

$$R_1 \le \frac{1}{2}\log\left(\frac{H_1(B+D)H_1^T + Q_1}{H_1DH_1^T + Q_1}\right)$$
$$R_2 \le \frac{1}{2}\log\left(\frac{H_2DH_2^T + N_2}{Q_2}\right)$$

are achievable, where $H_1 = (1, a_{21})$ and $H_2 = (a_{12}, 1)$. The convex hull of the union of these pairs over all possible S, B, and D matrices yields the capacity region of the 2×2 Gaussian MIMO broadcast channel with channels described by H_1 and H_2, and input covariance constraint matrix S. The 2×2 Gaussian MIMO broadcast channel is a channel in which two transmitters can cooperate in order to send messages to two independent, noncooperating receivers. This is equivalent to both $\mathcal{S}_1$ and $\mathcal{S}_2$ knowing each others' messages, whereas in the genie-

$$\begin{aligned} &COV(Y_1, Y_2, M_1, N_1, M_2, N_2, X_1) \\ &= E[\Theta^T\Theta] \\ &= \begin{pmatrix} P_1 + a_{21}^2P_2 + Q_1 & a_{12}P_1 + a_{21}P_2 & \lambda P_1 & \overline{\lambda}P_1 & \alpha P_1 + a_{21}\gamma P_2 & \beta P_1 + a_{21}\overline{\gamma}P_2 & P_1 \\ a_{12}P_1 + a_{21}P_2 & a_{12}^2P_1 + P_2 + Q_2 & a_{12}\lambda P_1 & a_{12}\overline{\lambda}P_1 & a_{12}\alpha P_1 + \gamma P_2 & a_{12}\beta P_1 + \overline{\gamma}P_2 & a_{12}P_1 \\ \lambda P_1 & a_{12}\lambda P_1 & \lambda P_1 & 0 & \alpha\lambda P_1 & \beta\lambda P_1 & \lambda P_1 \\ \overline{\lambda}P_1 & a_{12}\overline{\lambda}P_1 & 0 & \overline{\lambda}P_1 & \alpha\overline{\lambda}P_1 & \beta\overline{\lambda}P_1 & \overline{\lambda}P_1 \\ \alpha P_1 + a_{21}\gamma P_2 & a_{12}\alpha P_1 + \gamma P_2 & \alpha\lambda P_1 & \alpha\overline{\lambda}P_1 & \gamma P_2 + \alpha^2P_1 & \alpha\beta P_1 & \alpha P_1 \\ \beta P_1 + a_{21}\overline{\gamma}P_2 & a_{12}\beta P_1 + \overline{\gamma}P_2 & \beta\lambda P_1 & \beta\overline{\lambda}P_1 & \alpha\beta P_1 & \overline{\gamma}P_2 + \beta^2P_1 & \beta P_1 \\ P_1 & a_{12}P_1 & \lambda P_1 & \overline{\lambda}P_1 & \alpha P_1 & \beta P_1 & P_1 \end{pmatrix} \end{aligned} \tag{26}$$

aided cognitive radio channel, $\mathcal{S}_2$ knows $\mathcal{S}_1$'s message, but not *vice versa*. There is a lack of symmetry, and this is apparent in the plots, where it can be seen that the binning (dirty-paper coding) technique aims to eliminate the interference from $\mathcal{S}_1$, and thus boosts the rate of $\mathcal{S}_2$ more than that of $\mathcal{S}_1$. Although $\mathcal{S}_1$ also sees rate increases, it is unclear whether the interference mitigation performed by $\mathcal{S}_2$ is optimal for $\mathcal{S}_1$'s rate. An upper bound on the rate of $\mathcal{S}_2$ is provided by the interference-free channel in which $a_{12} = 0$. Thus, $R_2 \leq \frac{1}{2}\log(1 + P_2/Q_2)$. For small R_1 this provides a tighter bound than the MIMO channel outer bound. However, the rate R_1 cannot be similarly bounded, as $\mathcal{S}_2$, which knows $\mathcal{S}_1$'s message could aid $\mathcal{S}_1$ in sending it and thus boost $\mathcal{S}_1$'s rate above the interference-free channel case of $a_{21} = 0$. In fact, the point $(R_1^*, 0)$ of Corollary 2 is achievable, where

$$R_1^* = \frac{1}{2}\log\left(1 + (\sqrt{P_1} + a_{21}\sqrt{P_2})^2/Q_1\right).$$

Optimal choices of α and β (to optimize, for example, the sum rate) remains an open problem. However, some values of α and β are intuitive and worth noting. Rather than randomly generate α, β, γ, and λ values, we cycle through γ and λ systematically, and choose K, a parameter selected so as to demonstrate the effect of a gradual and systematic decrease in the parameters α and β as

$$\alpha = \gamma \frac{a_{12}^2 P_1}{K(a_{12}^2 P_1 + Q_2)} \tag{27}$$

$$\beta = \overline{\gamma} \frac{a_{12}^2 P_1}{K(a_{12}^2 P_1 + Q_2)}. \tag{28}$$

If we were to consider X_1 as interference and X_2 as the message which dirty-paper codes against X_1, this choice of parameters imitates Costa's choice of dirty-paper coding parameter, that is,

$$\alpha X_1 + \beta X_1 = \frac{a_{12}^2 P_1}{K(a_{12}^2 P_1 + Q_2)} X_1.$$

By selecting $K = \sqrt{2}$, a very large region was obtained, and accurately obtains most of the large R_2 values. This aligns well with our intuition, since a dirty-paper coding choice of parameter would seem to yield large rates for R_2, while the effect on R_1 remains unknown. For $K \geq 2$ and selected to be an integer for convenience, the larger the K, the flatter and wider the region, notably in the R_1 direction. Thus, we can also conclude that smaller α and β yield larger R_1 values. However, the choice of $\alpha = \beta = 0$ does not yield the largest R_1 values (since the region then reduces to the Han and Kobayashi region for the interference channel). Thus, our scheme constructed to dirty-paper code *against* $\mathcal{S}_1$ could actually benefit $\mathcal{S}_1$, as is evidenced by larger R_1 values for nonzero α and β values. We plot the regions obtained by choosing α and β as in (27) and (28) in Fig. 7.

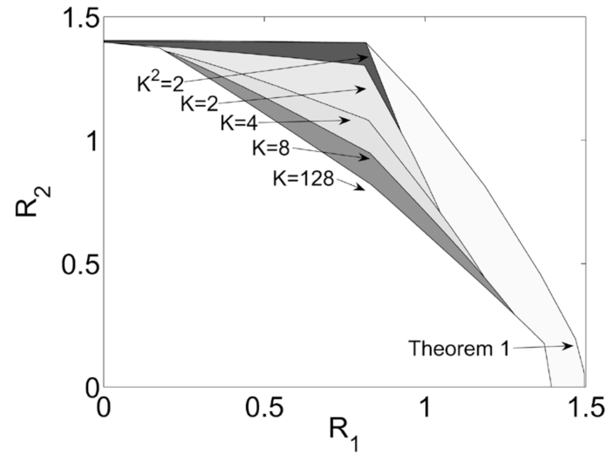


Fig. 7. Outermost region is the achievable region of Theorem 1. The next regions, in consecutive order (outermost to innermost) are those of Theorem 1 with $\alpha(K)$, $\beta(K)$ as in (27), (28), and having values K equal to $\sqrt{2}, 2, 4, 8,$ and 128.

