Fundamental Limits of Cognitive Networks

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Efficient, reliable communication
Efficient, reliable communication

M Tx antennas

N Rx antennas
Efficient, reliable communication
Efficient, reliable communication
Efficient, reliable communications
Radio

Software-defined Radio = SDR

Cognitive Radio = CR
Radio

Software-defined Radio = SDR

Cognitive Radio = CR
Source → Modulator → Channel → Demodulator → Destination

Software-defined Radio = SDR

Cognitive Radio = CR
Radio

Software-defined Radio = SDR

Cognitive Radio = CR

Source → Modulator → Channel → Demodulator → Destination
Radio

Software-defined Radio = SDR

Cognitive Radio = CR
Example: GNU Radio+USRP
Efficient, reliable communications

With cognition

M Tx antennas

N Rx antennas

M Tx antennas

N Rx antennas
Channel capacity

Channel: \( p(y|x) \)
Channel capacity

Channel: $p(y|x)$

Capacity

$$C = \max_{p(x)} I(X; Y) \text{ bits/channel use}$$
Capacity

\[ C = \max_{p(x)} I(X; Y) \text{ bits/channel use} \]

Channel capacity

\[ I(X; Y) = \sum_{x,y} p(x, y) \log \left( \frac{p(x, y)}{p(x)p(y)} \right) \]
Channel capacity

Channel: \( p(y|x) \)

Capacity

\[
C = \max_{p(x)} I(X; Y) \quad \text{bits/channel use}
\]

Highest rate (bits/channel use) that can communicate at reliably
Mathematical description of capacity

- Can achieve reliable communication for all transmission rates $R$: $R < C$
Mathematical description of capacity

- Can achieve reliable communication for all transmission rates $R$:

\[ R < C \]
Mathematical description of capacity

- Can achieve reliable communication for all transmission rates $R$: $R < C$

- BUT, probability of decoding error always bounded away from zero if $R > C$
Mathematical description of capacity

• Can achieve reliable communication for all transmission rates $R$:

$$R < C$$

• BUT, probability of decoding error always bounded away from zero if

$$R > C$$
AWGN channel capacity

Wireless channel with fading

\[ Y = hX + N \]

Gaussian noise \( \sim N(0, P_N) \)
AWGN channel capacity

Wireless channel with fading

Gaussian noise $\sim N(0, P_N)$

$$C = \frac{1}{2} \log \left( \frac{|h|^2 P + P_N}{P_N} \right)$$

$$= \frac{1}{2} \log (1 + SNR) \text{ (bits/channel use)}$$
Capacity and capacity regions

- Point to point capacity

\[ X_1 \xrightarrow{R} Y_1 \]
Capacity and capacity regions

- **Point to point capacity**

- **Multi-user capacity region**
Capacity and capacity regions

- Point to point capacity

- Multi-user capacity region
Capacity and capacity regions

- Point to point capacity

- Multi-user capacity region
Capacity regions

Outer bound
Capacity region
Achievable region
Fundamental Limits of Cognitive Networks

Motivation?
Motivation 1: smart cognitive devices
Motivation 2: spectral efficiency
Spectrum licensing: future

Primary users/ primary license holders
Spectrum licensing: future

Primary users/ primary license holders

Secondary users
Spectrum licensing: future

Primary users/ primary license holders

Secondary users ↔ Cognitive radios
Secondary spectrum usage

Transmitter 1 -> Receiver 1

Primary link
Secondary spectrum usage

Transmitter 1 → Receiver 1

Primary link

Transmitter 2 → Receiver 2

Secondary link (cognitive)
Secondary spectrum usage

Transmitter 1  
\(X_1\)  
\(\rightarrow\)  
\(Y_1\)  
Receiver 1  
\(\text{Primary link}\)

Transmitter 2  
\(X_2\)  
\(\rightarrow\)  
\(Y_2\)  
Receiver 2  
\(\text{Secondary link (cognitive)}\)
Secondary spectrum usage

What can the cognitive link do?
Cognition

- Assumptions on primary/secondary models will dictate behavior + performance
Cognition

- Assumptions on primary/secondary models will dictate behavior + performance
- Cognition boils down to **side-information** and how to use it
Cognition

- Assumptions on primary/secondary models will dictate behavior + performance

- Cognition boils down to **side-information** and how to use it

- Use information theory to tell us which techniques are most promising
1. White spaces

![Diagram showing time and frequency with 'Primary' blocks]

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1. White spaces
$I(X_2; Y_2)$

Side-info needed?

