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Fundamental Limits of Cognitive Radio Networks

Survey of recent results on information theoretic limits

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Summary. Cognitive radios have the potential to greatly improve spectral efficiency in wireless networks. In this chapter we explore the fundamental limits of communication in channels employing cognitive radios. We take an information-theoretic approach, making use of information theory's wide variety of tools and ability to characterize communication limits independent of actual implementations. The chapter first surveys information theoretic results on a simple wireless channel where a primary link and a secondary (or cognitive) link share the same spectrum: the cognitive radio channel. Our survey includes recent capacity and achievable rate region calculations for the case when the channel is known to all transmitters and receivers. We compare the rates achieved through new non-orthogonal schemes where the cognitive and primary user simultaneously use the channel to more traditional spectral gap filling solutions. In the second part of the chapter we outline new results on the limits of cognitive radio channels where the fading coefficients are known to different degrees at the nodes.

1.1 Introduction

The advent of cognitive radios, or software defined radios able to adapt to the sensed environment, along with recent FCC initiatives allowing for secondary spectrum access promise more flexible, and potentially more efficient spectrum access. There are many questions and aspects to be tackled before before cognitive radios can seamlessly and opportunistically employ spectrum licensed to primary user(s). Of both theoretical and practical importance is the question: what are the fundamental limits of communication in the presence of one or more cognitive radios? Information theory provides an ideal framework for analyzing this question. The capacity and rate regions achieved in a network with cognitive radios provide fundamental, unquestionable limits of the possible communication. Such results provide benchmarks for the communication field, where researchers may gauge the efficiency of any practical cognitive radio system. In this chapter, we outline some of the recent theoretical advances pertaining to the limits of cognitive radio systems, first assuming all channel gains are known to all involved nodes, and then extending to the case when fading coefficients are known to different extents at the involved wireless devices. In both cases we emphasize how the cognitive radios alter classical information theoretic communication scenarios, and what is gained by their introduction.

The FCC's recent Secondary Markets Initiative (SMI, [17]) was sparked by empirical measurements showing that most of the time certain licensed frequency bands remain unused, and the natural desire to remedy this and increase spectral efficiency. Currently, spectrum is either unlicensed, creating a spectral free-for-all (as, for example in the 2.4 GHz band), or is licensed to certain primary users (such as for example cellular providers). The goal of the SMI is to remove unnecessary regulatory barriers to new secondary market oriented policies. Of the multiple possible types of secondary leasing [16], in this chapter we will focus on *dynamic spectrum leasing*. There, licensed users (which we will use interchangeably with the term primary users) ultimately hold the right to the spectrum. However, the primary license holder may wish to re-distribute or share his spectrum with other devices not necessarily in his own network. The motivation, and fascinating game theoretical and economic models for doing so is beyond the scope of this chapter, and we refer the interested reader to [22,39,42] In *dynamic spectrum leasing*, the non-licensed secondary devices would opportunistically (dynamically) employ the spectrum according to the primary licensee's regulations. Three main types of opportunistic employment of the primary spectrum are possible (although by no means exhaustive).

1. **Interference-controlled:** the primary license holder could stipulate for example maximal permissible secondary user interference levels, in effect guaranteeing the primary users certain transmission rates. This could allow primary and secondary users to transmit in the same bands, that is, in the same time-frequency-space-code blocks. The concept of *interference temperature* [16] has been introduced with goal of avoiding the compromise of the primary users' spectrum: secondary devices should control their emissions such that the aggregate interference at the primary users is below a certain level (or interference temperature).
2. **Interference-avoiding:** a subset of the interference-controlled regime is that in which the primary licensee only allows secondary users to use its spectrum on the condition that its user suffer no interference whatsoever. Such systems are much more restrictive than *interference controlled* systems, but are common in cognitive radio literature [22, 25, 43, 49]. The secondary user could adhere to this strict requirement by filling in spectral holes. That is, a secondary user would transmit only in the absence of primary users.
3. **Interference-free:** when cognitive devices exist in a network but have no information of their own to transmit, they could potentially act as relays, and *collaborate* with the primary users [38]. Rather than cause interference

to the primary link, they boost it. Neglecting any other possibly active cognitive clusters [15], this system is interference-free.

