Frequency-Domain Bit-Flipping Equalizer for Wideband MIMO Channels

Toshiaki Koike-Akino, Natasha Devroye, and Vahid Tarokh

***Abstract*—We propose a low-complexity equalizer whose performance approaches that of the optimal maximum-likelihood estimators in wideband multiple-input multiple-output (MIMO) channels. The proposed algorithm makes use of a bit-flipping refinement procedure preceded by a frequency-domain equalizer and is based on local-optima searching algorithms. Through performance evaluations, it is demonstrated that the proposed equalizer can perform well when a large number of diversity branches are available in severely dispersive fading channels.**

***Index Terms*—MLSE, MIMO, frequency-domain equalizer.**

I. INTRODUCTION

THE recent demand for ubiquitous networking has necessitated breakthroughs realizing high-speed radio communications. One promising technique for achieving high data-rates is termed multiple-input multiple-output (MIMO), in which multiple antennas are used at both the transmitter and the receiver of a wireless communication system. MIMO systems have been shown, in theory, to significantly improve the channel capacity in rich-scattering wireless links[1, 2], and the achievable gains have been confirmed, in practice, through over-Gbps transmitting field experiments [3]. In the near future, signal bandwidth will likely increase to several GHz to accommodate even higher data-rate transmissions. For realizing the full benefits of such wideband MIMO systems, sophisticated equalization techniques are required to cope with severely dispersive channels.

In frequency-selective MIMO channels, optimal equalizers are based on maximum-likelihood sequence estimation (MLSE). The computational complexity required by MLSE increases exponentially in the number of antennas and channel memory taps. Therefore, equalizers which trade optimality for reduced complexity are typically employed in dispersive channels. Among low-complexity sub-optimum equalizers, frequency-domain (FD) equalization [4] based on the mean-square error (MMSE) or the zero-forcing (ZF) criterion offers a good tradeoff between performance and complexity, which is independent of the channel memory length.

In this letter, we propose a way to improve the FD equalizer by using a simple bit-flipping refinement, which is based on hill-climbing[5]. Although it requires almost no additional complexity and no arithmetic multiplications (the only multiplications used are those for the fast Fourier transform, FFT), its performance can be very close to that of the optimal MLSE equalizer. The key idea is to leverage the fact that the probability of hitting a local optimum in the ML search space decreases exponentially with the number of diversity branches as discussed by the authors in [6–8]. This implies that local-optima searching algorithms can approach the MLSE performance in wideband MIMO channels where we can exploit the inherently large number of diversity branches resulting from the dispersive channel and multiple receive antennas. As the search-space analysis suggests, the proposed bit-flipping equalizer offers excellent performance in wideband MIMO channels while achieving a computational complexity almost comparable to that of the linear FD-MMSE equalizer.

The similar idea of using a bit-flipping refinement to improve performance has been considered in, for example, [9, 10]. Our chief contribution lies in the algorithm derivation: we reduce the complexity to one that is comparable to low-complexity FD-MMSE equalizers and which does not require multiplications. We show the significant advantage of the proposed equalizer in wideband MIMO systems for severely dispersive channels with 100-path delays.

Notation: Throughout the letter, we describe matrices and vectors in capital and lower-case italic boldfaces, respectively. The transpose, the conjugate transpose, and the Frobenius norm of a matrix $\boldsymbol{X}$ are denoted as $\boldsymbol{X}^{\mathrm{T}}$, $\boldsymbol{X}^{\dagger}$ and $\|\boldsymbol{X}\|$, respectively. The real part of a complex number x is represented as $\Re[x]$. The matrix $\boldsymbol{I}_p$ denotes the p-dimensional identity matrix. The notations $\mathbb{C}$ and $\mathbb{Z}_p = \{0, 1, \ldots, p-1\}$ designate the set of complex numbers and the set of non-negative integers less than p, respectively.

II. NEAR-MLSE BIT-FLIPPING MIMO EQUALIZER

In previous works [6–8], we have shown that the average number of undesired local optima that appear in the ML search space decreases exponentially with the number of diversity branches. This suggests that sub-optimal local-optima searching algorithms may approximate MLSE with almost no performance loss in wideband MIMO channels, as these channels exhibit a high level of diversity degree.