$I(X_1; Y_1)$
2. Just transmit

Interfere with each other!
2. Just transmit

\[ R_1 \leq \frac{1}{2} \log_2 \left( 1 + \frac{\text{Power of signal 1}}{\text{Interference from signal 2 + Noise}} \right) \]

\[ R_2 \leq \frac{1}{2} \log_2 \left( 1 + \frac{\text{Power of signal 2}}{\text{Interference from signal 1 + Noise}} \right) \]
``Interference temperature''

Maximum interference

Current interference

\[ [0, P_2] \]

P_1

X_1 \rightarrow Y_1

X_2 \rightarrow Y_2
``Interference temperature''

Maximum interference
Cognitive Tx can add!
Current interference
Side-info needed?

"Interference temperature"

Maximum interference
Cognitive Tx can add!
Current interference

$[0, P_2]$
3. Opportunistic “cognitive” decoding
3. Opportunistic “cognitive” decoding
3. Opportunistic “cognitive” decoding
3. Opportunistic “cognitive” decoding

Side-info needed?
4. Simultaneous **Cognitive** Transmission

Assumption: Tx 2 knows message encoded by X₁ a-priori
4. Simultaneous Cognitive Transmission

Cognitive Tx may obtain primary’s message in a fraction of the time
4. Simultaneous Cognitive Transmission

Cognitive Tx may overhear primary’s message
"Competitive"

Interference channel
“Cooperative”

2 Tx antenna
Broadcast channel
“Cognitive”

Cognitive channel
What rates \((R_1, R_2)\) are achievable?
Information theoretic abstraction
Interference channel

\[ W_1 \quad \xrightarrow{Tx~1} \quad X_1(W_1) \quad \xrightarrow{p_{Y_1,Y_2|X_1,X_2}} \quad Y_1 \quad \xrightarrow{Rx~1} \quad \hat{W}_1 \]

\[ W_2 \quad \xrightarrow{Tx~2} \quad X_2(W_2) \quad \xrightarrow{p_{Y_1,Y_2|X_1,X_2}} \quad Y_2 \quad \xrightarrow{Rx~2} \quad \hat{W}_2 \]
DM Cognitive interference channel

\[ X_1(W_1, W_2) \quad p_{Y_1,Y_2|X_1,X_2} \quad Y_1 \quad \hat{W}_1 \]

\[ X_2(W_2) \quad Y_2 \quad \hat{W}_2 \]
DM Cognitive interference channel

\[ W_1 \rightarrow \text{Tx 1} \rightarrow X_1(W_1, W_2) \quad \rightarrow \text{Rx 1} \rightarrow \hat{W}_1 \]

\[ W_2 \rightarrow \text{Tx 2} \rightarrow X_2(W_2) \quad \rightarrow \text{Rx 2} \rightarrow \hat{W}_2 \]

\[ p_{Y_1, Y_2|X_1, X_2} \]

Gaussian Cognitive interference channel

\[ W_1 \rightarrow \text{Tx 1} \rightarrow 1 \rightarrow \text{Rx 1} \rightarrow \hat{W}_1 \]

\[ W_2 \rightarrow \text{Tx 2} \rightarrow b \rightarrow \text{Rx 2} \rightarrow \hat{W}_2 \]

\[ a \rightarrow + \rightarrow N_1 \rightarrow + \rightarrow \text{Rx 1} \]

\[ 1 \rightarrow + \rightarrow N_2 \rightarrow + \rightarrow \text{Rx 2} \]

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Introduction

Introduction


Capacity in very weak interference

Introduction


Capacity in very weak interference


Capacity in very strong interference

Introduction


Capacity in very weak interference


Capacity in very strong interference


Large unified region

Introduction


Capacity in very weak interference


Capacity in very strong interference


Large unified region


Broadcast channel is contained

Introduction


Capacity in very weak interference


Capacity in very strong interference


Large unified region


Broadcast channel is contained


Interference channel with cognitive relay

Causal cognitive interference channel

Causal cognitive interference channel

Semi-deterministic cognitive interference channel
Causal cognitive interference channel