IV. Cognitive Radio Channels: The Causal Case

In practice, the encoded message x_1^n that $\mathcal{S}_1$ wants to transmit cannot be noncausally given to $\mathcal{S}_2$. The transmitter $\mathcal{S}_2$ must obtain the message in real time, and one possible way to do so is by exploiting its proximity to $\mathcal{S}_1$. As in [21], this proximity is modeled by a reduction G in path loss, or equivalently, an increase in capacity between $\mathcal{S}_1$ and $\mathcal{S}_2$, relative to the channels between the senders and the receivers. If, for example, the channel between $\mathcal{S}_1$ and $\mathcal{S}_2$ is an AWGN channel, then the capacity increases to $C = \frac{1}{2}\log\left(1 + G \cdot \frac{P_1}{Q}\right)$, where Q is the additive Gaussian noise power. Alternatively, if $\mathcal{S}_1$ and $\mathcal{S}_2$ are base stations, then it may be possible for S_2 to obtain S_1's message through a high-bandwidth wired connection (if one exists) in real time. In the Gaussian cognitive radio channel model, all receivers know the channel(s) between themselves and the relevant sender(s). In addition, both senders and receivers know the interference channel parameters a_{12} and a_{21}. We propose four protocols, two of which follow a two-phase scheme as shown in Fig. 8 and derive the corresponding achievable regions shown in Fig. 9 (Protocol 1), and Fig. 10 (Protocol 2), which allow S_2 to obtain S_1's message in a causal manner. Protocol 3 is formally defined as a protocol in which no message knowledge is required by $\mathcal{S}_2$, that is, the two senders remain independent. This protocol has the same achievable region as that of the interference channel shown as the innermost (dotted) region in Fig. 6. Protocol 4 is again causal in nature, and produces a point of the form $(R, 0)$, and is not shown explicitly, but whose addition to the convex hull of the four achievable regions is shown in Fig. 11 (which is again achievable), and forms the final inner bound on the causal achievable region. We assume that $\mathcal{S}_1$ knows the channel between itself and $\mathcal{S}_2$ in all cases. We note that the genie-aided cognitive radio channel achievable region provides an outer bound on a causal achievable region which employs the same underlying coding strategy. This is because any causal scheme utilizing the coding strategy of Corollary 2 may be imitated by a noncausal scheme. The genie-aided achievable region computed in the previous section will thus serve as an outer bound for the causal schemes presented here.

We first propose a two-phase protocol, Protocol 1, for which Phase 1 consists of a Gaussian broadcast channel $\mathcal{S}_1 \to (\mathcal{S}_2, \mathcal{R}_1)$. During Phase 1, $\mathcal{S}_2$ is in "listening" mode, while $\mathcal{S}_1$ transmits some portion of the M_1 message, $\mu n R_{11}$ bits of the total nR_1 bits to Y_1, and all of M_1 (nR_{11} bits) and N_1 (nR_{12} bits) to $\mathcal{S}_2$. Once $\mathcal{S}_2$ has obtained the message M_1

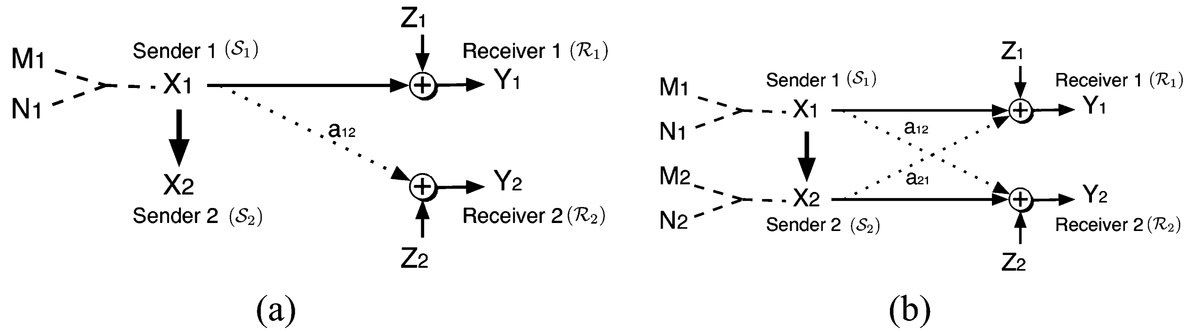


Fig. 8. (a) illustrates the listening phase of the cognitive radio channel and (b) illustrates the cognitive radio channel phase.

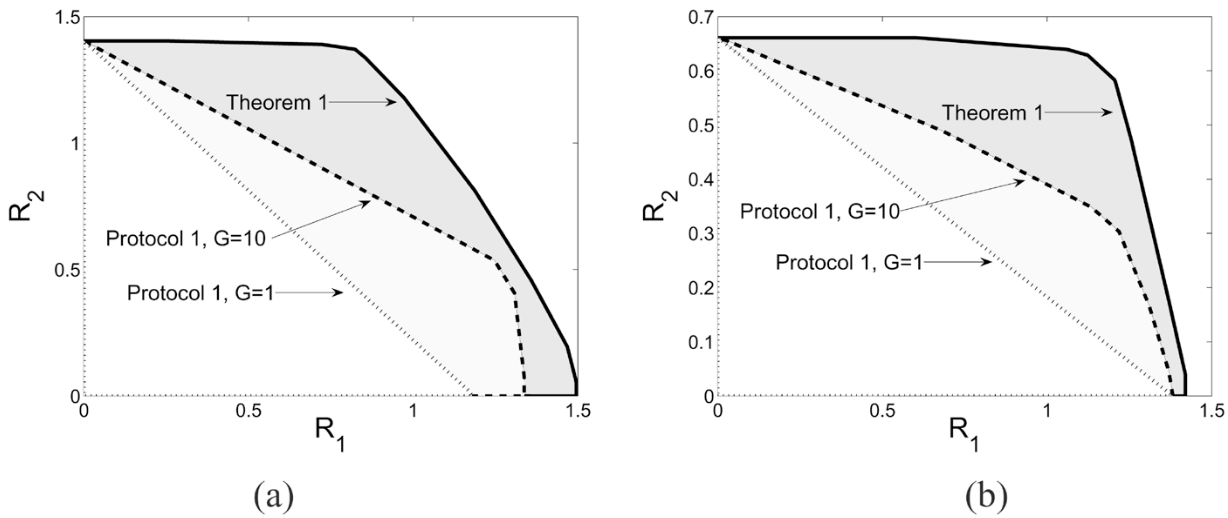


Fig. 9. The outermost (solid) curve is the overall genie-aided achievable region of the genie-aided cognitive radio channel as in Theorem 1. Both plots demonstrate the various regions causally attained using Protocol 1 for gain factor values $G = 1$ (innermost, dotted) and 10 (middle, dashed). Both graphs illustrate the regions with parameters $Q_1 = Q_2 = 1$ and $a_{12} = a_{21} = 0.55$, and in (a) $P_1 = P_2 = 6$, in (b) $P_1 = 6$, $P_2 = 1.5$.

of $\mathcal{S}_1$, transmission follows the Gaussian modified genie-aided cognitive radio channel scheme during Phase 2. The rationale behind Protocol 1 is to have two clear and distinct phases: one in which $\mathcal{S}_1$'s message is transmitted to $\mathcal{S}_2$, and a second in which both transmitters send their messages and the genie-aided rates are achievable. However, during Phase 1, we use a broadcast channel approach to the problem in order to allow the receiver $\mathcal{R}_1$ to decode at least part of the message of $\mathcal{S}_1$. That is, during the period in which $\mathcal{S}_2$ is obtaining $\mathcal{S}_1$'s message, the receiver "overhears" some of the message and is able to decode it partially. More precisely, we have the following.

Lemma 3: (*Protocol 1*) Let the 4-tuple $(R_{11}, R_{12}, R_{21}, R_{22})$ be an achievable rate pair of the modified genie-aided cognitive radio channel. Define

$$R_s(\alpha) = \frac{1}{2}\log\left(1 + \frac{G\alpha P_1}{Q}\right)$$

and

$$R_y(\alpha) = \frac{1}{2}\log\left(1 + \frac{(1-\alpha)P_1}{\alpha P_1 + Q_1}\right)$$

where Q is the additive Gaussian noise power at $\mathcal{S}_2$, and G is the gain factor and $Q_1 > Q/G$. Let $\hat{\alpha} \in [0, 1]$ be such that

$$\frac{n\mu R_{11}}{R_y(\hat{\alpha})} = \frac{n(R_{11} + R_{12})}{R_y(\hat{\alpha}) + R_s(\hat{\alpha})} \tag{29}$$

and define $f \triangleq \frac{\mu R_{11}}{R_y(\hat{\alpha})}$. Then, if $\frac{1}{1-f}((1-\mu)R_{11}, R_{12}, R_{21}, R_{22})$ is achievable for the modified genie-aided cognitive radio channel, then the rate pair $(R_1 = R_{11} + R_{12}, R_2 = R_{21} + R_{22})$ is achievable for the causal case. □