In this section, we propose a low-complexity local-optima searching equalizer which uses simple bit-flipping operations. The proposed scheme uses the low-complexity FD-MMSE equalizer as its initial estimate. This initial estimate can be significantly refined by the bit-flipping procedure. Since the required complexity for the bit-flipping refinement is much lower than the FD-MMSE equalizer, the overall complexity can be almost comparable to that of the FD-MMSE equalizer.

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A. Frequency-Domain MMSE Equalizer

We first describe the conventional FD-MMSE equalizer[4], which is one of the most promising equalizers for wideband MIMO channels because its complexity is basically independent of the channel memory length. We use a transmission frame which is K symbols long, preceded by a P-symbol cyclic prefix to avoid inter-frame interference. We suppose that delayed versions of the transmitted signal (multi-path) all arrive within a P-symbol delay interval. The M-antenna transmitter sends the modulated signals vector $\boldsymbol{x}_k \in \mathbb{C}^{M\times 1}$ during the k-th symbol ($0 \leq k \leq K-1$) to the N-antenna receiver. The received signal vector $\boldsymbol{y}_k \in \mathbb{C}^{N\times 1}$ is modeled as

$$\boldsymbol{y}_k = \sum_{p=0}^{P-1} \boldsymbol{H}_p \boldsymbol{x}_{k-p} + \boldsymbol{w}_k, \tag{1}$$

where $\boldsymbol{H}_p \in \mathbb{C}^{N\times M}$ and $\boldsymbol{w}_k \in \mathbb{C}^{N\times 1}$ are the channel gain matrices of the p-th delayed versions of the transmitted signal and the Gaussian noise vector with a variance of σ^2 at the k-th symbol, respectively.

Due to the cyclic prefix, we can represent the received signal in the frequency domain, for the l–th frequency index ($0 \leq l \leq K-1$), as

$$\boldsymbol{r}_l = \boldsymbol{G}_l \boldsymbol{u}_l + \boldsymbol{v}_l, \tag{2}$$

where $\boldsymbol{r}_l \in \mathbb{C}^{N\times 1}$, $\boldsymbol{G}_l \in \mathbb{C}^{N\times M}$, $\boldsymbol{u}_l \in \mathbb{C}^{M\times 1}$, and $\boldsymbol{v}_l \in \mathbb{C}^{N\times 1}$ are the received signal vector, the channel matrix, the transmitted signal vector and the noise vector in the frequency domain, respectively. Those are obtained from the time domain signals using an FFT. Channel estimation may be performed in the time domain or in the frequency domain. The FD-MMSE equalizer employs the MMSE nulling over each l-th signal as follows:

$$\tilde{\boldsymbol{u}}_l = \left(\boldsymbol{A}_l + \sigma^2 \boldsymbol{I}_M\right)^{-1} \boldsymbol{G}_l^\dagger \boldsymbol{r}_l = \boldsymbol{Q}_l^{-1} \boldsymbol{z}_l, \tag{3}$$

where $\boldsymbol{A}_l = \boldsymbol{G}_l^\dagger \boldsymbol{G}_l \in \mathbb{C}^{M\times M}$, $\boldsymbol{Q}_l = \boldsymbol{A}_l + \sigma^2 \boldsymbol{I}_M \in \mathbb{C}^{M\times M}$, and $\boldsymbol{z}_l = \boldsymbol{G}_l^\dagger \boldsymbol{r}_l \in \mathbb{C}^{M\times 1}$. The MMSE outputs $\tilde{\boldsymbol{u}}_l$ are transformed to the time domain $\tilde{\boldsymbol{x}}_k$ using an inverse FFT (IFFT), after which the receiver decodes $\boldsymbol{x}_k$ from $\tilde{\boldsymbol{x}}_k$.

Although the MMSE equalizer outperforms the ZF equalizer in general, it is considerably inferior to the optimal MLSE equalizer if the transmitter and receiver are equipped with a comparable number of antennas. To improve performance, we may use successive interference cancellation (SIC) as done in the V-BLAST algorithm [1, 11], which uses frame-by-frame decision feedback to reduce the inter-stream interference. The most likely sequence (corresponding to the smallest diagonal entry of $\sum \boldsymbol{Q}_l^{-1}$) is successively subtracted from the received signals, and the updated MMSE nulling then yields better estimates due to increased diversity gains. In this letter, we propose a more efficient scheme to achieve near optimal MLSE performance through the use of a local-optima searching algorithm.