Semi-deterministic cognitive interference channel


Cognitive interference channels with secrecy


Causal cognitive interference channel


Semi-deterministic cognitive interference channel


Cognitive interference channels with secrecy


Cognitive Z interference channel

Causal cognitive interference channel


Semi-deterministic cognitive interference channel


Cognitive interference channels with secrecy


Cognitive Z interference channel


Degrees of Freedom of Cognitive Channels

Causal cognitive interference channel

Semi-deterministic cognitive interference channel

Cognitive interference channels with secrecy

Cognitive Z interference channel

Degrees of Freedom of Cognitive Channels

Wyner-type cognitive networks
Causal cognitive interference channel

Semi-deterministic cognitive interference channel

Cognitive interference channels with secrecy

Cognitive Z interference channel

Degrees of Freedom of Cognitive Channels

Wyner-type cognitive networks
A. Lapidoth, S. Shamai (Shitz), and M. A. Wigger, “A Linear Interference Network with Local Side-Information”, in Proc. ISIT 2007, Nice, France, June 24-29, 2007.

Interference channel with cognitive relay
Cognition

Non-causal side information at Tx/Rxs
Introduction


Contributions

Capacity in very weak interference


Capacity in very strong interference


Large unified region


Broadcast channel is contained


Special case of broadcast channel with cognitive radios


Our recent result [Rini, Tuninetti, Devroye IZS 2010] = largest known achievable rate region
Contributions


Capacity in very weak interference


Capacity in very strong interference


Large unified region


Broadcast channel is contained


Special case of broadcast channel with cognitive radios


In Gaussian noise, achieves within 1.87 bits from outer bound derived in
Introduction


Contributions

Capacity in very weak interference


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Broadcast channel is contained


Special case of broadcast channel with cognitive radios


Capacity in certain (new) cases and channels - high SNR deterministic channel
Achievable scheme

- rate-splitting
- superposition coding
- Gel’fand-Pinkser binning
Rate splitting

\[
R_1 = R_{1c} + R_{1pb},
\]

\[
R_2 = R_{2c} + R_{2pa} + R_{2pb}.
\]
Rate splitting

\[ R_1 = R_{1c} + R_{1pb}, \]
\[ R_2 = R_{2c} + R_{2pa} + R_{2pb}. \]

\( c = \text{common}, p = \text{private}, \)
\( a = \text{alone}, b = \text{broadcast} \)
Superposition coding
c = common, p = private,
a = alone, b = broadcast

Superposition coding
Superposition coding

$c = \text{common}, p = \text{private}, 
a = \text{alone}, b = \text{broadcast}$
c = common, p = private, 
a = alone, b = broadcast

Superposition coding

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Superposition coding
Superposition coding

\( c = \text{common}, p = \text{private}, \)
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Superposition coding
Superposition coding

c = common, p = private,
a = alone, b = broadcast

$p_{u_{2pb}}|u_{1c},u_{2c},u_{2pa},x_2$

$p_{u_{2pa}}|u_{2c}$

$p_{u_{2c}}$

$p_{x_2}|u_{2pa},u_{2c}$

$p_{x_1}|u_{2pb},u_{2pa},u_{1pb},u_{1c},u_{2c},x_2$

$p_{u_{1pb}}|u_{1c},u_{2c}$

$p_{u_{1c}}|u_{2c}$
$c = \text{common}, \ p = \text{private},$

$a = \text{alone}, \ b = \text{broadcast}$

**Gel’fand-Pinsker binning**
Gel’fand-Pinsker binning
\[ C_{1pb} = \max_{p_{u_{1pb}, x_1 | u_{2pa}}} I(U_{1pb}; Y_1) - I(U_{1pb}; U_{2pa}) \]

**Gel’fand-Pinsker binning**
In Gaussian noise with "proper" choice of auxiliary RV
Gel’fand-Pinsker binning = Dirty Paper Coding in Gaussian noise!

