Stability Analysis for Cognitive Radio with Multi-Access Primary Transmission

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***Abstract*—This letter analyzes the impact, from a network-layer perspective, of having a single cognitive radio transmitter-receiver pair share the spectrum with multiple primary users wishing to communicate to a single receiver in a multi-access channel (MAC). In contrast to previous work which assumes a time division multi-access strategy, here, we assume the set of primary users *simultaneously* access the channel to deliver their packets to a common destination. We derive the symmetric stable throughput regions, consisting of maximal arrival rates for primary and secondary (or cognitive radio) users under two investigated protocols. The first protocol is a conventional MAC scheme where the primary and secondary nodes operate independently. The second protocol corresponds to a multi-access relay channel (MARC) which exploits user cooperation between primary and secondary nodes. We prove that cooperation is beneficial in the considered MARC as it enables higher throughputs for both primary and secondary users.**

***Index Terms*—Cooperative diversity, cognitive radio, multi-access relay channel, queueing theory.**

I. Introduction

It has recently been shown that allowing secondary users, often thought of as cognitive radios (CRs), to cooperatively relay messages for a primary user while transmitting their own information improves performance for both primary and secondary users [1]-[4]. Intuitively, by relaying messages and thereby increasing the throughput for and emptying the queues of the primary sources, the secondary user also creates more opportunities for its own transmission. Recent work has considered multi-access configurations in which several primary users communicate with a common destination and a single cognitive node acts solely as a relay for the primary users [4]. However, these existing cooperative solutions assume time division multi-access (TDMA) policies where the primary users transmit in dedicated orthogonal channels, resulting in the suboptimal use of the available bandwidth [4], [5]. These systems have been analyzed in a cross-layer fashion, using tools from information theory (Physical layer) and queueing theory (Network layer) [3]-[5].

Similar cross-layer analysis for random multi-access channel (MAC) systems has been addressed in the literature in different contexts, most of which do not deal with cognitive or cooperative systems [6]-[9]. In this letter we incorporate and analyze the impact of CR with and without relaying capabilities in primary MAC schemes under the assumption that transmitters do not have channel state information. In particular, two protocols are compared: the non-cooperative CR MAC and the cooperative CR multi-access relay channel (MARC). The first considered protocol does not allow for cooperation between primary and secondary users and consists of a logical extension of the TDMA non-cooperative protocol to a MAC [4] in which users simultaneously transmit. In contrast, the second investigated protocol allows for cooperation between primary and secondary users; the cognitive node acts as a common relay in order to assist primary transmissions [10] while also communicating its own data. We show that the simultaneous transmission of the primary sources uses the available resources of the system more efficiently and improves performance for both non-cooperative and cooperative protocols. The proposed analysis takes into account the bursty nature of transmission (networking standpoint) and the stability region for each protocol is derived using queueing theory. To the best of our knowledge the analysis of a CR system for a MAC primary network with and without relaying ability has not been reported in the literature.

The remainder of the paper is organized as follows: Section II introduces the system model and presents the basic assumptions required for the analysis. Section III deals with the cognitive MARC with and without relaying and analyzes the associated stable throughput region. Numerical results are presented and discussed in Section IV, followed by concluding remarks in Section V.

II. System Model and Problem Formulation

We assume a simple MARC configuration consisting of two primary users A, B, one common cognitive relay S, one common primary destination D and one cognitive destination D'. All the transmitters (A, B, S) have buffers of infinite capacity to store incoming packets, and Q_i denotes the queue length in number of packets of the i-th node's buffer ($i \in \{A, B, S\}$). Packets are transmitted with a transmission rate R bits per channel use (BPCU)[1]. Time is considered to be slotted and both sources transmit simultaneously, forming a standard MAC [10]. The packet arrivals at each node are independent and stationary Bernoulli processes with mean $\lambda_A = \lambda_B \triangleq \lambda_P$ (packets per slot) for both primary users and λ_S (packets per slot) for the cognitive node. In this case, the stability of the system can be checked by using Loynes' theorem [11]. This states that the i-th queue is stable, if the average arrival rate λ_i is less than the average departure rate μ_i. Whenever Loynes'

[1]Transmission rates of the three transmitters are identical for simplicity; allowing for different rates is the subject of future work.