B. Bit-Flipping Equalizer

The key idea of the proposed hill-climbing method, referred to as a bit-flipping equalizer, is as follows: Given a tentative decision sequence, we compute its likelihood. For all possible single bit flips, we determine whether the flip will increase the likelihood or not. Assuming (2^{2q})-ary square QAM, we have a total of $2qMK$ bits per frame. If more than one bit-flips can increase the likelihood, we first flip the bit which gives the maximum likelihood; hence the *hill-climbing* terminology. After several bit flips (at most $2qMK$ times), this algorithm converges to a local optimum that may offer a better estimate than the first tentative decision. Since we have shown that the number of local optima decreases exponentially with the number of diversity branches in [6–8], our hill-climbing method may approach the optimum MLSE performance with high probability in wideband MIMO channels, which inherently experience a large amount of diversity.

In this algorithm, after each bit flip, we must compute the likelihood. In the following, we propose a computationally efficient likelihood calculation and bit-flipping procedure that requires no multiplications at all for bit-flipping, but does require multiplications in the form of an FFT. (Note that FFT operations are much more computationally efficient than regular multiplications.) For simplicity, we consider BPSK modulations for now and extend our results to different modulation schemes later on. Suppose that we have a tentative decision $\hat{\boldsymbol{x}} \in \mathbb{C}^{MK\times 1}$. The Euclidean metric in the MLSE equalizer is expressed over the frequency domain as follows

$$\mu(\hat{\boldsymbol{x}}) = \left\|\boldsymbol{r} - \boldsymbol{G}\boldsymbol{F}\hat{\boldsymbol{x}}\right\|^2, \tag{4}$$

where $\boldsymbol{r} \in \mathbb{C}^{NK\times 1}$, $\boldsymbol{G} \in \mathbb{C}^{NK\times MK}$, $\hat{\boldsymbol{x}} \in \mathbb{C}^{MK\times 1}$, and $\boldsymbol{F} \in \mathbb{C}^{MK\times MK}$ are written as

$$\boldsymbol{r} = \begin{bmatrix} \boldsymbol{r}_0 \\ \boldsymbol{r}_1 \\ \vdots \\ \boldsymbol{r}_{K-1} \end{bmatrix}, \boldsymbol{G} = \begin{bmatrix} \boldsymbol{G}_0 & & \boldsymbol{0} \\ & \boldsymbol{G}_1 & \\ & & \ddots & \\ \boldsymbol{0} & & & \boldsymbol{G}_{K-1} \end{bmatrix}, \hat{\boldsymbol{x}} = \begin{bmatrix} \hat{\boldsymbol{x}}_0 \\ \hat{\boldsymbol{x}}_1 \\ \vdots \\ \hat{\boldsymbol{x}}_{K-1} \end{bmatrix},$$

$$\boldsymbol{F} = \frac{1}{\sqrt{K}} \begin{bmatrix} \rho_{0,0}\boldsymbol{I}_M & \rho_{0,1}\boldsymbol{I}_M & \cdots & \rho_{0,K-1}\boldsymbol{I}_M \\ \rho_{1,0}\boldsymbol{I}_M & \rho_{1,1}\boldsymbol{I}_M & \cdots & \rho_{1,K-1}\boldsymbol{I}_M \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{K-1,0}\boldsymbol{I}_M & \rho_{K-1,1}\boldsymbol{I}_M & \cdots & \rho_{K-1,K-1}\boldsymbol{I}_M \end{bmatrix}.$$

The matrix $\boldsymbol{F}$ is known as the FFT block matrix where $\rho_{k,l} = \exp(-\mathrm{j}2\pi kl/K)$.