In Gaussian noise with "proper" choice of auxiliary RV

Gel’fand-Pinsker binning = Dirty Paper Coding in Gaussian noise!
Dirty-paper coding

[Gel’fand, Pinsker, 1980]
[Costa, 1983]
Dirty-paper coding
Dirty-paper coding
Dirty-paper coding

write in black ink?
Dirty-paper coding

adjust your ink ✓
Example of dirty-paper coding

Send 2 bits:

- 11
- 10
- 01
- 00
Example of dirty-paper coding

Send 2 bits:

Power limited

- 11
- 10
- 01
- 00
Example of dirty-paper coding

Interference
Example of dirty-paper coding

Do NOT have enough power to subtract off the interference!
Example of dirty-paper coding

How to send 01?
Example of dirty-paper coding

How to send 01?
Example of dirty-paper coding

Interference

How to send 01?
Example of dirty-paper coding

How to send 11?
Example of dirty-paper coding

How to send 11?
Example of dirty-paper coding

How to send 11?

Interference
Dirty-paper coding

\[ X_1 \rightarrow Y_1 \] \[ X_2 \rightarrow Y_2 \]

NO power penalty!
NOT subtracting off interference!
\( W_2 \) \( U_{2c} \) \( U_{2pa} \) \( U_{2pb} \) \( W_1 \) \( U_{1c} \) \( U_{1pb} \)

\( c = \text{common}, \ p = \text{private}, \ a = \text{alone}, \ b = \text{broadcast} \)

Gel’fand-Pinsker binning

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The dotted lines indicate binning. We see rate splits are use the fact that in certain regimes, this strategy was shown to b...
Analytically shown to be largest known region

\[
\begin{align*}
R'_0 & \geq I(U_{1c}; U_{2pa}, X_2|U_{2c}) \\
R'_0 + R'_1 + R'_2 & \geq I(U_{1c}; U_{2pa}, X_2|U_{2c}) + I(U_{1pb}; U_{2pa}, U_{2pb}, X_2|U_{2c}, U_{1c}) \\
& \quad + I(U_{2pb}; X_2|U_{2c}, U_{2pa}, U_{1c}) \\
R_{2c} + R_{1c} + R_{2pa} + R_{2pb} + R'_0 + R'_2 & \leq I(Y_2; U_{1c}, U_{2c}, U_{2pa}, U_{2pb}) + I(U_{1c}; U_{2pa}|U_{2c}) \\
R_{1c} + R_{2pa} + R_{2pb} + R'_0 + R'_2 & \leq I(Y_2; U_{1c}, U_{2pa}, U_{2pb}|U_{2c}, U_{1c}) + I(U_{1c}; U_{2pa}|U_{2c}) \\
R_{2pa} + R_{2pb} + R'_2 & \leq I(Y_2; U_{2pa}, U_{2pb}|U_{2c}, U_{1c}) + I(U_{1c}; U_{2pa}|U_{2c}) \\
R_{1c} + R_{2pb} + R'_0 + R'_2 & \leq I(Y_2; U_{1c}, U_{2pb}|U_{2c}, U_{2pa}) + I(U_{1c}; U_{2pa}|U_{2c}) \\
R_{2pb} + R'_2 & \leq I(Y_2; U_{2pb}|U_{2c}, U_{1c}, U_{2pa}) + I(U_{2pa}; U_{1c}|U_{2c}) \\
R_{2c} + R_{1c} + R_{1pb} + R'_0 + R'_1 & \leq I(Y_1; U_{2c}, U_{1c}, U_{1pb}) \\
R_{1c} + R_{1pb} + R'_0 + R'_1 & \leq I(Y_1; U_{1c}, U_{1pb}|U_{2c}) \\
R_{1pb} + R'_1 & \leq I(Y_1; U_{1pb}|U_{2c}, U_{1c})
\end{align*}
\]

over all input distributions of the form \( pX_1, X_2, U_{1c}, U_{2c}, U_{2pa}, U_{1pb}, U_{2pb} \)
Analytically shown to be largest known region