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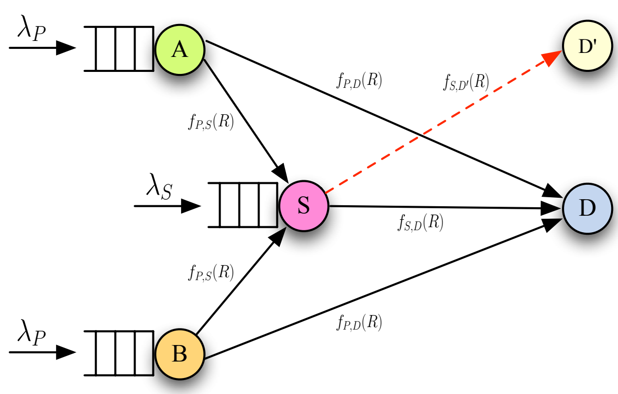

Fig. 1. The system model. A, B are terminal nodes with destination node D, while S is a cognitive relaying node which relays for A and B as well as transmits independent information to D'.

theorem is applicable, the average departure rate μ_i is defined as the *maximum stable throughput* of the i-th queue [3].

All wireless links are assumed to be stationary, frequency non-selective and Rayleigh block fading. This means the fading coefficients $\alpha_{i,j}$ (for the $i \rightarrow j$ link) remain constant during one packet, but change independently from one packet time to another according to a circularly symmetric complex Gaussian distribution with zero mean and variance $\sigma_{i,j}^2$. Furthermore, additive white Gaussian noise (AWGN) with zero mean and unit variance is assumed which corresponds to an average signal-to-noise ratio (SNR) equal to $\rho_{i,j} = P_0\sigma_{i,j}^2$ for the link $i \rightarrow j$, where P_0 denotes the transmitted power for each node. For the sake of presentation a symmetric configuration is assumed which corresponds to $\rho_{A,S} = \rho_{B,S} \triangleq \rho_{P,S}$ and $\rho_{A,D} = \rho_{B,D} \triangleq \rho_{P,D}$. This simplified model is sufficient in order to show the enhancements of the proposed schemes and is a guideline for more general network models. The channel coefficients $\alpha_{i,j}$ are assumed to be known only at the receivers (not at the transmitters) and perfect radio sensing is assumed for the cognitive relay [4] which allows the node S to access the channel only when it becomes available.

When the transmission rate cannot be adapted properly to the every instantaneous channel state (as is the case here), outages may occur. An outage occurs when the instantaneous capacity of the link $i \rightarrow j$ is lower than the transmitted spectral efficiency R. In such cases, re-transmission of the lost data is required. The retransmission process is based on an Acknowledgement/Negative-Acknowledgement (ACK/NACK) mechanism, in which short length error-free packets are broadcast by the destinations to inform the network of that packet's reception status. Each link $i \rightarrow j$ is characterized by the probability $f_{i,j}(R) \triangleq \mathbb{P}\{\log_2(1 + P_0|\alpha_{i,j}|^2) > R\} = \exp(-\frac{2^R-1}{\rho_{i,j}})$ which denotes the probability that the link $i \rightarrow j$ is not in outage. In this expression we have employed the information theoretic capacity of an additive white Gaussian noise link, which provides an upper bound to what can be achieved in practical systems.

Fig. 1 presents the system configuration. The objective of this letter is to study the stable throughput region of the described system with and without relaying capabilities.

III. Cognitive MARC with and without Relaying

A. Non-cooperative protocol

The non-cooperative protocol does not allow for cooperation between primary and secondary users. The cognitive node transmits its own data only when no primary transmissions are sensed, that is, whenever both primary users are idle. The stability region for the primary users is studied through the use of a dominant system where a primary user transmits "dummy" packets if the other primary queue is not empty (primary transmission always consists of two simultaneous links) [3], [9]. This dominant system de-couples the interaction between the two primary queues and as a consequence, they are empty simultaneously. Although this assumption provides an inner bound on the stability region of the original system, it is efficient for the considered symmetric scenario and enables the use of Little's theorem.