We now determine whether a single bit-flip can reduce the metric or not. For instance, suppose we flip a bit in the k-th symbol from the m-th transmit antenna. We let $\hat{\boldsymbol{x}}'$ be the bit-flipped version of $\hat{\boldsymbol{x}}$, whose entries are equivalent to $\hat{\boldsymbol{x}}$ except for the d-th entry, where $d = kM + m$. Then, we can express the bit flipped signal as

$$\hat{\boldsymbol{x}}' = \hat{\boldsymbol{x}} + 2\delta_d \boldsymbol{e}_d, \tag{5}$$

where $\delta_d = -1$ when $\hat{x}_d = 1$, otherwise $\delta_d = 1$. The vector $\boldsymbol{e}_d \in \mathbb{C}^{MK\times 1}$ is a unit vector whose d-th entry is one and the others are all zeros. The corresponding metric is given by

$$\begin{aligned}\mu(\hat{\boldsymbol{x}}') &= \left\|\boldsymbol{r} - \boldsymbol{G}\boldsymbol{F}\hat{\boldsymbol{x}}'\right\|^2 = \left\|(\boldsymbol{r} - \boldsymbol{G}\boldsymbol{F}\hat{\boldsymbol{x}}) - 2\delta_d \boldsymbol{G}\boldsymbol{F}\boldsymbol{e}_d\right\|^2 \\ &= \mu(\hat{\boldsymbol{x}}) + 4\alpha_{d,d} - 4\delta_d\Big(\Re\left[\eta_d\right] - \sum_{i=0}^{MK-1} \Re\left[\alpha_{d,i}\right]\hat{x}_i\Big), \end{aligned}\tag{6}$$

where

$$\alpha_{i,j} = \boldsymbol{e}_i^\dagger \boldsymbol{F}^\dagger \boldsymbol{G}^\dagger \boldsymbol{G}\boldsymbol{F}\boldsymbol{e}_j, \qquad \eta_i = \boldsymbol{e}_i^\dagger \boldsymbol{F}^\dagger \boldsymbol{G}^\dagger \boldsymbol{r}. \tag{7}$$

Therefore, we need only to examine whether the following term is positive or not:

$$\nu_d = \delta_d \left(\Re[\eta_d] - \sum \Re[\alpha_{d,i}]\hat{x}_i \right) - \alpha_{d,d}. \quad (8)$$

When $\nu_d > 0$, we should flip the corresponding bit since the Euclidean metric decreases by $4\nu_d$ after the flipping. We first flip the bit of index $D = \mathrm{argmax}_{d \in \mathbb{Z}_{MK}} \nu_d$ which results in the the largest ν_d among all possible bits ($d \in \mathbb{Z}_{MK}$). Although we need to compute ν_d for all $0 \le d \le MK - 1$, as shown in (8), assuming $\alpha_{i,j}$ and η_i are obtained in advance (pre-processing), the computation requires only additions, as $\hat{x}_i \in \{-1, 1\}$. After flipping the D-th bit, we update all the values ν_d as follows:

$$\nu_d \leftarrow \begin{cases} \nu_d - 2\delta_D \delta_d \Re[\alpha_{d,D}], & d \neq D, \\ -\nu_d, & d = D. \end{cases} \quad (9)$$

We note that this update procedure does not require any multiplications (multiplying by 2^q can be done by adding or bit-shifting for floating-point precision or fixed-point precision).

For reliable initial estimates, we may exploit the decision outputs of the FD-MMSE equalizer. Although we can use any equalizer to obtain initial estimates, the bit-flipping equalizer is particularly suited to the FD-MMSE equalizer because $\alpha_{i,j}$ and η_i can be obtained using only an IFFT, as we will show next. As seen in (7), $\alpha_{i,j}$ is the (i, j)-th entry of a matrix $\boldsymbol{\mathcal{A}} = \boldsymbol{F}^\dagger \boldsymbol{G}^\dagger \boldsymbol{G} \boldsymbol{F}$, and η_i is the i-th element of a vector $\boldsymbol{\eta} = \boldsymbol{F}^\dagger \boldsymbol{G}^\dagger \boldsymbol{r}$. Those may be written as

$$\boldsymbol{\mathcal{A}} = \boldsymbol{F}^\dagger \boldsymbol{G}^\dagger \boldsymbol{G} \boldsymbol{F} = \mathrm{Toep}_M(\boldsymbol{B}), \quad (10)$$

$$\boldsymbol{B} = \frac{\boldsymbol{F}^\dagger}{\sqrt{K}} \begin{bmatrix} \boldsymbol{A}_0 \\ \boldsymbol{A}_1 \\ \vdots \\ \boldsymbol{A}_{K-1} \end{bmatrix}, \quad \boldsymbol{\eta} = \boldsymbol{F}^\dagger \boldsymbol{G}^\dagger \boldsymbol{r} = \boldsymbol{F}^\dagger \begin{bmatrix} \boldsymbol{z}_0 \\ \boldsymbol{z}_1 \\ \vdots \\ \boldsymbol{z}_{K-1} \end{bmatrix}. \quad (11)$$