In this work, in order to obtain fundamental limits of communication in the presence of cognitive radios, we turn our attention mostly to the more general *interference-controlled* model.

For both the *interference-controlled* as well as the *interference-avoiding* models, the secondary user must be able to perform some fundamental tasks. Specifically, we require a device which is able to sense the communication opportunities, and then take actions based on the sensed information. In this chapter, such actions will include transmitting (or refraining from transmitting) and adapting their modulation and/or coding strategies so as to "better" employ the sensed spectral environment. For the most part, "better" will mean larger rates. Cognitive radios, a term coined by Mitola [36] are special types of software-defined radios (SDR), and are natural candidates for secondary users. SDRs are wireless communication devices with the ability to transmit and receive using a variety of protocols and modulation schemes (enabled by reconfigurable software rather than hardware). Cognitive radios are SDRs which can furthermore become "cognitive", and, as dictated by the software, adapt their behavior to the wireless surroundings without user intervention. Such radios can make decisions based on the availability of nearby collaborative nodes, or on the regulations dictated by their current location and/or spectral conditions. In this chapter, secondary users will be assumed to have abilities similar to cognitive radios. We use the terms secondary user and cognitive user interchangeably.

1.1.1 Chapter outline

The theoretical limits of cognitive radio systems are still being explored. As such, most models considered thus far are fairly simplified. In this chapter, we focus on wireless networks in which only a single primary user-primary receiver pair is present. First, in Section 1.2 we will explore achievable rate and capacity regions in which a single cognitive, or secondary, user is present. We define the cognitive radio channel (also known in the information theory community as the interference channel with degraded message sets) and review results on the achievable and capacity regions for both the discrete channel and Gaussian noise channel. For the first part, the channel parameters are assumed to be known to all involved nodes. In the second part of this chapter, we survey recent results on information theoretic limits of cognitive channels in which the fading coefficients are known to different degrees at the four nodes.

1.2 Fundamental Limits of Cognitive Radio Channels: Perfect CSI

Consider first the simplest scenario in which a primary user and a secondary user co-exist in the same wireless channel, as shown in Fig.. 1.1. The primary (sender, receiver) pair is denoted by ($\mathcal{S}_1$, $\mathcal{R}_1$), while the cognitive (sender, receiver) pair is denoted by ($\mathcal{S}_2, \mathcal{R}_2$). Each sender wishes to transmit its own independent information to its receiver. We wish to determine the fundamental limits of the communication possible, as a function of both the wireless channel, and the transmitter and receiver capabilities (such as power constraints, and the cognitive nature of $\mathcal{S}_2$). We assume perfect channel state information (CSI), that is, all parameters characterizing the channel are assumed to be known perfectly to the senders and receivers. Before turning to definitions of variables and rates in this channel, we look qualitatively at different possible communication scenarios in this simple channel.

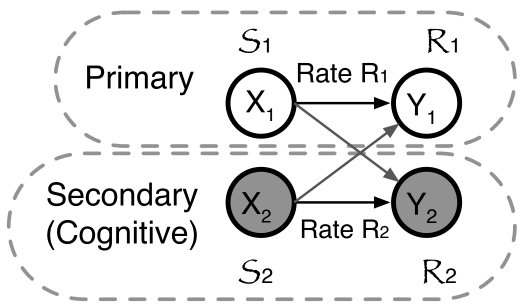

Fig. 1.1. A simple channel in which the primary transmitter $\mathcal{S}_1$ wishes to transmit a message to the primary receiver $\mathcal{R}_1$ and the secondary (or cognitive) transmitter $\mathcal{S}_2$ wishes to transmit a message to its receiver $\mathcal{R}_2$. We explore the rates R_1 and R_2 that are achievable in this channel.