The instantaneous capacity region for the primary users is the set of all rates (R_A, R_B) satisfying the three constraints [12, Ch. 6]

$$R_A < \mathcal{C}_{AD} \triangleq \log_2\left(1 + P_0|\alpha_{A,D}|^2\right), \tag{1}$$

$$R_B < \mathcal{C}_{BD} \triangleq \log_2\left(1 + P_0|\alpha_{B,D}|^2\right), \tag{2}$$

$$R_A + R_B < \mathcal{C}_{AD+BD} \triangleq \log_2\left(1 + P_0|\alpha_{A,D}|^2 + P_0|\alpha_{B,D}|^2\right). \tag{3}$$

Without loss of generality we study the stability of user A but the resulting expressions hold for both primary users due to the considered symmetry. The common destination can succesfully decode the packet of the primary user A in two basic cases. The first case assumes that all the above three constraints are satisfied and thus the packets from both primary users can be decoded at the destination. Alternatively, the second case assumes that the destination cannot decode both users (the three constraints are not satisfied jointly) but it can decode user A by treating user B as noise (single user decoding). The stability of the primary user A is given by

$$\begin{aligned}\lambda_A < \mu_A^{(\max)} &= \underbrace{\mathbb{P}\left\{2R < \mathcal{C}_{AD+BD} \bigcap R < \mathcal{C}_{AD} \bigcap R < \mathcal{C}_{BD}\right\}}_{\triangleq \zeta_{P,D}(R), \text{(Both users can be decoded)}} \\ &+ \underbrace{(1-\zeta_{P,D}(R))\mathbb{P}\left\{\log_2\left(1 + \frac{P_0|\alpha_{A,D}|^2}{P_0|\alpha_{B,D}|^2+1}\right) > R\right\}}_{\text{(User } A \text{ can be decoded)}} \\ &= \zeta_{P,D}(R) + [1-\zeta_{P,D}(R)]h_{P,D}(R) \\ &\triangleq \Psi_{P,D}(R),\end{aligned} \tag{4}$$

where $\zeta_{P,D}(R) \triangleq F\left(\frac{2^R-1}{P_0}, \frac{2^{2R}-1}{P_0}, \frac{1}{\sigma_{P,D}^2}\right)$, the function F is defined in Appendix A and $h_{P,D}(R) = \mathbb{P}\left\{\log_2\left(1 + \frac{P_0|\alpha_{A,D}|^2}{P_0|\alpha_{B,D}|^2+1}\right) > R\right\} = \exp(-\frac{2^R-1}{\rho_{P,D}})\frac{1}{2^R}$ (ratio of two independent and identically-distributed (i.i.d.) exponential random variables)[2]. The cognitive user accesses the channel when both primary queues become empty (based on

[2]If a, b are exponential random variables with parameters λ_a and λ_b, respectively, $P(z) = \mathbb{P}\left\{\frac{aP_0}{bP_0+1} < z\right\} = 1 - e^{-\frac{\lambda_a z}{P_0}} \frac{\lambda_b}{\lambda_b + \lambda_a z}$.

the assumed dominant system). The stability of the cognitive queue requires

$$\lambda_S < \mu_S^{(\max)} = \mathbb{P}\{Q_A = 0\}\mathbb{P}\{Q_B = 0\} f_{S,D'}(R) = \left[1 - \frac{\lambda_P}{\mu_P^{(\max)}}\right]^2 f_{S,D'}(R), \quad (5)$$

where $\mu_A^{(\max)} = \mu_B^{(\max)} \triangleq \mu_P^{(\max)}$ and $\mu_S^{(\max)}$ denote the service rates for the primary queues (Q_A, Q_B) and secondary queue (Q_S), respectively, and for the above expression we have used Little's theorem [13].

The dominant system- discussion: The stable throughput analysis of the considered MAC system is based on the construction of a dominant system, in which a primary user transmits "dummy" packets if the other primary queue is not empty. The stochastic dominance is a well-known technique for decoupling the interaction between queues and ensures that the stationary distribution of the original system is stochastically dominated by that of the new system [8]. The proposed dominant system allows a simple analysis of the considered MAC protocols and enables the derivation of a useful inner bound of the stability region. This bound reveals the gain of the proposed protocols against previous techniques and is sufficient for the purposes of this paper. It is worth noting that due to the symmetric data-rate and channel assumption, the primary queues follow the same behavior (i.e. both queues contain the same average number of packets) and thus "dummy" transmission does not dominate the primary transmission. However, the investigation of tighter inner and outer bounds to the stability region could be considered for future work.