Here, we denote $\mathrm{Toep}_M(\cdot)$ as a Hermite-symmetry block-Toeplitz matrix generated by the argument matrices as follows:

$$\mathrm{Toep}_M\left(\begin{bmatrix} \boldsymbol{B}_0 \\ \boldsymbol{B}_1 \\ \vdots \\ \boldsymbol{B}_{K-1} \end{bmatrix} \right) = \begin{bmatrix} \boldsymbol{B}_0 & \boldsymbol{B}_1^\dagger & \cdots & \boldsymbol{B}_{K-1}^\dagger \\ \boldsymbol{B}_1 & \boldsymbol{B}_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{B}_1^\dagger \\ \boldsymbol{B}_{K-1} & \cdots & \boldsymbol{B}_1 & \boldsymbol{B}_0 \end{bmatrix}, \quad (12)$$

where $\boldsymbol{B}_k \in \mathbb{C}^{M \times M}$ is the k-th block of $\boldsymbol{B}$. (Note that $\boldsymbol{B}$ is a sparse matrix; specifically, $\boldsymbol{B}_k$ becomes the zero matrix for all $P \le k \le K - P$ as long as the effective channel memory length is within the cyclic prefix duration.) Also note that $\mathrm{Toep}_M(\cdot)$ does not require any arithmetic computations. As in (11), both $\alpha_{i,j}$ and η_i can be derived as the IFFT of $\boldsymbol{A}_k$ and $\boldsymbol{z}_k$, both of which are already calculated for the FD-MMSE equalizer in (3). As a consequence, the bit-flipping equalizer does not require any high-complexity computations when the FD-MMSE equalizer is employed to yield the initial estimates.

Our proposed bit-flipping equalizer has two main advantages: 1) No multiplications (besides those of an FFT) are required, and 2) the computational complexity is basically independent of the channel memory length. This implies that the proposed equalizer is suitable for a dispersive channel. Because $\boldsymbol{B}$ is quite sparse, we only need to update ν_d for $2P - 2$ consecutive symbols around $d = D$. The system configuration of the bit-flipping equalizer for MIMO channels is depicted in Fig. 1. Note that the single-bit flipping can be extended to flipping multiple bits at the same time for improving performance. Our approach can be readily applied to higher (2^{2q})-ary square QAMs such as 4QAM, 16QAM, 64QAM and so on. The main idea of the extension is similar to the method introduced in [12], in which the QAM constellation is regarded as a superposition of $2q$ BPSK signals.

III. Performance Evaluations

In this section, we demonstrate the gains achieved by the proposed bit-flipping equalizer. We evaluate the bit-error-rate (BER) performance for uncoded QPSK and 16QAM signals using a Gray mapping in 4×4 MIMO systems. We assume that the channel follows a frequency-selective Rayleigh fading with exponentially decaying 100-path spikes (0.2 dB decay per symbol delay). We suppose that the channel is perfectly known at the receiver for simplicity. The transmission frame has a 1024-symbol information, preceded by a 128-symbol cyclic prefix.

Figs. 2 and 3 show BER performance for uncoded 4QAM and 16QAM 4×4 MIMO systems, respectively. We present the performance curves of the proposed bit-flipping equalizer (denoted as BF), the FD-ZF equalizer, the conventional FD-MMSE equalizer, the SIC equalizer, and the optimum MLSE equalizer. Because the exact MLSE equalization requires prohibitively high complexity, we present the performance curve of the matched filter bound (MFB) which is obtained by assuming all the interfering signals are optimally combined and whose performance is known to be comparable to the optimal MLSE performance in principle. The figures demonstrate that the performance of the bit-flipping equalizer preceded by FD-MMSE approaches that of the MLSE. We can see that none of the other sub-optimum equalizers (the ZF equalizer, the FD-MMSE equalizer, and the SIC non-linear equalizer) can achieve a comparable performance; more-than 8 dB loss is observed at a BER of 10^{-5} even for the SIC equalizer.