\[
R'_{0} \geq I(U_{1c}; U_{2pa}, X_{2}|U_{2c}) \tag{3a}
\]
\[
R'_{0} + R'_{1} + R'_{2} \geq I(U_{1c}; U_{2pa}, X_{2}|U_{2c}) + I(U_{1pb}; U_{2pa}, U_{2pb}, X_{2}|U_{2c}, U_{1c}) + I(U_{2pb}; X_{2}|U_{2c}, U_{2pa}, U_{1c}) \tag{3b}
\]
\[
R_{2c} + R_{1c} + R_{2pa} + R_{2pb} + R'_{0} + R'_{2} \leq I(Y_{2}; U_{1c}, U_{2c}, U_{2pa}, U_{2pb}) + I(U_{1c}; U_{2pa}|U_{2c}) \tag{3c}
\]
\[
R_{1c} + R_{2pa} + R_{2pb} + R'_{0} + R'_{2} \leq I(Y_{2}; U_{1c}, U_{2pa}, U_{2pb}|U_{2c}) + I(U_{1c}; U_{2pa}|U_{2c}) \tag{3d}
\]
\[
R_{2pa} + R_{2pb} + R'_{2} \leq I(Y_{2}; U_{2pa}, U_{2pb}|U_{2c}, U_{1c}) + I(U_{1c}; U_{2pa}|U_{2c}) \tag{3e}
\]
\[
R_{1c} + R_{2pb} + R'_{0} + R'_{2} \leq I(Y_{2}; U_{1c}, U_{2pb}|U_{2c}, U_{2pa}) + I(U_{1c}; U_{2pa}|U_{2c}) \tag{3f}
\]
\[
R_{2pb} + R'_{2} \leq I(Y_{2}; U_{2pb}|U_{2c}, U_{1c}, U_{2pa}) + I(U_{2pa}; U_{1c}|U_{2c}) \tag{3g}
\]
\[
R_{2c} + R_{1c} + R_{1pb} + R'_{0} + R'_{1} \leq I(Y_{1}; U_{2c}, U_{1c}, U_{1pb}) \tag{3h}
\]
\[
R_{1c} + R_{1pb} + R'_{0} + R'_{1} \leq I(Y_{1}; U_{1c}, U_{1pb}|U_{2c}) \tag{3i}
\]
\[
R_{1pb} + R'_{1} \leq I(Y_{1}; U_{1pb}|U_{2c}, U_{1c}) \tag{3j}
\]

over all input distributions of the form \( pX_{1}, X_{2}, U_{1c}, U_{2c}, U_{2pa}, U_{1pb}, U_{2pb} \)

Reduces to capacity whenever capacity is known.

\[c = \text{common, } p = \text{private,} \]
\[a = \text{alone, } b = \text{broadcast}\]
Reduces to capacity whenever capacity is known

- Discrete memoryless channels - weak interference, very strong interference, **new classes of weak and strong interference**
Reduces to capacity whenever capacity is known

- Discrete memoryless channels - weak interference, very strong interference, **new classes of weak and strong interference**

- Deterministic channels - classes of semi-deterministic channels and **deterministic channels**
Reduces to capacity whenever capacity is known

- Discrete memoryless channels - weak interference, very strong interference, new classes of weak and strong interference

- Deterministic channels - classes of semi-deterministic channels and deterministic channels

- High-SNR deterministic approximation - capacity
Reduces to capacity whenever capacity is known

- Discrete memoryless channels - weak interference, very strong interference, **new classes of weak and strong interference**

- Deterministic channels - classes of semi-deterministic channels and **deterministic channels**

- High-SNR deterministic approximation - capacity

- Gaussian channels - weak interference, very strong interference, **constant gap in all other regimes**
Reduces to capacity whenever capacity is known

- Discrete memoryless channels - weak interference, very strong interference, **new classes of weak and strong interference**

- Deterministic channels - classes of semi-deterministic channels and **deterministic channels**

- High-SNR deterministic approximation - capacity

- Gaussian channels - weak interference, very strong interference, **constant gap in all other regimes**

Monday, February 22, 2010
High-SNR linear deterministic cognitive interference channel

To show that the last sum rate is loose we write

\[ R_1 + R_2 \leq n_{22} + n_{11} \]

We now show how the point can be achieved in 11.

XII. CONCLUSION

Say translation to Gaussian noise is next step, along with full capacity region (if not obtained) for the general case with interfering links. Ideas about what the teaser may correspond to in Gaussian noise may be nice to put in.