Proof: In Phase 1, consider the Gaussian broadcast channel between $\mathcal{S}_1 \to \mathcal{S}_2$ and $\mathcal{S}_1 \to \mathcal{R}_1$, and let R_s denote the rate between $\mathcal{S}_1$ and $\mathcal{S}_2$, and R_y denote the rate between $\mathcal{S}_1$ and $\mathcal{R}_1$ (output Y_1). Let the noise at $\mathcal{S}_2$ be additive Gaussian noise of power Q, and the gain factor between $\mathcal{S}_1$ and $\mathcal{S}_2$ be G. Assuming Q_1, the noise power at $\mathcal{R}_1$, is greater than Q/G, we have a degraded broadcast channel, and then the following broadcast rates [4] are achievable for $0 \le \alpha \le 1$:

$$R_s(\alpha) < \frac{1}{2}\log\left(1 + \frac{G\alpha P_1}{Q}\right)$$
$$R_y(\alpha) < \frac{1}{2}\log\left(1 + \frac{(1-\alpha)P_1}{\alpha P_1 + Q_1}\right).$$

Notice that $\mathcal{R}_1$ can only decode its own message, whereas $\mathcal{S}_2$ can decode both its message and that of $\mathcal{R}_1$ due to the degraded nature of the broadcast channel. Now, consider trying to achieve a rate of $R_1 = R_{11} + R_{12}$ for the causal Gaussian cognitive radio channel, where R_{11} and R_{12} are achievable rates for the modified Gaussian cognitive radio channel. For a given $\mu \in [0, 1]$, we try to find $\hat{\alpha}$ such that the messages at Y_1 and $\mathcal{S}_2$ are

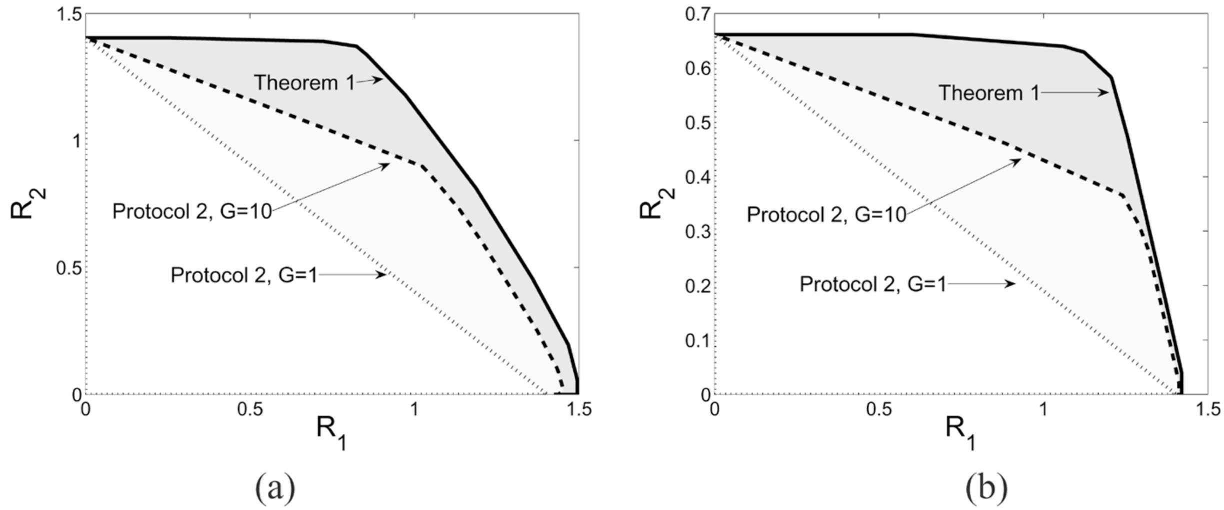


Fig. 10. The outermost (solid) curve is the genie-aided cognitive channel achievable region of Theorem 1. Both plots demonstrate the various regions causally attained using Protocol 2 for values of $G = 1$ (innermost, dotted), $G = 10$ (middle, dashed), and the genie-aided achievable region (outermost, solid). Both graphs illustrate the regions for parameters $Q_1 = Q_2 = 1$ and $a_{12} = a_{21} = 0.55$, and in (a) $P_1 = P_2 = 6$, in (b) $P_1 = 6, P_2 = 1.5$.

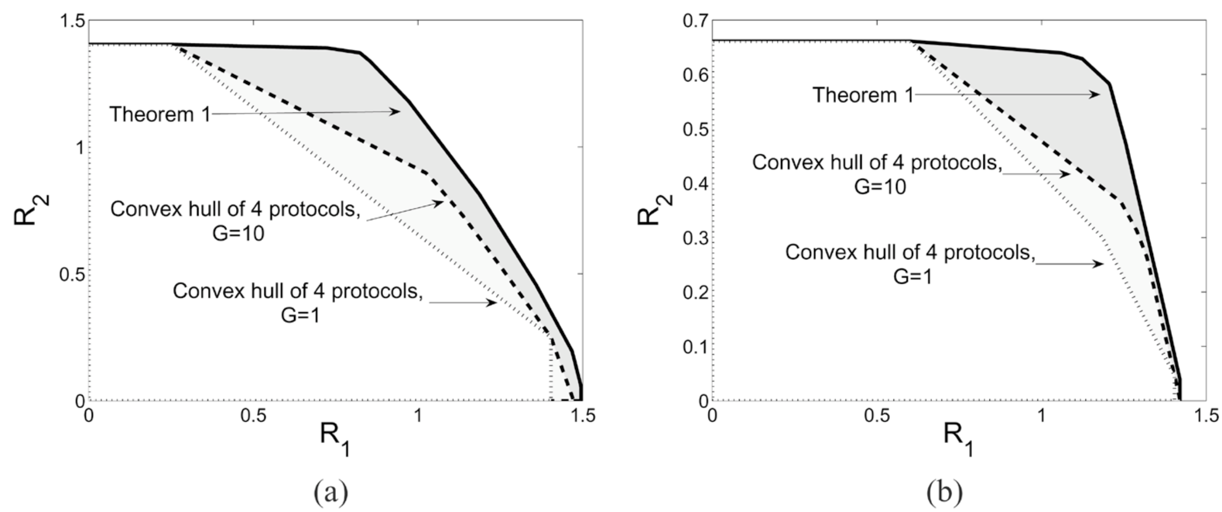


Fig. 11. The outermost (solid) region is the achievable region of Theorem 1 for the genie-aided cognitive radio channel. Both plots demonstrate the various regions attained using a convex combination of the Protocols 1, 2, 3, and 4 for values of $G = 1$ (innermost, dotted) and $G = 10$ (middle, dashed) and parameters $Q_1 = Q_2 = 1$, $a_{12} = a_{21} = 0.55$, and in (a) $P_1 = P_2 = 6$, in (b) $P_1 = 6, P_2 = 1.5$. Note that since $\mathcal{S}_2$ knows $\mathcal{S}_1$'s message, it could aid $\mathcal{S}_1$ in sending it, and boost R_1 above the interference-free channel case of $a_{21} = 0$.

fully received *simultaneously*. Thus, we try to find α such that, $\frac{n\mu R_{11}}{R_y(\hat{\alpha})} = \frac{n(R_{11}+R_{12})}{R_y(\hat{\alpha})+R_s(\hat{\alpha})}$. We let

$$f \triangleq \frac{\mu R_{11}}{R_y(\hat{\alpha})} = \frac{R_{11}+R_{12}}{R_y(\hat{\alpha})+R_s(\hat{\alpha})}.$$

This is the fraction of the transmission duration $\mathcal{S}_1$ spends in the broadcast channel phase. During this phase, Y_1 has obtained μR_{11}. Thus, in order to send the overall rates R_{11}, and R_{12}, during Phase 2, of duration $(1-f)$ of the total transmission length, the rate $\frac{1}{1-f}((1-\mu)R_{11}, R_{12}, R_{21}, R_{22})$ must be achievable for the modified Gaussian cognitive radio channel. If this is the case, then the rate $(R_{11}, R_{12}, R_{21}, R_{22})$ has been achieved for the overall causal modified cognitive radio channel, leading to a rate of $(R_{11}+R_{12}, R_{21}+R_{22})$ for the causal cognitive radio channel. ∎

Next consider Protocol 2 which also consists of two phases, and in which $\mathcal{S}_1$ transmits using the distribution for C_{cog}^m during both phases. This scheme is conceptually similar to that found in [21]. The two phases can still be viewed as in Fig. 8, however, the underlying distributions differ from those of Protocol 1. In Phase 1, $\mathcal{S}_1$ transmits to Y_1 and Y_2, while $\mathcal{S}_2$ is in "listening" mode, and refrains from transmission until it has completely overheard and decoded the message of $\mathcal{S}_1$. At this point, the scheme enters Phase 2, in which $\mathcal{S}_2$ starts transmission as well, according to the scheme of Section II. In Phase 2, $\mathcal{S}_1$ continues transmitting according to the same distribution, however a reduced rate of information transfer will be necessary, due to the added interference from the now-transmitting $\mathcal{S}_2$. The rationale behind this scheme is to simplify the coding scheme at $\mathcal{S}_1$: it transmits using the same distribution for both phases. It is just $\mathcal{S}_2$ who must first listen, then once it has the message, proceed to join $\mathcal{S}_1$ in transmitting.