B. Cooperative protocol

This protocol takes into account the relaying ability of the cognitive user and allows for cooperation between primary and secondary users. In contrast with the non-cooperative protocol where a primary node removes a packet from its queue when it is successfully received at the destination, here it can also drop a packet when it is successfully received by the cognitive node (a modified ACK/NACK is available). To support relaying, the node S is equipped with two relaying queues Q_{RA} and Q_{RB} for the primary users A and B, respectively.

The cognitive node S serves both primary relaying queues using a superposition coding technique when a primary slot becomes idle. The resulting capacity region is that of a symmetric[3] broadcast channel (a single destination with two data flows) [12, Sec. 6.2.1] defined by:

$$R_{RA} + R_{RB} < \log_2(1 + P_0|\alpha_{S,D}|^2), \quad (6)$$

where R_{RA} and R_{RB} denote the transmission rate in BPCU for the relaying queues Q_{RA} and Q_{RB}, respectively. As the maximum transmission power for each node is equal to P_0, a symmetric power allocation which assigns power $P_0/2$ to each data flow is assumed. The considered symmetric broadcast channel introduces two decoding scenarios at the destination: (1) If the above constraint is satisfied, the destination can decode both primary data flows, (2) if the above constraint does not hold, the destination may be able to decode one user by treating the other user as noise (single user decoding).

[3]Recall that we assume symmetric channel gains.

When at least one of the two primary queues at the cognitive relay is empty we construct a dominant system for the service of the relaying queues as follows. When only one relaying queue (Q_{RA} or Q_{RB}) becomes empty and the cognitive node has data to transmit to its destination ($Q_S \neq 0$), the superposition coding technique serves one relaying queue and the cognitive queue. When the cognitive queue is also empty and thus a relaying queue is the only non-empty queue at the cognitive node, the node S superimposes the primary relaying data with "dummy" packets in order to decouple queue interaction (dominant system). Therefore, the departure rate of a relaying queue always corresponds to a symmetric broadcast channel with two superimposed data flows. The stability of the primary user A requires

$$\lambda_A < \mu_A^{(\max)} = \Psi_{P,D}(R) + \underbrace{[1 - \Psi_{P,D}(R)]}_{\text{User } A \text{ cannot be decoded}} \Psi_{P,S}(R), \quad (7)$$

where $\Psi_{P,S}(R)$ follows the definition in Eq. (4). Furthermore, the stability of the relaying queue gives one more constraint for the primary throughpout which is written as

$$\lambda_{RA} = \underbrace{\frac{\lambda_A}{\mu_A^{(\max)}}}_{Q_A \neq 0}[1 - \Psi_{P,D}(R)]\Psi_{P,S}(R) < \mu_{RA}^{(\max)} = \underbrace{\left[1 - \frac{\lambda_A}{\mu_A^{(\max)}}\right]^2}_{Q_A=0, Q_B=0} \Big[\underbrace{f_{S,D}(2R)}_{\text{Both data flows are decoded}} + \underbrace{[1 - f_{S,D}(2R)] u_{S,D}(P_0/2, R)}_{Q_{RA} \text{ is decoded}} \Big] \quad (8)$$

$$\Rightarrow \frac{\frac{\lambda_A}{\mu_A^{(\max)}}}{\left[1 - \frac{\lambda_A}{\mu_A^{(\max)}}\right]^2} < \frac{f_{S,D}(2R) + [1 - f_{S,D}(2R)] u_{S,D}(P_0/2, R)}{[1 - \Psi_{P,D}(R)]\Psi_{P,S}(R)}. \quad (9)$$

where $u_{i,j}(m, n) = \mathbb{P}\left\{\log_2\left(1 + \frac{m|\alpha_{i,j}|^2}{m|\alpha_{i,j}|^2+1}\right) > n\right\}$.