The average number of iterations (bit-flips) required to reach a local optimum was at most 0.7 and 2.6 for QPSK and 16QAM cases, respectively, for all E_b/N_0. (For the high SNR regime, it is much smaller.) Since the total number of bits in a frame is $2MK = 8192$ for QPSK and $4MK = 16384$ for 16QAM, we see that the number of bit-flips is much less than the number of transmitted bits on average. The BER performance curves for several values of the maximum allowable bit-flips I_max are plotted in Fig. 4 for QPSK transmissions. The curves of $I_\mathrm{max} = 0$ are equivalent to the performance of the FD-MMSE equalizer. One can see that limiting the maximum number of bit-flips may significantly degrade the BER performance even thought the average number of bit-flips may remain quite small. Comparing Figs. 2 and 4, we note that even the proposed equalizer with a maximum of eight-bit flips outperforms the SIC equalizer at a BER of 10^{-5}. We conclude that the bit-flipping equalizer performs significantly better than other sub-optimal equalizers.

The FD-MMSE equalizer has a complexity of $\mathcal{O}[2(N + M)(M^2 + \log K)K]$, whereas the bit-flipping equalizer uses

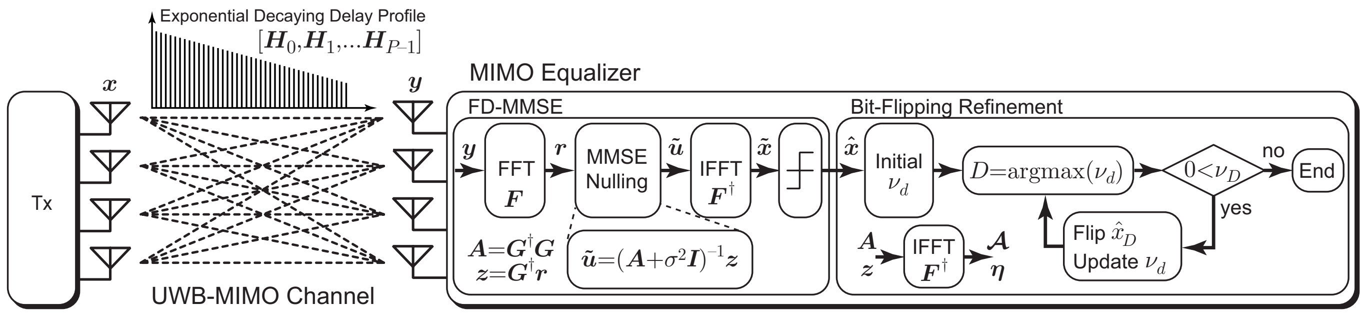

Fig. 1. Wideband MIMO system employing the proposed bit-flipping equalizer preceded by FD-MMSE initial estimation. (4×4 MIMO system, severely dispersive channels.)

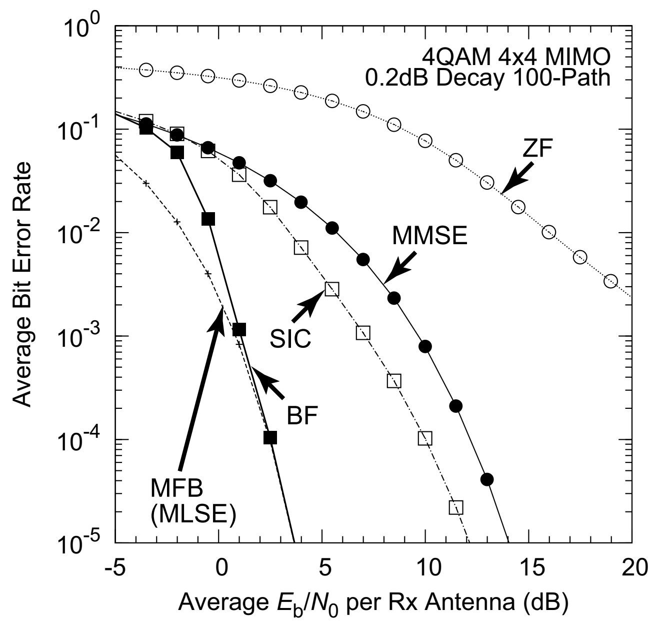

Fig. 2. Average BER performance versus average E_b/N_0. (4QAM 4×4 MIMO system, 100-path Rayleigh, 0.2dB decaying per symbol delay.)