In order to determine the rate pairs (R_1, R_2) achievable in this causal scheme, let nR_1 be the total number of bits to be transmitted by $\mathcal{S}_1$. Define $f = \frac{n_1}{n}$, where $0 \le n_1 \le n$ is the number of symbols for Phase 1, and $(n-n_1)$ is the number of remaining symbols for Phase 2. Then we have the following.

Lemma 4: (*Protocol 2*) Fix a distribution according to (2). Let R'_{11} be the rate achieved by $\mathcal{S}_1$ to $\mathcal{R}_1$ and R'_{12} be the rate achieved by $\mathcal{S}_1$ to $(\mathcal{R}_1,\mathcal{R}_2)$ in Phase 1, and $R''_{11}, R''_{12}, R''_{21}, R''_{22}$ be the rates achieved by $\mathcal{S}_1, \mathcal{S}_2$, respectively, during the second (cognitive) phase. If the following equations are satisfied:

$$\frac{R''_{11}}{R''_{11}+I(M_1;S_2|N_1)-R'_{11}} \le f \le 1 \quad (30)$$

$$\frac{R''_{12}}{R''_{12}+I(N_1;S_2|M_1)-R'_{12}} \le f \le 1 \quad (31)$$

$$\frac{R''_{11}+R''_{12}}{R''_{11}+R''_{12}+I(M_1,N_1;S_2)-R'_{11}-R'_{12}} \le f \le 1 \quad (32)$$

then the rate pair $(R_1 = f(R'_{11}+R'_{12})+(1-f)(R''_{11}+R''_{12}),\ R_2=(1-f)(R''_{21}+R''_{22}))$ is achieved by Protocol 2. □

Proof: Let $n_1 = fn$. Phase 1 and Phase 2 have durations n_1 and $(n-n_1)$ symbols, respectively. By definition of $R'_{11}, R'_{12}, R''_{11}, R''_{12}$, the overall data transmitted by $\mathcal{S}_1$ during the n symbols is

$$nR_1 = n_1(R'_{11}+R'_{12})+(n-n_1)(R''_{11}+R''_{12}).$$

However, in order for $\mathcal{S}_2$ to reliably obtain the message of $\mathcal{S}_1$ in the first n_1 symbols over the channel $(M_1,N_1)\to\mathcal{S}_2$, using the distribution employed in Section II, the multiple-access channel (MAC) constraints must be satisfied. This requires choosing n_1 large enough to simultaneously satisfy all three constraints (we abuse notation and let S_2 denote the received signal at $\mathcal{S}_2$)

$$\begin{aligned} n_1 I(M_1;S_2|N_1) &\ge nR_{11} = n_1R'_{11}+(n-n_1)R''_{11}\\ n_1 I(N_1;S_2|M_1) &\ge nR_{12} = n_1R'_{12}+(n-n_1)R''_{12}\\ n_1 I(M_1,N_1;S_2) &\ge nR_1 = n_1(R'_{11}+R'_{12})\\ &\quad +(n-n_1)(R''_{11}+R''_{12}). \end{aligned}$$

Note that these mutual informations are evaluated according to the distribution for S_1 given in Section II. This leads to the requirement of (30)–(32). During Phase 2, of length $(n-n_1)$ symbols, the rates $R''_{11}+R''_{12}$ and $R''_{21}+R''_{22}$ are achievable for a fraction $(1-f)$ of the total transmission length. Thus, weighting the two portions yields the achievable rate pair

$$\Big(R_1 = f(R'_{11}+R'_{12})+(1-f)(R''_{11}+R''_{12}),\quad R_2=(1-f)(R''_{21}+R''_{22})\Big) \quad ■$$

Finally, two more obvious protocols are achievable: let Protocol 3 denote a scheme in which $\mathcal{S}_2$ starts transmission immediately and does not obtain the message of $\mathcal{S}_1$, as is the case for the classical interference channel. Any point achievable for the interference channel is causally achievable here. Qualitatively, when the channel between $\mathcal{S}_1$ and $\mathcal{S}_2$ is not significantly better than that between $\mathcal{S}_1$ and $\mathcal{R}_1$ (i.e., there is little or no geometric gain) it becomes more beneficial, in terms of achievable rates, for $\mathcal{S}_1$ and $\mathcal{S}_2$ to simply independently transmit their messages, rather than use a cognitive scheme, which would require unnecessarily large overhead.

Our final protocol, Protocol 4, describes a causal way of achieving a rate pair of the form $(R,0)$, where $\mathcal{S}_2$ sends no information of its own and simply aids $\mathcal{S}_1$ in sending $\mathcal{S}_1$'s message. Let Protocol 4 be a two-phase protocol, for which Phase 1 consists of a Gaussian broadcast channel $\mathcal{S}_1 \to (\mathcal{S}_2,\mathcal{R}_1)$. For any $\alpha\in[0,1]$, let $R_1(\alpha)$ and $R_2(\alpha)$ denote the broadcast rates [4] between $\mathcal{S}_1\to\mathcal{R}_1$ and $\mathcal{S}_1\to\mathcal{S}_2$, respectively. Let the additive Gaussian noise at sender $\mathcal{S}_2$ be of power Q, and the gain factor between $\mathcal{S}_1$ and $\mathcal{S}_2$ be G. Let

$$R_1^* = \frac{1}{2}\log\left(1+\frac{(\sqrt{P_1}+a_{21}\sqrt{P_2})^2}{Q_1}\right)$$

the rate achievable in Phase 2 during which $\mathcal{S}_2$ collaborates to transmit $\mathcal{S}_1$'s message according to the optimal distribution for the vector channel $(\mathcal{S}_1,\mathcal{S}_2)\to\mathcal{R}_1$ (Corollary 2). Let $f=\frac{1}{1+R_2(\alpha)/R_1^*}$. Then the rate pair

$$(f(R_1(\alpha)+R_2(\alpha)),0) \quad (33)$$

is achievable in a causal fashion. To see why this is indeed the case, in Phase 1, consider the Gaussian broadcast channel $\mathcal{S}_1\to(\mathcal{S}_2,\mathcal{R}_1)$, and let R_1 denote the rate between $\mathcal{S}_1$ and $\mathcal{R}_1$, and R_2 denote the rate between $\mathcal{S}_1$ and $\mathcal{S}_2$. Let the noise at $\mathcal{S}_2$ be additive Gaussian noise of power Q, and the gain factor between $\mathcal{S}_1$ and $\mathcal{S}_2$ be G, and $Q/G<Q_1$. Then the following broadcast rates [4] are achievable for any given $0\le\alpha\le1$:

$$\begin{aligned} R_1(\alpha) &< \frac{1}{2}\log\left(1+\frac{(1-\alpha)P_1}{\alpha P_1+Q_1}\right)\\ R_2(\alpha) &< \frac{1}{2}\log\left(1+\frac{G\alpha P_1}{Q}\right). \end{aligned}$$

Again, let Phase 1 be of duration n_1 symbols, and Phase 2 be of duration $1-n_1$ symbols. During Phase 1, $\mathcal{R}_1$ receives $R_1(\alpha)n_1$ bits, while $\mathcal{S}_2$ receives $(R_1(\alpha)+R_2(\alpha))n_1$ bits. We also require that $\mathcal{S}_2$ receives the total number of bits to be sent to $\mathcal{R}_1$ during the first n_1 symbols. Thus, if the overall rate (from $\mathcal{S}_1$ to $\mathcal{R}_1$) achieved is denoted by R, then $n_1(R_1(\alpha)+R_2(\alpha))=R$. During Phase 2, both senders form a vector channel in order to send the remaining $n_1R_2(\alpha)$ bits. They do so at the maximal rate possible for this vector channel, given by

$$\begin{aligned} R_1^* &= \max_{p(x_1,x_2)} I(X_1,X_2;Y_1)\\ &= \frac{1}{2}\log\left(1+\frac{(\sqrt{P_1}+a_{21}\sqrt{P_2})^2}{Q_1}\right). \end{aligned}$$

Thus, equating the number of bits sent during Phase 2 we obtain $(1-n_1)R_1^* = n_1R_2(\alpha)$. Defining $f=\frac{n_1}{n}$ to be the fraction of the total transmission duration spent in Phase 1, we have $f=\frac{1}{1+R_2(\alpha)/R_1^*}$ and $R=f(R_1(\alpha)+R_2(\alpha))$.