Regarding the stability of the cognitive queue, the cognitive user transmits its own data in two ways: (1) via a superposition technique (simultaneous with primary schemes) or (2) via single user transmission when all the primary and relaying queues are empty. Therefore the stability of the cognitive queue is written as

$$\lambda_S < \mu_S^{(\max)} = \underbrace{\left[1-\frac{\lambda_P}{\mu_P^{(\max)}}\right]^2}_{Q_A=0,Q_B=0}\underbrace{\left[1-\frac{\lambda_R}{\mu_R^{(\max)}}\right]^2}_{Q_{RA}=0,Q_{RB}=0} f_{S,D'}(R)$$
$$+\underbrace{\left[1-\frac{\lambda_P}{\mu_P^{(\max)}}\right]^2}_{Q_A=0,Q_B=0}\underbrace{\left[2\left(1-\frac{\lambda_R}{\mu_R^{(\max)}}\right)\frac{\lambda_R}{\mu_R}\right]}_{Q_{RA}=0 \text{ or } Q_{RB}=0}$$
$$\times\left[f_{S,D'}(2R)+[1-f_{S,D'}(2R)]u_{S,D'}(R)\right], \quad (10)$$

where $\lambda_{RA} = \lambda_{RB} \triangleq \lambda_R$ and $\mu_{RA}^{(\max)} = \mu_{RB}^{(\max)} \triangleq \mu_R^{(\max)}$ denote the arrival rate and the service rate, respectively, for the relaying queues. It is worth noting that the considered dominant system decouples the interaction between the queues and allows the use of Little's theorem in Eq. (10).

Asymptotic behavior: Cognitive cooperation is a useful concept for scenarios where the relaying link provides a better path than the direct links [4]. At high SNRs ($P_0 \rightarrow \infty$), the destination can decode both users with a high probability and therefore cooperation is no longer useful. For the MAC protocol with simultaneous transmissions, this fundamental conclusion can be seen by Eq. (4) where $\Psi_{P,D}(R) \rightarrow 1$ and therefore the relaying queues do not receive data for retransmission ($\lambda_{RA} \rightarrow 0$ in Eq. (8)). Furthermore, as far as the maximum primary stable throughput is concerned, the MAC scheme without cooperation provides a $\mu_P^{(\max)} \rightarrow 1$ packets/slots/user and therefore is the most efficient scheme for high SNRs ($\mu_P^{(\max)} \rightarrow 1/2$ packets/slots/user for a symmetric TDMA case). Therefore, a MAC strategy with cooperation at low SNRs and without cooperation at high SNRs seems to be a unified scheme regardless of SNR for the primary system, and thus the cognitive user's behavior is the only one that needs to change depending on the SNR.

It is worth noting that from an information theory standpoint, non-cooperative TDMA and non-cooperative MAC rate regions touch at one point for all SNRs, which is corresponding to a fraction of time $1/2$ for each user (symmetric TDMA scheme). Therefore, MAC and TDMA schemes become equivalent for the considered symmetric configuration at high SNRs. However, from a networking perspective, under the assumption that the packet sizes are fixed (do not scale with SNR) and slots cannot be sub-divided, TDMA results in a rate of $R/2$ BPCU per user while MAC eventually provides a rate equal to R BPCU per user.