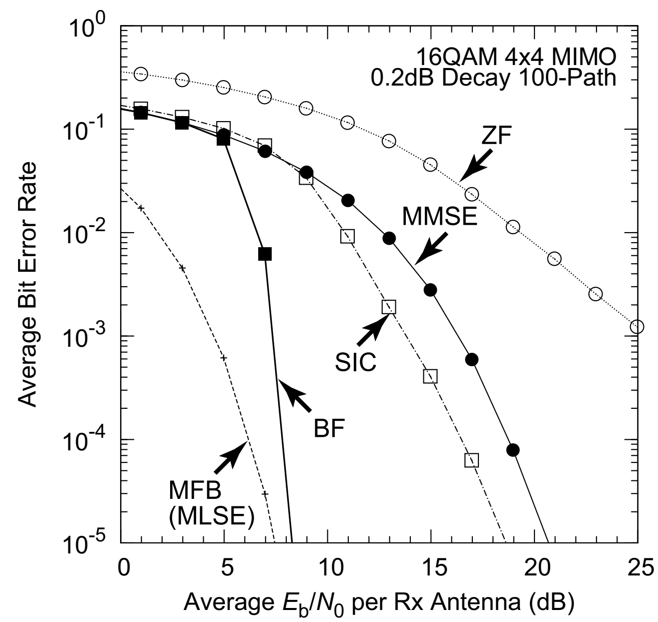

Fig. 3. Average BER performance versus average E_b/N_0. (16QAM 4×4 MIMO system, 100-path Rayleigh, 0.2dB decaying per symbol delay.)

$\mathcal{O}[M(M+2)K \log K]$ multiplications for pre-processing to obtain η_i and $\alpha_{i,j}$ from an IFFT and $\mathcal{O}[4qM^2P]$ additions per bit-flip to update ν_d. The MLSE equalizer requires a minimum of $\mathcal{O}[2^{2qM(P-1)}]$ trellis-states. We can thus see that the proposed bit-flipping equalizer has considerably reduced complexity yet still performs close to the MLSE optimum in this highly dispersive channel.

We have confirmed that our proposed bit-flipping equalizer performs well in an extremely dispersive channel (0.2 dB decaying 100-path fading), whose root mean-square (RMS) delay spread is approximately $28\,T_\mathrm{s}$ where T_s is the symbol duration. Since our proposed scheme exploits the path diversity in wideband channels, its performance deteriorates when the channel delay spread is small. Figs. 5 and 6 show BER performance in 0.6 dB decaying 20-path fading channels, whose RMS delay spread becomes approximately $7\,T_\mathrm{s}$, for uncoded 4QAM and 16QAM, respectively. In this figure, we also present the performance of the similar local-optima searching equalizer, which is proposed in [7]. This scheme utilizes the Hopfield neural network (NN). Comparing with 100-path channels, the 20-path channels can degrade the BER performance of the proposed bit-flipping equalizer especially for 16QAM case. Although the bit-flipping equalizer still enjoys a 5 dB performance gain over the SIC equalizer at a BER of 10^{-5} for 16QAM, it has an approximately 5 dB performance loss over the MLSE performance. This suggests that our proposed scheme is more advantageous for wideband channels which experience a large delay spread, and that it may be useless in frequency-flat fading channels or with OFDM signalling. The equalizer based on the Hopfield NN can outperform the SIC but not the bit-flipping equalizer. Since the NN equalizer and the bit-flipping equalizer have almost comparable complexity, the bit-flipping equalizer may be more useful.

IV. SUMMARY

In this letter, we proposed a novel low-complexity equalizer which uses simple bit-flipping without any multiplications, that is of particular use in severely dispersive MIMO channels. The presented algorithm is based on the hill-climbing method, which searches for local optima. Since the average number of local optima decreases with the number of diversity branches