In summary, we have provided four possible causal protocols for cognitive radio channels, as summarized in Table II.

These four Protocols can be combined to form an overall causal achievable region.

Theorem 5: The convex hull of the regions achieved under Protocols 1, 2, 3, and 4 is causally achievable. □

In order to demonstrate the effect of causality on the achievable region in the Gaussian noise case, for Protocol 1, consider Fig. 9. For values of the gain factor $G=1$ and $G=10$, a finite set of μ taken from $[0,1]$, and for a certain genie-aided achievable rate-tuple $(R_{11},R_{12},R_{21},R_{22})$, we solve for $\hat{\alpha}$ such that (29) is satisfied. If such an $\hat{\alpha}$ exists, form f and verify whether

TABLE II
SUMMARY OF THE FOUR CAUSAL COGNITIVE RADIO PROTOCOLS

Protocol 1	Phase 1: broadcast channel $\mathcal{S}_1 \to (\mathcal{S}_2, \mathcal{R}_1)$, phase 2: cognitive transmission.
Protocol 2	$\mathcal{S}_1$ transmits using the cognitive transmission distribution during both phases, $\mathcal{S}_2$ starts transmitting immediately once it has obtained the message.
Protocol 3	No cognitive transmission: a pure interference channel.
Protocol 4	Phase 1: broadcast channel $\mathcal{S}_1 \to (\mathcal{S}_2, \mathcal{R}_1)$, phase 2: relay channel $(\mathcal{S}_1, \mathcal{S}_2) \to \mathcal{R}_1$

$\frac{1}{1-f}((1-\mu)R_{11}, R_{12}, R_{21}, R_{22})$ lies in the achievable region of the modified genie-aided Gaussian cognitive radio channel. If so, $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable in the causal case. Fig. 9 demonstrates the regions attained by Protocol 1 for $G = 1$ (innermost, dotted) and $G = 10$ (middle, dashed) as compared to the achievable region of the genie-aided cognitive radio channel of Theorem 1 (outer, solid). For Protocol 2, the regions of Fig. 10 are achievable for $G = 1$ (innermost) and $G = 10$ (middle), and are compared to the genie-aided achievable region of the cognitive radio channel of Theorem 1 (outer). In order to calculate these regions, we use the same assumptions on the forms of the relevant random variables as in Section III. To calculate R'_{11} and R'_{12} one can use the equations of Theorem 1, ignoring all of $\mathcal{S}_2$'s signals, as it is not transmitting anything during Phase 1. That is, (R'_{11}, R'_{12}) satisfy

$$\begin{aligned} R'_{11} &\leq I(Y_1, N_1; M_1) \\ R'_{12} &\leq I(Y_1, M_1; N_1) \\ R'_{11} + R'_{12} &\leq I(Y_1; M_1, N_1) \\ R'_{12} &\leq I(Y_2; N_1). \end{aligned}$$

These mutual information terms are evaluated using the assumed Gaussian forms on the random variables of Section III. Finally, R''_1 and R''_2 are exactly the rates calculated in Section III. However, these rates are only achieved for a fraction of the total n symbols. Evaluating the region numerically yields Fig. 10. Protocol 3 yields the same region as the interference channel, as computed in [10], and whose achievable region corresponds to the innermost (dotted) region of Fig. 6. Protocol 4 yields points of the form $(R, 0)$ for each selected value of α, the power tradeoff parameter for the broadcast channel, and for each gain factor value G. For $G = 1$, the maximal Protocol 4 point was 1.4037 bits/channel use and for $G = 10$, the maximal point achieved by Protocol 4 was 1.4730 bits/channel use for $P_2 = 6$ and 1.4026 bits/second for $P_2 = 1.5$. The overall causal achievable region is then the convex hull of the regions achieved under Protocols 1, 2, 3, and 4. This region is shown in Fig. 11.

V. CONCLUSION

Although interest in cognitive radio technology has exploded recently, theoretical knowledge concerning its limits is still being acquired. In this paper, we contribute to this emerging field by defining and proving an achievable region for a more flexible and potentially more efficient transmission model for cognitive radio channels. In contrast to the traditional cognitive radio channel model or protocol in which a sender fills voids in time/spectrum (i.e., waits for silence or unused frequency bands), a second sender may transmit with an existing sender at the same time or in the same frequency band. Thus, the generalized cognitive radio channel is modeled as an interference channel in which two senders communicate over a common medium to two independent, noncooperating receivers, and the second sender is given, or causally obtains the messages of the first sender. We note that this cognitive channel definition can be extended to multiple transmitters in which the kth transmitter receives or obtains the messages of the $k-1$ preceding transmitters and could thus perform successive dirty-paper coding. We computed an achievable region for the genie-aided cognitive radio channel in which one sender is noncausally given the other's message. We then removed the noncausal constraint, and four protocols which allow $\mathcal{S}_2$ to causally obtain $\mathcal{S}_1$'s message were proposed. Three of the four protocols use a two-phase technique. During the first phase, $\mathcal{S}_2$ obtains $\mathcal{S}_1$'s message while during the second phase the genie-aided rates are achievable. In this genie-aided scheme, the sender with the noncausal interference knowledge uses a binning technique as in Gel'fand and Pinsker [9] (or dirty-paper coding in the Gaussian case, as in [2]), to cancel the interference from $\mathcal{S}_1$ to $\mathcal{S}_2$. Gel'fand–Pinsker coding is performed on top of the information-separating technique first proposed by Han and Kobayashi in [10], which yields, in most cases [16], the largest to date known achievable region for the interference channel. Simulations in a Gaussian noise case compare the region achieved to the 2×2 MIMO channel upper bound intersected with the ideal upper bound on R_2 provided by an interference-free channel, and an inner bound provided by the interference channel.

APPENDIX
PROBABILITY OF ERROR ANALYSIS

Consider P_e, the sum of the average probability of errors of the two senders. The average is taken over all random codes generated as described in Section II. It is assumed that all messages $s \in \mathcal{S}$ are equiprobable. Without loss of generality it is assumed that $(s_{11}, s_{12}, s_{21}, s_{22}) = (1,1,1,1)$ is sent with dither w^n. Notice that the first two components, s_{11} and s_{12}, are message indices, whereas the last two components, s_{21} and s_{22}, are bin indices. Then P_e may be bounded by, for each dither sequence w^n, as

$$\begin{aligned} P_e \leq\ & \Pr\{\psi_1(y_1^n, w^n) \\ &\quad \triangleq (\hat{s_{11}}, \hat{s_{12}}, \hat{s_{21}}) \neq (1,1,1) | (s_{11}, s_{12}, s_{21}, s_{22}) \\ &\qquad = (1,1,1,1)\} + \Pr\{\psi_2(y_2^n, w^n) \\ &\quad \triangleq (\hat{s_{12}}, \hat{s_{21}}, \hat{s_{22}}) \neq (1,1,1) | (s_{11}, s_{12}, s_{21}, s_{22}) \\ &\qquad = (1,1,1,1)\}. \end{aligned}$$