IV. Numerical Results

Computer simulations were carried-out in order to validate the performance of the proposed schemes. In order to aid the clarity of presentation, a symmetric configuration is assumed with $R = 2$ BPCU for all nodes, $\rho_{A,D} = \rho_{B,D} = \rho_{P,D} = 7$ dB, $\rho_{A,S} = \rho_{B,S} = \rho_{P,S} = 20$ dB and $\rho_{S,D} = \rho_{S,D'} = 20$ dB. Fig. 2 plots the primary throughput (λ_P) versus the maximum cognitive stable throughput ($\mu_S^{(\max)}$) for the considered protocols: non-cooperative TDMA, cooperative TDMA, non-cooperative MAC and cooperative MAC. For the TDMA-based protocols [4], a scheduling process (i.e. Round-Robin scheduling) allows the users A and B to access the channel over disjoint and symmetric fractions of time (a fraction of time $1/2$). On the other hand, the MAC scheme follows the description of Section III and thus allows both users to access the channel simultaneously. The first important observation is that the non-cooperative MAC protocol provides the worst performance for the cognitive node for low primary data rates. The combination of poor primary direct links with multi-user interference corresponds to a high MAC outage probability, resulting in the slow emptying of the primary queues and very few secondary transmission opportunities. However, its performance is improved as the primary data rate increases and outperforms the non-cooperative TDMA scheme for $\lambda_P > 0.25$ (packets/slot). Due to the division of the available degrees of freedom, TDMA techniques become inefficient as the primary data rate increases. Alternatively, cooperation is beneficial for the MAC case and significantly overcomes the limitations of the non-cooperative MAC case by offering a better link between the primary users and the common destination. Cooperation improves the non-cooperative MAC protocol by providing a maximum stable primary throughput equal to $\mu_P^{(\max)} \approx 0.45$ rather than $\mu_P^{(\max)} \approx 0.35$ packets/slot for the non-cooperative case. Furthermore, the cooperative MAC scheme outperforms the cooperative TDMA protocol ($\mu_P^{(\max)} \approx 0.34$) as it more efficiently uses the available resources of the system and thus is an appropriate solution for the assumed symmetric configuration.

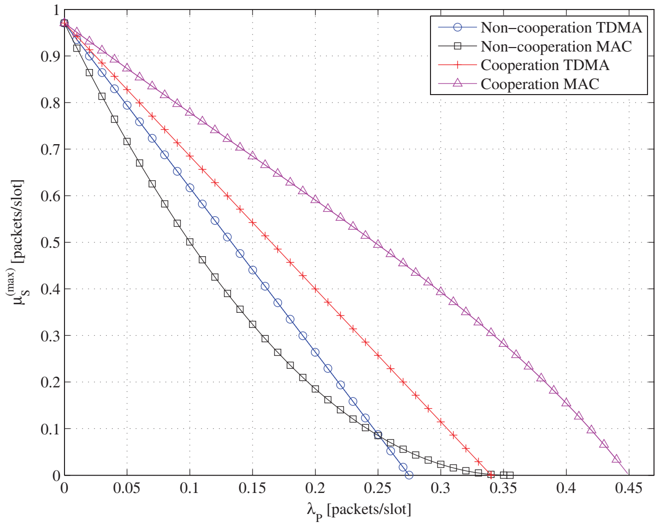

Fig. 2. Primary throughput (λ_P) versus the maximum secondary throughput ($\mu_P^{(\max)}$) for non-cooperation TDMA, non-cooperation MAC, cooperation TDMA, cooperation MAC; $R = 2$ BPCU, $\rho_{P,D} = 7$ dB, $\rho_{P,S} = \rho_{S,D} = \rho_{S,D'} = 20$ dB.

In order to generalize the above conclusions and show the limitations of the cognitive cooperation, Fig. 3 presents the stable throughput region of the proposed protocols for different SNR regions of the relaying links. More specifically, in Fig. 3(a), we assume a symmetric channel configuration with $\rho_{P,D} = \rho_{P,S} = \rho_{S,D} = \rho_{S,D'} = 7$ dB and $R = 2$ BPCU. As can be seen, for this configuration, cooperation is not useful for the TDMA technique and thus cooperative TDMA provides the same stable throughput as the non-cooperative TDMA