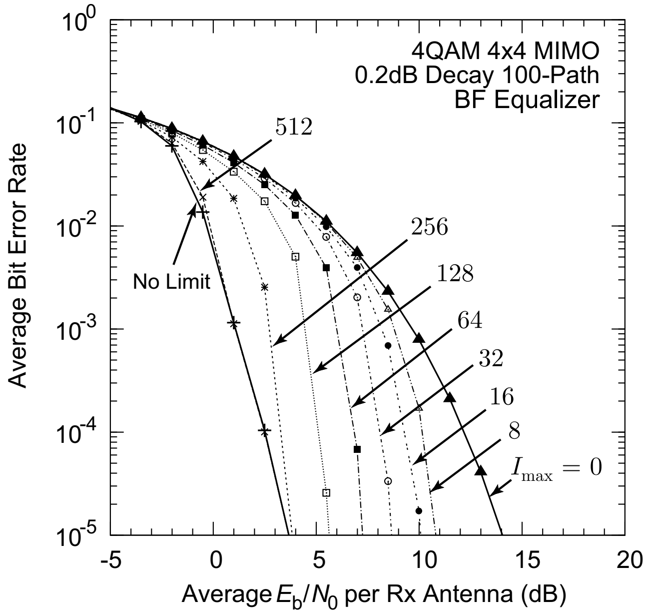

Fig. 4. Average BER performance versus average E_b/N_0 for several parameters $I_{\max}$. (4QAM 4×4 MIMO system, 100-path Rayleigh, 0.2dB decaying per symbol delay.)

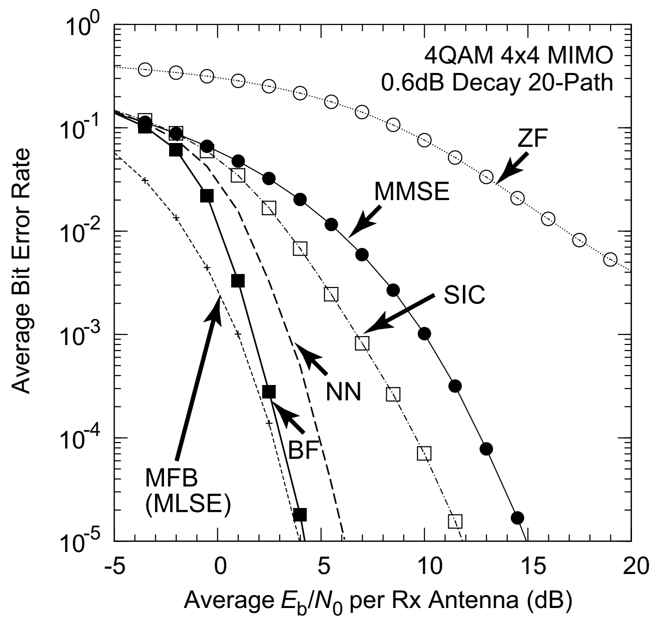

Fig. 5. Average BER performance versus average E_b/N_0. (4QAM 4×4 MIMO system, 20-path Rayleigh, 0.6dB decaying per symbol delay.)

as discussed in our previous works, the bit-flipping refinement followed by the FD-MMSE equalizer can achieve near-ML performance in wideband MIMO channels.

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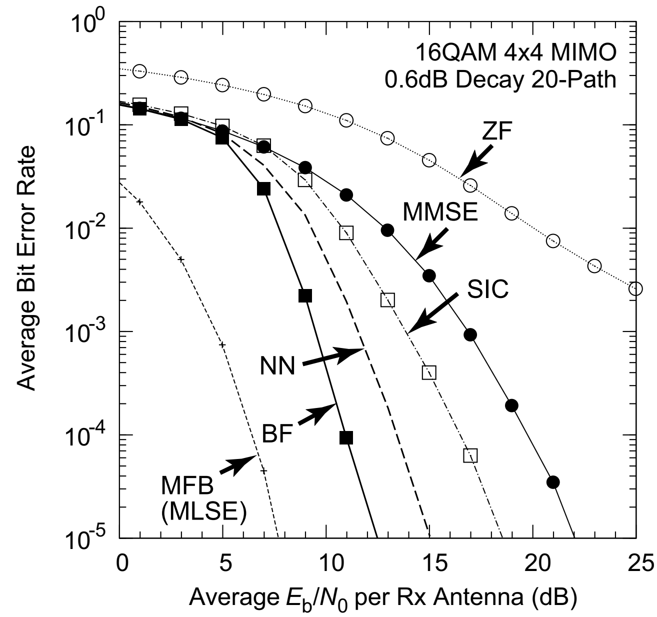

Fig. 6. Average BER performance versus average E_b/N_0. (16QAM 4×4 MIMO system, 20-path Rayleigh, 0.6dB decaying per symbol delay.)

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