Although the decoding at $\mathcal{R}_1$ and $\mathcal{R}_2$ is described in terms of $\psi_1(\cdot,\cdot)$ and $\psi_2(\cdot,\cdot)$, it is more convenient to write certain probabilities of error events directly in terms of the n-sequence encodings $m_1^n, n_1^n, m_2^n, n_2^n$. One type of decoding error occurs when a decoded message/bin index does not equal the sent message/bin index. Recall that in order to send message indices s_{11} and s_{12}, the n-sequences $m_1^n(s_{11})$ and $n_1^n(s_{12})$ are selected and used to obtain x_1^n. $\mathcal{S}_2$ is then given x_1^n. It employs a joint typicality based decoder to obtain m_1^n, n_1^n from x_1^n, that is, look for (m_1^n, n_1^n) such that

$$(x_1^n, m_1^n, n_1^n, w^n) \in A_\epsilon^n(X_1, M_1, N_1, W).$$

We thus have a MAC between $(M_1, N_1) \to X_1$, and thus, (3)–(5) ensure that $\mathcal{S}_2$ is able to obtain m_1^n, n_1^n from x_1^n. Once it has these, the bin indices s_{21} and s_{22} are used to find n-sequences $m_2^n(s_{11}, s_{12}, s_{22})$ and/or $n_2^n(s_{11}, s_{12}, s_{21})$, the first n-sequences in bins s_{22} and s_{21}, respectively, that are jointly typical with $(m_1^n(s_{11}), n_1^n(s_{12}), w^n)$. If no such m_2^n or n_2^n exist then $m_2^n(s_{11}, s_{12}, s_{22})$ and $n_2^n(s_{11}, s_{12}, s_{21})$ are not defined and an encoding error occurs. It is also convenient to define $m_2^n(s_{22}, k)$, the kth n-sequence in bin s_{22}, and similarly, $n_2^n(s_{21}, l)$, the lth n-sequence in bin s_{21} (these are always well defined). Hence, when n_2 or m_2 are triple indexed, without loss of generality, it is assumed the selected n-sequences are $m_1^n(s_{11} = 1)$, $n_1^n(s_{12} = 1)$, $m_2^n(s_{11} = 1, s_{12} = 1, s_{22} = 1) = m_2^n(s_{22}, \hat{k})$, and $n_2^n(s_{11} = 1, s_{12} = 1, s_{21} = 1) = n_2^n(s_{21}, \hat{l})$ for some $\hat{k}$, $\hat{l}$. A sufficient condition for correct decoding of the message $(s_{11}, s_{12}, s_{21}, s_{22}) = (1,1,1,1)$ is that $S_1(y_1^n, w^n)$ and $S_2(y_2^n, w^n)$ only contain the tuple $(m_1^n(1), n_1^n(1), n_2^n(1, \hat{l}), w^n)$ and $(n_1^n(1), m_2^n(1, \hat{k}), n_2^n(1, \hat{l}), w^n)$, respectively. Then the probabilities of error can be upper-bounded as

$$\begin{aligned}
&\Pr\{\psi_1^n(y_1^n, w^n) \triangleq (\hat{s_{11}}, \hat{s_{12}}, \hat{s_{21}}) \neq (1,1,1) | s = (1,1,1,1)\} \\
&\quad\leq \Pr\{(m_1^n(1), n_1^n(1), n_2^n(1,1,1), w^n) \\
&\qquad \text{is not the only element in } S_1(y_1^n, w^n) | s = (1,1,1,1)\} \\
&\Pr\{\psi_2^n(y_2^n, w^n) \triangleq (\hat{s_{12}}, \hat{s_{21}}, \hat{s_{22}}) \neq (1,1,1) | s = (1,1,1,1)\} \\
&\quad\leq \Pr\{(n_1^n(1), m_2^n(1,1,1), n_2^n(1,1,1), w^n) \\
&\qquad \text{is not the only element in } S_2(y_2^n, w^n) | s = (1,1,1,1)\}.
\end{aligned}$$

The indices i, j, k, l are associated with the choice of the n-sequence encodings transmitted by the random variables M_1, N_1, M_2, N_2, respectively. Define the error events as in the first equation at the bottom of the page. Then, if $\overline{X}$ denotes the complement of event X, we have the second equation at the bottom of the page.

We examine each error event separately. As $n \to \infty$, $\Pr\{E_{\mathrm{MAC}}^1\} \to 0$ by the Asymptotic Equipartition Property. Provided the multiple-access equations(3)–(5) are satisfied, the three probabilities $\Pr\{E_{\mathrm{MAC}}^2\}$, $\Pr\{E_{\mathrm{MAC}}^3\}$, $\Pr\{E_{\mathrm{MAC}}^4\}$ all vanish as $n \to \infty$. Furthermore

$$\begin{aligned}
&\Pr\{E_0^1\} \\
&\quad\leq \Pr\{(m_1^n(1), n_1^n(1), w^n) \notin A_\epsilon^n(M_1, N_1|W)\} \\
&\qquad + \prod_{1 \leq l \leq 2^{\lceil n(L_{21}-R_{21})\rceil}} \Pr\big\{(m_1^n(1), n_1^n(1), n_2^n(1,l), w^n) \\
&\qquad\qquad \notin A_\epsilon^n(M_1, N_1, N_2|W) | (m_1^n, n_1^n, w^n) \\
&\qquad\qquad \in A_\epsilon^n(M_1, N_1|W)\big\} \\
&\quad\leq \epsilon + (1 - 2^{-n(I(N_2;M_1,N_1|W)+3\epsilon)})^{2^{\lceil n(L_{21}-R_{21})\rceil}} &\text{(34)}\\
&\quad\leq \epsilon + e^{-2^{-n(I(N_2;M_1,N_1|W)+3\epsilon - L_{21}+R_{21}+1/n)}}. &\text{(35)}
\end{aligned}$$

$$\begin{aligned}
E_{\mathrm{MAC}}^1 &= \{(m_1^n(1), n_1^n(1), x_1^n, w^n) \notin A_\epsilon^n(M_1, N_1, X_1|W)\} \\
E_{\mathrm{MAC}}^2 &= \{\exists i \neq 1 | (m_1^n(i), n_1^n(1), x_1^n, w^n) \in A_\epsilon^n(M_1, N_1, X_1|W)\} \\
E_{\mathrm{MAC}}^3 &= \{\exists j \neq 1 | (m_1^n(1), n_1^n(j), x_1^n, w^n) \in A_\epsilon^n(M_1, N_1, X_1|W)\} \\
E_{\mathrm{MAC}}^4 &= \{\exists (i,j) \neq (1,1) | (m_1^n(i), n_1^n(j), x_1^n, w^n) \in A_\epsilon^n(M_1, N_1, X_1|W)\} \\
E_{\mathrm{MAC}} &= E_{\mathrm{MAC}}^1 \cup E_{\mathrm{MAC}}^2 \cup E_{\mathrm{MAC}}^3 \cup E_{\mathrm{MAC}}^4 \\
E_0^1 &= \{(m_1^n(1), n_1^n(1), n_2^n(1, \hat{l}), w^n) \notin A_\epsilon^n(M_1, N_1, N_2|W)), \forall \hat{l} \in \{1, 2, \ldots, 2^{\lceil n(L_{21}-R_{21})\rceil}\}\} \\
E_1^1 &= \{(y_1^n, m_1^n(1), n_1^n(1), n_2^n(1,1,1), w^n) \notin A_{\epsilon_d}^n(Y_1, M_1, N_1, N_2|W)\} \\
E_{ij(s_{21},l)}^1 &= \{(y_1^n, m_1^n(i), n_1^n(j), n_2^n(s_{21}, l), w^n) \in A_{\epsilon_d}^n(Y_1, M_1, N_1, N_2|W)\} \\
E_0^2 &= \{(m_1^n(1), n_1^n(1), m_2^n(1, \hat{k}), w^n) \notin A_\epsilon^n(M_1, N_1, M_2|W), \forall \hat{k} \in \{1, 2, \ldots, 2^{\lceil n(L_{22}-R_{22})\rceil}\}\} \\
E_1^2 &= \{(y_2^n, n_1^n(1), m_2^n(1,1,1), n_2^n(1,1,1), w^n) \notin A_{\epsilon_d}^n(Y_2, M_2, N_1, N_2|W)\} \\
E_{j(s_{22},k)(s_{21},l)}^2 &= \{(y_2^n, n_1^n(j), m_2^n(s_{22}, k), n_2^n(s_{21}, l), w^n) \in A_{\epsilon_d}^n(Y_2, N_1, M_2, N_2|W)\}.
\end{aligned}$$