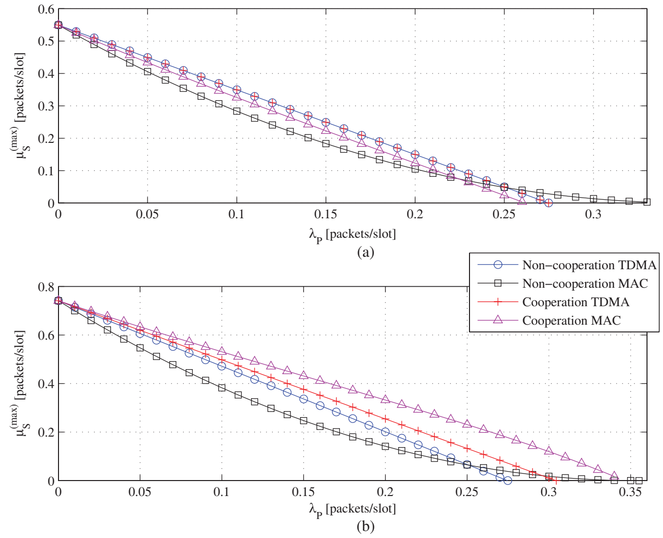

Fig. 3. Primary throughput (λ_P) versus the maximum secondary throughput ($\mu_P^{(\max)}$) for non-cooperation TDMA, non-cooperation MAC, cooperation TDMA, cooperation MAC; (a) $R = 2$ BPCU, $\rho_{P,D} = \rho_{P,S} = \rho_{S,D} = \rho_{S,D'} = 7$ dB, (b) $R = 2$ BPCU, $\rho_{P,D} = 7$ dB, $\rho_{P,S} = \rho_{S,D} = \rho_{S,D'} = 10$ dB.

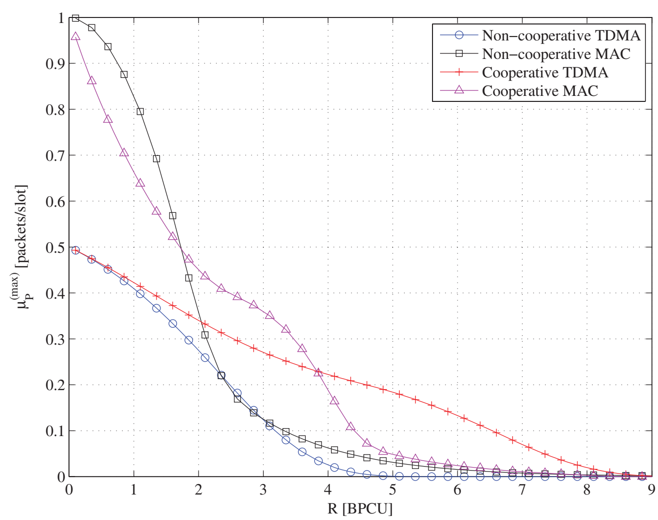

Fig. 4. Maximum primary throughput ($\mu_P^{(\max)}$) versus the transmission rate R for non-cooperation TDMA, non-cooperation MAC, cooperation TDMA, cooperation MAC; $\rho_{P,D} = 7$ dB, $\rho_{P,S} = \rho_{S,D} = \rho_{S,D'} = 20$ dB.

scheme. This behavior is in line with the results presented in [4], where cognitive cooperation is beneficial only when the relay-destination link is better than the source-destination link. As far as the MAC technique is concerned, it can be seen in Fig. 3(a) that the cooperative MAC outperforms the non-cooperative MAC for low primary data rates. However, as the primary data rate increases ($\lambda_P > 0.225$) the non-cooperative MAC provides a higher stable throughput. This observation is based on the fact that for high primary data rates, the probability that the primary queues become empty decreases which results in less opportunities for the relay node to access the channel. Furthermore, from the simulation parameters in Figure 3(a) we are able to conclude that TDMA is suitable at low primary data rates. The MAC performs relatively poorly in this regime due to the poor relay-destination link as well as the power split induced by the assumed superposition scheme. However, for high data rates, the non-cooperative MAC schemes is the appropriate solution, as we have concluded in the previous simulation results. Accordingly, Fig. 3(b) plots the stable throughput for a configuration with $R = 2$ BPCU, $\rho_{P,D} = 7$ dB and $\rho_{P,S} = \rho_{S,D} = \rho_{S,D'} = 10$ dB, which corresponds to a slight improvement of the relaying links. It can be seen that as the relaying links become stronger (in comparison with the direct links), cooperation is beneficial for the system and our observations follow our conclusions of Fig. 2.