REFERENCES


High-SNR linear deterministic cognitive interference channel

\[ R_1 \leq n_{11} \]
\[ R_2 \leq n_{22} \]
\[ R_1 + R_2 \leq n_{22} + n_{11} \]
\[ R_1 + R_2 \leq \max\{n_{11} - n_{21}, n_{1c}\} + \max\{n_{22} - n_{12}, n_{2c}\} + n_{12} + n_{21} \]

To show that the last sum rate is loose we write
\[ \max\{n_{11} - n_{21}, n_{1c}\} + \max\{n_{22} - n_{12}, n_{2c}\} + n_{12} + n_{21} \geq n_{11} + n_{22} \]

We now show how the point can be achieved in 11.

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Say translation to Gaussian noise is next step, along with full capacity region (if not obtained) for the general case with interfering links. Ideas about what the teaser may correspond to in Gaussian noise may be nice to put in.

REFERENCES


High-SNR deterministic models


Constant gap results using deterministic intuition


High-SNR linear deterministic cognitive interference channel

\[ \text{To show that the last sum rate is loose we write} \]
\[ \max\{n_{11} - n_{21} + n_{12} + n_{22}, n_{1c} + n_{2c}\} + \max\{n_{22} - n_{12} + n_{21} + n_{12}, n_{2c}\} + n_{12} + n_{21} \geq n_{11} + n_{22} \]

Image 11.

XII. CONCLUSION


REFERENCES


High-SNR linear deterministic cognitive interference channel

To show that the last sum rate is loose we write
\[
\max\{n_{11} - n_{21}, n_{1c}\} + \max\{n_{22} - n_{12}, n_{2c}\} + n_{12} + n_{21} \geq n_{11} + n_{22}
\]

We now show how the point can be achieved in 11.

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Say translation to Gaussian noise is next step, along with full capacity region (if not obtained) for the general case with interfering links. Ideas about what the teaser may correspond to in Gaussian noise may be nice to put in.

REFERENCES


High-SNR linear deterministic
cognitive interference channel

\[ R_1 \leq n_{11} \]
\[ R_2 \leq \max\{n_{21}, n_{22}\} \]
\[ R_1 + R_2 \leq \max\{n_{21}, n_{22}\} + [n_{11} - n_{21}]^+ \]

We have capacity!

High-SNR linear deterministic cognitive interference channel

\[
R_1 \leq n_{11} \quad R_2 \leq n_{22} \quad R_1 + R_2 \leq n_{22} + n_{11} \quad R_1 + R_2 \leq \max\{n_{11} - n_{21}, n_{1c}\} + \max\{n_{22} - n_{12}, n_{2c}\} + n_{12} + n_{21}
\]

To show that the last sum rate is loose we write

\[
\max\{n_{11} - n_{21}, n_{1c}\} + \max\{n_{22} - n_{12}, n_{2c}\} + n_{12} + n_{21} \geq n_{11} + n_{22}
\]

We now show how the point can be achieved in 11.

XII. CONCLUSION

Say translation to Gaussian noise is next step, along with full capacity region (if not obtained) for the general case with interfering links. Ideas about what the teaser may correspond to in Gaussian noise may be nice to put in.

REFERENCES


High-SNR deterministic C-IFC

Gaussian C-IFC

Intuition
Known Gaussian results

\[ \begin{align*}
|a| & \quad \text{weak interference} \\
|b| & \quad \text{very strong interference}
\end{align*} \]

<table>
<thead>
<tr>
<th>a</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td></td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\text{very strong interference} & = \text{very strong interference} \\
\text{weak interference} & = \text{weak interference}
\end{align*} \]
Known Gaussian results

We prove a finite gap

We obtain Equation 4.1c after substitution in Equation 4.2 of the optimal value for $a$ by the definition of $S_i$. Since the sum rate bound is valid for any $R_{out}$, the bound Equation 4.1b is redundant. Furthermore, the outer bound in Equation 4.1 is known to be achievable in certain cases. Notice that in strong interference, receiver 2 can decode both messages without imposing any rate penalty on the rate for user 1. We conclude that

$|a| \leq \alpha$,

very strong interference

weak interference

$|b| > 1$ and

We prove a finite gap
Proving a finite gap

MISO strategy

Outer bound

Straight line approximation of outer bound

Some strategy

Bound this maximum distance
Proving a finite gap

<table>
<thead>
<tr>
<th></th>
<th>weak int.</th>
<th>strong int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>strong sign.</td>
<td>1, \sqrt{\frac{P_2}{P_1}}</td>
<td>\min {1, \sqrt{\frac{P_2}{P_1}}}</td>
</tr>
<tr>
<td>weak sign.</td>
<td>0</td>
<td>\max {1, \sqrt{\frac{P_2}{P_1}}}</td>
</tr>
</tbody>
</table>

\[ |a| \]

\[ \begin{align*}
|ab| &= 1 \\
\text{superposition: } c + c \\
\text{broadcast: } c + p \\
\text{DPC: } c + p
\end{align*} \]
What rates \((R_1, R_2)\) are achievable?
Extensions of "cognition" in multi-user IT