$$\begin{aligned}
P_e \leq\ & \Pr\{E_{\mathrm{MAC}}^1\} + \Pr\{E_{\mathrm{MAC}}^2\} \\
&+ \Pr\{E_{\mathrm{MAC}}^3\} + \Pr\{E_{\mathrm{MAC}}^4\} \\
&+ \Pr\{E_0^1 | \overline{E_{\mathrm{MAC}}}\} + \Pr\{E_1^1 | \overline{E_0^1} \cap \overline{E_{\mathrm{MAC}}}\} \\
&+ \sum_{ij(s_{21},l) \neq 11(1,\hat{l})} \Pr\{E_{ij(s_{21},l)}^1 | \overline{E_1^1} \cap \overline{E_0^1} \cap \overline{E_{\mathrm{MAC}}}\} \\
&+ \Pr\{E_0^2 | \overline{E_{\mathrm{MAC}}}\} + \Pr\{E_1^2 | \overline{E_0^2} \cap \overline{E_{\mathrm{MAC}}}\} \\
&+ \sum_{j(s_{22},k)(s_{21},l) \neq 1(1,\hat{k})(1,\hat{l})} \Pr\{E_{j(s_{22},k)(s_{21},l)}^2 | \overline{E_1^2} \cap \overline{E_0^2} \cap \overline{E_{\mathrm{MAC}}}\}.
\end{aligned}$$

$$\begin{aligned}
&\sum_{ij(s_{21},l)\neq 11(1,\hat{l})} \Pr\{E^1_{ij(s_{21},l)}|T_1\}\\
&\quad= \sum_{ij(s_{21},l)\neq 11(1,\hat{l})} \Pr\{(y_1^n, m_1^n(i), n_1^n(j), n_2^n(s_{21},l), w^n) \in A^n_{\epsilon_d}(Y_1, M_1, N_1, N_2|W)|T_1\}\\
&\quad\le (2^{\lceil n(R_{11}-10\epsilon)\rceil}-1)\cdot \Pr\{E^1_{21(1,\hat{l})}|T_1\}\\
&\qquad+ (2^{\lceil n(R_{12}-10\epsilon)\rceil}-1)\cdot \Pr\{E^1_{12(1,\hat{l})}|T_1\}\\
&\qquad+ (2^{\lceil n(L_{21}-10\epsilon)\rceil}-1)\cdot \Pr\{E^1_{11(2,1)}|T_1\}\\
&\qquad+ (2^{\lceil n(R_{11}-10\epsilon)\rceil}-1)(2^{\lceil n(R_{12}-10\epsilon)\rceil}-1)\cdot \Pr\{E^1_{22(1,\hat{l})}|T_1\}\\
&\qquad+ (2^{\lceil n(R_{11}-10\epsilon)\rceil}-1)(2^{\lceil n(L_{21}-10\epsilon)\rceil}-1)\cdot \Pr\{E^1_{21(2,1)}|T_1\}\\
&\qquad+ (2^{\lceil n(R_{12}-10\epsilon)\rceil}-1)(2^{\lceil n(L_{21}-10\epsilon)\rceil}-1)\cdot \Pr\{E^1_{12(2,1)}|T_1\}\\
&\qquad+ (2^{\lceil n(R_{11}-10\epsilon)\rceil}-1)(2^{\lceil n(R_{12}-10\epsilon)\rceil}-1)(2^{\lceil n(L_{21}-10\epsilon)\rceil}-1)\cdot \Pr\{E^1_{22(2,1)}|T_1\}\\
&\quad\le 2^{n(R_{11}-I(M_1;Y_1,N_1,N_2|W)-10\epsilon+2\epsilon_d+1/n)}\\
&\qquad+ 2^{n(R_{12}-I(N_1;Y_1,M_1,N_2|W)-10\epsilon+2\epsilon_d+1/n)}\\
&\qquad+ 2^{n(L_{21}-I(N_2;Y_1,M_1,N_1|W)-10\epsilon+2\epsilon_d+1/n)}\\
&\qquad+ 2^{n(R_{11}+R_{12}-I(M_1,N_1;Y_1,N_2|W)-10\epsilon+2\epsilon_d+2/n)}\\
&\qquad+ 2^{n(R_{11}+L_{21}-I(M_1,N_2;Y_1,N_1|W)-10\epsilon+2\epsilon_d+2/n)}\\
&\qquad+ 2^{n(R_{12}+L_{21}-I(N_1,N_2;Y_1,M_1|W)-10\epsilon+2\epsilon_d+2/n)}\\
&\qquad+ 2^{n(R_{11}+R_{12}+L_{21}-I(M_1,N_1,N_2;Y_1|W)-10\epsilon+2\epsilon_d+3/n)}
\end{aligned}$$

$$\begin{aligned}
&\sum_{j(s_{22},k)(s_{21},l)\neq 1(1,\hat{k})(1,\hat{l})} \Pr\{E^2_{j(s_{22},k)(s_{21},l)}|T_2\}\\
&\quad= \sum_{j(s_{22},k)(s_{21},l)\neq 1(1,\hat{k})(1,\hat{l})} \Pr\{(y_2^n, n_1(j), m_2^n(s_{22},k), n_2^n(s_{21},l), w^n) \in A^n_{\epsilon}(Y_2, M_2, N_1, N_2|W)|T_2\}\\
&\quad\le (2^{\lceil n(R_{12}-10\epsilon)\rceil}-1)\cdot \Pr\{E^2_{2(1,\hat{k})(1,\hat{l})}|T_2\}\\
&\qquad+ (2^{\lceil n(L_{22}-10\epsilon)\rceil}-1)\cdot \Pr\{E^2_{1(2,1)(1,\hat{l})}|T_2\}\\
&\qquad+ (2^{\lceil n(L_{21}-10\epsilon)\rceil}-1)\cdot \Pr\{E^2_{1(1,\hat{k})(2,1)}|T_2\}\\
&\qquad+ (2^{\lceil n(L_{22}-10\epsilon)\rceil}-1)(2^{\lceil n(R_{12}-10\epsilon)\rceil}-1)\cdot \Pr\{E^2_{2(2,1)(1,\hat{l})}|T_2\}\\
&\qquad+ (2^{\lceil n(L_{22}-10\epsilon)\rceil}-1)(2^{\lceil n(L_{21}-10\epsilon)\rceil}-1)\cdot \Pr\{E^2_{1(2,1)(2,1)}|T_2\}\\
&\qquad+ (2^{\lceil n(R_{12}-10\epsilon)\rceil}-1)(2^{\lceil n(L_{21}-10\epsilon)\rceil}-1)\cdot \Pr\{E^2_{2(1,\hat{k})(2,1)}|T_2\}\\
&\qquad+ (2^{\lceil n(L_{22}-10\epsilon)\rceil}-1)(2^{\lceil n(R_{12}-10\epsilon)\rceil}-1)(2^{\lceil n(L_{21}-10\epsilon)\rceil}-1)\cdot \Pr\{E^2_{2(2,1)(2,1)}|T_2\}\\
&\quad\le 2^{n(R_{12}-I(N_1;Y_2,M_2,N_2|W)-10\epsilon+2\epsilon_d+1/n)}\\
&\qquad+ 2^{n(L_{22}-I(M_2;Y_2,N_1,N_2|W)-10\epsilon+2\epsilon_d+1/n)}\\
&\qquad+ 2^{n(L_{21}-I(N_2;Y_2,M_2,N_1|W)-10\epsilon+2\epsilon_d+1/n)}\\
&\qquad+ 2^{n(L_{22}+R_{12}-I(M_2,N_1;Y_2,N_2|W)-10\epsilon+2\epsilon_d+2/n)}\\
&\qquad+ 2^{n(L_{22}+L_{21}-I(M_2,N_2;Y_2,N_1|W)-10\epsilon+2\epsilon_d+2/n)}\\
&\qquad+ 2^{n(R_{12}+L_{21}-I(N_1,N_2;Y_2,M_2|W)-10\epsilon+2\epsilon_d+2/n)}\\
&\qquad+ 2^{n(L_{22}+R_{12}+L_{21}-I(M_2,N_1,N_2;Y_2|W)-10\epsilon+2\epsilon_d+3/n)}.
\end{aligned}$$