The amount of data that can be transmitted from the senders $\mathcal{S}_1, \mathcal{S}_2$ to the receivers $\mathcal{R}_1, \mathcal{R}_2$ naturally depends on the amount of cooperation possible between transmitters and receivers. In this chapter, we consider different possibilities for transmitter cooperation[1], and so three categories of transmitter cooperation may be as shown in Fig. 1.2, and can be summarized as:

1. **Competitive behavior:** The two transmitters transmit independent messages. There is no cooperation in sending the messages, and thus the two users *compete* for the channel. This is the same channel as the 2

[1] Transmitter cooperation seems to be of greater theoretical interest [26] than receiver cooperation. The case for cognitive receivers is also interesting and could form a nice topic for future research.

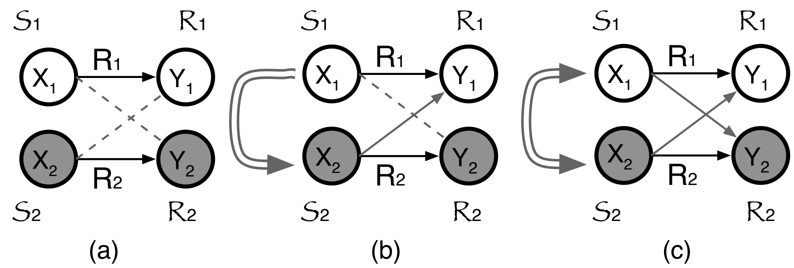

Fig. 1.2. (a) Competitive behavior, the interference channel. The transmitters may not cooperate. (b) Cognitive behavior, the cognitive radio channel. Asymmetric transmitter cooperation. (c) Cooperative behavior, the two antenna broadcast channel. The transmitters, but not the receivers, may fully and symmetrically cooperate.

sender, 2 receiver interference channel. This channel, introduced in [1,45] was consequently studied by, among many others, [5,6,21,34,40,41]. Although the capacity region of this channel is known in a few cases, its capacity in its most general setting remains an open problem.

2. **Cognitive behavior:** In this channel, the one way double arrow from $\mathcal{S}_1$ to $\mathcal{S}_2$ indicates asymmetric cooperation between the transmitters. This asymmetric cooperation is a result of $\mathcal{S}_2$ knowing $\mathcal{S}_1$'s message, but not vice-versa. We idealize the concept of message knowledge: whenever $\mathcal{S}_2$ is able to hear and decode the message of $\mathcal{S}_1$, we assume it has full *a-priori*, or *non-causal* knowledge. We use the term cognitive behavior to emphasize the need for $\mathcal{S}_2$ to be a "smart" device capable of altering its transmission strategy according to the message of the primary user. We can motivate considering asymmetric side information in practice in three ways:
   - **Cognitive networks:** depending on the device capabilities, as well as the geometry and channel gains between the various nodes, certain cognitive nodes may be able to hear and/or obtain the messages to be transmitted by other nodes. These messages would need to be obtained in real time, and could exploit the geometric gains between cooperating transmitters relative to receivers in, for example, a 2 phase protocol [14].
   - **Automatic Repeat reQuest (ARQ) system:** a cognitive transmitter, under suitable channel conditions (if it has a better channel to the primary transmitting node than the primary receiver), could decode the primary user's transmitted message during an initial transmission attempt. In the event that the primary receiver was not able to correctly decode the message, and it must be re-transmitted, the cognitive user would already have the to-be-transmitted message, or asymmetric

side information, at no extra cost in terms of overhead in obtaining the message.

- **Heterogeneous sensors:** consider a network of wireless sensors in which a sensor $\mathcal{S}_2$ has a better sensing capability than another sensor $\mathcal{S}_1$ and thus is able to sense two events, while $\mathcal{S}_1$ is only able to sense one. Thus, when they wish to transmit, they must do so under an asymmetric side-information assumption: sensor $\mathcal{S}_2$ has two messages, and the other has just one. This scenario is considered in [48].