- causal versus non-causal cognition
Extensions of "cognition" in multi-user IT

- causal versus non-causal cognition

- cognitive relay: interference, relay channels
Extensions of "cognition" in multi-user IT

- causal versus non-causal cognition

- cognitive relay: interference, relay channels

- more cognitive users, more scenarios....
Degrees of freedom: classical

\[ DOF = \# \text{"clean" channels in a multi-stream network} \]

\[ M \text{ Tx antennas} \quad N \text{ Rx antennas} \]

DOF = \min(M,N) = 2

\[ \text{MIMO} \]

\[ \text{Interference channel} \]

DOF = 1

Monday, February 22, 2010
Degrees of freedom: classical

\[ \text{DOF} = \# \text{“clean” channels in a multi-stream network} \]

**MIMO**

\[ \text{DOF} = \min(M,N)=2 \]

**Broadcast channel**

\[ \text{DOF} = 2 \]

**Multiple-access channel**

\[ \text{DOF} = 2 \]

**Interference channel**

\[ \text{DOF} = 1 \]
Degrees of freedom: cognitive, M antennas

MIMO interference channel

DOF = M

Degrees of freedom: cognitive, M antennas

DOF = M

MIMO interference channel

MIMO cognitive channel, cases a,b,c

DOF = M

Degrees of freedom: cognitive, M antennas

MIMO interference channel

DOF = M

MIMO cognitive channel, cases a,b,c

DOF = M

MIMO cognitive channel, cases d,e,f

DOF = 2M

Scaling laws

# nodes n → ∞
Scaling laws  \# nodes n → ∞

- [Gupta+Kumar 2000]: Non-cooperative ad hoc networks
  - per-node throughput \sim O(1/√n \log(n))
  - Degradation is due to multi-hop and interference between nodes
Scaling laws \( \# \text{ nodes } n \rightarrow \infty \)

- [Gupta+Kumar 2000]: Non-cooperative ad hoc networks
  - per-node throughput \( \sim O(1/\sqrt{n \log(n)}) \)
  - Degradation is due to multi-hop and interference between nodes

- [Franceschetti et al. 2000]: ad hoc networks
  - per-node throughput \( \sim O(1/\sqrt{n}) \)
  - percolation theory
Scaling laws  \( n \to \infty \)

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- [Ozgur, Leveque, Tse 2007]: Cooperative ad hoc networks
  - Nodes may cooperate as in a MIMO system
  - Per-node throughput \( \sim O(1) \) (constant)
Scaling laws \# nodes $n \rightarrow \infty$

- [Gupta+Kumar 2000]: Non-cooperative ad hoc networks
  - per-node throughput $\sim O(1/\sqrt{n \log(n)})$
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- [Franseschetti et al. 2000]: ad hoc networks
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- [Ozgur, Leveque, Tse 2007]: Cooperative ad hoc networks
  - nodes may cooperate as in a MIMO system
  - per-node throughput $\sim O(1)$ (constant)

- Many many more...
Scaling laws: with cognition

• What we guarantee:

Primary nodes act as if cognitive network does not exist
Primary nodes achieve same scaling law as if cognitive network does not exist
Scaling laws: with cognition

- What we guarantee:

  *Primary nodes act as if cognitive network does not exist*

  *Primary nodes achieve same scaling law as if cognitive network does not exist*

- What we prove:

  \[ T_p(n) = \Theta \left( \sqrt{\frac{1}{n \log n}} \right), \quad T_s(m) = \Theta \left( \sqrt{\frac{1}{m \log m}} \right) \]
Efficient, reliable communications

With cognition

M Tx antennas
N Rx antennas

M Tx antennas
N Rx antennas
Efficient, reliable communications

With cognition

M Tx antennas  N Rx antennas
Thank you

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