The inequalities (34), (35) follow the form of [4, p. 356]. Thus, $\Pr\{E_0^1\}$ will decay to 0 as $n \to \infty$ as long as $L_{21}-R_{21} > I(N_2; M_1, N_1|W) + 3\epsilon$. Similarly, $\Pr\{E_0^2\} \to 0$ as $n \to \infty$ provided $L_{22}-R_{22} > I(M_2; M_1, N_1|W) + 3\epsilon$ (see (7)). Thus, we have shown that provided (6) and (7) hold, m_2^n and n_2^n may be found in the desired bins such that $(m_1^n, n_1^n, m_2^n, w^n)$ and $(m_1^n, n_1^n, n_2^n, w^n)$ are strong jointly typical triples in the ϵ sense. We now wish to show that the 5-tuple $(m_1^n, n_1^n, m_2^n, n_2^n, w_1^n)$ is also strongly jointly typical in the 2ϵ sense. This is not the case in general, but when the distribution $p(m_2, n_2|m_1, n_1, w^n)$ is of the form $p(m_2|m_1, n_1, w^n)p(n_2|m_1, n_1, w^n)$, then with high probability this will be the case, as proven in Lemma 6 at the end of this section.

Next, consider the error events E_1^1 and E_1^2. Since $(M_1, N_1, M_2, N_2, W) \to (X_1, X_2) \to (Y_1, Y_2)$ is a Markov chain, the probability that $(y_1^n, y_2^n, m_1^n, n_1^n, n_2^n, x_1^n, w^n)$ are not 3ϵ jointly typical goes to 0 as $n \to \infty$ (and so any subset thereof is also jointly typical with high probability).

Finally, consider all the possible joint decoding errors, given that the channel inputs are jointly 2ϵ typical. We decode with a ϵ_d joint typicality decoder, where $\epsilon_d > 3\epsilon$. We suppose indices $\hat{l}$ and $\hat{k}$ have been chosen and that

$$\begin{aligned} T_1 = \big\{ &(y_1^n, m_1^n(1), n_1^n(1), n_2^n(1,\hat{l}), w^n) \\ &\in A_{\epsilon_d}^n(Y_1, M_1, N_1, N_2|W) \big\} \\ T_2 = \big\{ &(y_2^n, n_1^n(1), m_2^n(1,\hat{k}), n_2^n(1,\hat{l}), w^n) \\ &\in A_{\epsilon_d}^n(Y_2, N_1, M_2, N_2|W) \big\}. \end{aligned}$$

Then, for all $(s_{21}, l) \neq (1, \hat{l})$ since

$$\Pr\{E_{11(2,1)}^1 | T_1\} = \Pr\{E_{11(s_{21},l)}^1 | T_1\}$$

and analogously for $\Pr\{E_{1(s_{22},k)(s_{21},l)}^2 | T_2\}$ at $\mathcal{R}_2$ we get the equations at the top of the preceding page. Taking $\epsilon_d = 4\epsilon$, and $(R_{11}, R_{12}, R_{21}, R_{22})$ and (L_{21}, L_{22}) as in the theorem statement, each of these quantities will tend to zero as the block length $n \to \infty$.

Lemma 6: Let sequences x_1^n, y_1^n, and z_1^n be generated independently with each letter distributed i.i.d. according to $p(x)$, $p(y)$, and $p(z)$. If x_1^n and y_1^n are ϵ-strongly jointly typical according to $p(x)q(y|x)$ ($q(y|x)$ not necessarily equal to $p(y)$) and x_1^n and z_1^n are ϵ-jointly typical according to $p(x)q(z|x)$ ($q(z|x)$ not necessarily equal to $p(z)$) then with probability $\to 1$ as $n \to \infty$, (x_1^n, y_1^n, z_1^n) are jointly 2ϵ-typical according to $p(x)q(y|x)q(z|x)$. □

Proof: For each $x' \in \mathcal{X}$, consider the subsequences $(x_{i_1}, x_{i_2}, \ldots, x_{i_M})$ of x_1^n such that $x_{i_1} = x_{i_2} = \cdots = x_{i_M} = x'$, where M denotes the number of occurrences of the letter x' in x_1^n. Then the subsequences $(y_{i_1}, y_{i_2}, \ldots, y_{i_M})$ of y_1^n and $(z_{i_1}, z_{i_2}, \ldots, z_{i_M})$ of z_1^n have distribution near (in the ϵ-strongly typical sense) $q(y|x=x')$ and $q(z|x=x')$, respectively. By the independence of the choice of these sequences, the joint distribution is near (in the strongly 2ϵ-typical sense) $q(y|x=x')q(z|x=x')$ with probability $1-\epsilon_{x'}$, with $\epsilon_{x'} \to 0$ as $n \to \infty$. Since the alphabet is finite, $\prod_{x' \in \mathcal{X}}(1-\epsilon_{x'}) \to 1$ as $n \to \infty$. ■

REFERENCES

[1] A. Carleial, "Interference channels," *IEEE Trans. Inf. Theory*, vol. IT-24, no. 1, pp. 60–70, Jan. 1978.

[2] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. IT-29, no. 3, pp. 439–441, May 1983.

[3] T. Cover and M. Chiang, "Duality between channel capacity and rate distortion," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1629–1638, Jun. 2002.

[4] T. Cover and J. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[5] FCC. [Online]. Available: http://www.fcc.gov/oet/cognitiveradio/

[6] "FCC ET Docket 03-108: Facilitating Opportunities for Flexible, Efficient, and Reliable Spectrum Use Employing Cognitive Radio Technologies," FCC , Tech. Rep., 2003.

[7] "FCC Report of the Spectrum Efficiency Working Group," F. C. C. S. P. T. Force, FCC, Tech. Rep., 2002.

[8] R. G. Gallagher, *Information Theory and Reliable Communication*. New York: Wiley, 1968, ch. 7.

[9] S. Gel'fand and M. Pinsker, "Coding for channels with random parameters," *Probl. Contr. and Inf. Theory*, vol. 9, no. 1, pp. 19–31, 1980.

[10] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 1, pp. 49–60, Jan. 1981.

[11] W. D. Horne. Adaptive Spectrum Access: Using the Full Spectrum Space. [Online]. Available: http://tprc.org/papers/2003/225/Adaptive_Spectrum_Horne.pdf

[12] T. Hunter, A. Hedayat, M. Janani, and A. Nostratinia, "Coded cooperation with space-time transmission and iterative decoding," in *Proc. WNCG Wireless Networking Symp.*, Austin, TX, Oct. 2003.

[13] T. Hunter and A. Nostratinia, "Coded cooperation under slow fading, fast fading, and power control," in *Proc. Asilomar Conf. Signals, Systems, and Computers*, vol. 1, Pacific Grove, CA, Nov. 2002, pp. 118–122.

[14] J. Mitola, "The software radio architecture," *IEEE Commun. Mag.*, vol. 33, no. 5, pp. 26–38, May 1995.

[15] ——, "Cognitive Radio," Ph.D. dissertation, Royal Institute of Technology (KTH), Stockholm, Sweden, 2000.

[16] G. Kramer, "Outer bounds on the capacity of Gaussian interference channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 3, pp. 581–586, Mar. 2004.

[17] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[18] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[19] J. Mitola, "Cognitive radio for flexible mobile multimedia communications," in *Proc. IEEE Mobile Multimedia Conf.*, San Dieg, CA, Nov. 1999, pp. 3–10.

[20] ——, "Future of signal processing—Cognitive radio," in *IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, May 1999.

[21] P. Mitran, H. Ochiai, and V. Tarokh, "Space-time diversity enhancements using collaborative communication," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2041–2057, Jun. 2005.

[22] I. Sason, "On achievable rate regions for the Gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 1345–1356, Jun. 2004.

[23] H. Sato, "The capacity of Gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 6, pp. 786–788, Nov. 1981.

[24] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[25] ——, "User cooperation diversity-Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.

[26] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz), "The capacity region of the Gaussian MIMO broadcast channel," *IEEE Trans. Inf. Theory*, submitted for publication.