Fig. 4 demonstrates the impact of the transmission rate on the maximum stable primary throughput[4] $\mu_P^{(\max)}$ for the different considered multi-access protocols. The simulation parameters are $\rho_{P,D} = 7$ dB, $\rho_{P,S} = 20$ dB and $\rho_{S,D} = \rho_{S,D'} = 20$ dB. As can be seen the non-cooperative MAC protocol outperforms the non-cooperative TDMA scheme for almost all the cases. This result indicates that the non-cooperative MAC scheme is more efficient than non-cooperative TDMA for cognitive applications where the maximization of the primary throughput is the goal, as long as the relaying links to both sources and destinations are sufficiently strong. A second important observation is that the cooperative MAC protocol outperforms the corresponding cooperative TDMA scheme for low and intermediate data rates but is outperformed by the cooperative TDMA for the high data rates. More specifically, the cooperative MAC outperforms the cooperative TDMA case for $R \leq 3.8$ BPCU with an inverse behavior for $R > 3.8$ BPCU. This behavior follows from the fact that in the cooperative MAC scheme, the transmission for both primary users is performed via the same relay-destination link by using a power split (superposition approach). The negative impact of a poor relay-destination link (in deep fades) on the performance of the cooperative MAC protocol is stronger due to the power division and the related interference of the superposition technique. These results motivate rate adaptation techniques where transmitters adapt their transmission rates to the instantaneous channel conditions [6] as well hybrid protocols which switch between different multi-access techniques.

It is worth noting that cooperation for cognitive systems is an interesting solution only when the direct links are in deep fades and both branches of the relaying link are strong enough in order to establish communication [4], which motivated our particular choice of simulation parameters.

V. Conclusion

In this letter, we studied the interplay between CR and cooperative diversity for a standard primary multi-access system. Based on a simple five-node CR MARC structure, we characterized both the stability and the throughout region under non-cooperative and cooperative protocols separately. The analysis revealed that cooperation is beneficial for the MAC case by providing higher stable throughput for all the users in the network. Furthermore, results indicated that the consideration of the MAC as primary transmisison policy can provide significant gains over previously reported TDMA

[4]The maximization of the primary throughput is the main objective of a standard CR system.

schemes. Future work includes the generalization of our analysis to a larger asymetric network with random multi-access channel as well as the consideration of delay performance issues.

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APPENDIX A
SUCCESS PROBABILITY FOR MAC (DECODING BOTH USERS)

Let X, Y be two i.i.d. exponential random variables with parameter λ and let η and η_0 with $\eta_0 > 2\eta$ be two deterministic random variables. We define the region

$$\begin{aligned}\mathcal{R}(\eta,\eta_0) &\triangleq \Big\{(x,y): x>\eta \bigcap y>\eta \bigcap x+y>\eta_0\Big\}\\ &= \Big\{(x,y): x>\eta \bigcap y>\eta \bigcap y>\eta_0-x\Big\}\\ &= \Big\{(x,y): x>\eta \bigcap y>\max\big[\eta,\eta_0-x\big]\Big\}\\ &= \Big\{(x,y): \eta_0-\eta \geq x>\eta \bigcap y>\eta_0-x\Big\}\\ &\quad \bigcup \Big\{(x,y): x>\eta_0-\eta \bigcap y>\eta\Big\}, \qquad (11)\end{aligned}$$

and thus the probability under question is written as

$$\begin{aligned}F(\eta,\eta_0,\lambda) &= \int\int_{\mathcal{R}(\eta,\eta_0)} p(x,y)dxdy\\ &= \int\int_{\mathcal{R}(\eta,\eta_0)} p_0(x)p_0(y)dxdy\\ &= \int_{\eta}^{\eta_0-\eta} p_0(x)\left(\int_{\eta_0-x}^{\infty} p_0(y)dy\right)dx\\ &\quad + \int_{\eta_0-\eta}^{\infty} p_0(x)\left(\int_{\eta}^{\infty} p_0(y)dy\right)dx\\ &= \lambda\big(\eta_0-2\eta\big)\exp(-\lambda\eta_0)+\exp(-\lambda\eta_0)\\ &= \exp(-\lambda\eta_0)\big[\lambda\big(\eta_0-2\eta\big)+1\big], \qquad (12)\end{aligned}$$

where $p(x, y)$ denotes the joint probability density function (PDF) of the random variables X, Y and $p_0(\cdot)$ denotes the PDF of the i.i.d. random variables X, Y. It is worth noting that the condition $\eta_0 > 2\eta$ always holds for the considered symmetric configuration (both users transmit with the same rate).

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