3. **Cooperative behavior:** The two transmitters know each others' messages (two way double arrows) and can thus fully and symmetrically cooperate in their transmission. The channel pictured in Fig.1.2 (c) may be thought of as a two antenna sender, two single antenna receivers broadcast channel [46]. The capacity region of the general broadcast channel is still unknown, save for certain cases [8, 9, 18, 32]. Of course, many achievable regions have been developed; the largest to date was computed in [31]. In Gaussian noise, much more can be said about the broadcast channel. The capacity region of a broadcast channel with single antennas coincides with the region of a degraded broadcast channel and is a classical result, outlined in Section 14.6.3 of [9]. In contrast, the capacity region of the Gaussian MIMO broadcast channel was recently found in the most non-trivial result of [46]. The gain of dirty-paper coding over time division multiple access (TDMA, the users time share the channel) when broadcasting information from a single base station to multiple users is explored in [27]. There, the authors find the sum-rate of broadcasting using the optimal dirty-paper coding strategy is at most min(number transmit antennas, number receivers) that of TDMA.

When $\mathcal{S}_2$ is a cognitive radio and $\mathcal{S}_1$ is not, both the competitive and cognitive behaviors seem plausible. Cooperative behavior would require the primary and secondary transmitters to cooperate, a situation which is unlikely in the event that the primary user is non-cognitive in nature. In the context of secondary markets, it is also more reasonable to assume that primary users should be able to continue transmitting in the same way whether secondary users are present or not. This rules out the cooperative behavior. We will however make use of cooperative behavior as an outer bound, and interesting comparison, to what can be hoped to be achieved by channels employing cognitive radios.

The cooperative behavior corresponds to the classical broadcast channel, while the competitive behavior reduces to the classical interference channel. We thus turn our attention to the much less studied behavior which spans and in a sense interpolates between the symmetric cooperative and competitive behaviors. We call this behavior *asymmetric cognitive behavior.* In this section we will consider one example of cognitive behavior: a two sender, two receiver (with two independent messages) interference channel with asymmetric, non-causal message knowledge at one of the transmitters, as shown in Fig.1.2(b).

Certain asymmetric (in transmitter cooperation) channels have been considered in the literature: for example in [44], the capacity region of a multiple access channel with asymmetric cooperation between the two transmitters is computed. The authors in [24] consider a channel which could involve asymmetric transmitter cooperation, and explore the conditions under which the capacity of this channel coincides with the capacity of the channel in which *both messages are decoded at both receivers.* In [12, 13] the authors introduced the *cognitive radio channel*, which captures the most basic form of asymmetric transmitter cooperation for the interference channel. This same model was subsequently studied in [28], where the capacity region for the Gaussian cognitive radio channel under weak interference is found, as well as [48], with an analogous result for the discrete memoryless case.

The fundamental way a cognitive transmitter differs from an interference channel is the presence of partial transmitter side-information. This is, intuitively speaking, where all gains are expected to come from, as it allows the cognitive transmitter to mitigate, or cancel interference from the primary user. As argued before, a cognitive radio, before it encodes and transmits its information, possesses both the primary user's message (we assume it also knows this primary user's encoding of the message) as well as its own message. Thus, if the cognitive radio were to simultaneously transmit with the primary user, it would know what interference (the primary user signal) its receiver would suffer. Note that in order to fully know the interference at the cognitive receiver, the cognitive transmitter also needs to know the channel between $\mathcal{S}_1$ and $\mathcal{R}_2$.[2] All nodes possess the conditional distribution $p(y_1, y_2|x_1, x_2)$ which fully describes the wireless channel. The channel capacity of a channel with input X, non-causal transmitter side information S, output Y, is given by the well-known formula obtained by Gel'fand and Pinsker [19] as

$$C = \max_{p(u,x|s)} \left[I(U;Y) - I(U;S) \right], \tag{1.1}$$

where U is an auxiliary random variable chosen to make the channel $U \to Y$ appear causal. We refer to the coding technique used in [19] as Gel'fand-Pinkser coding. By applying Gal'fand-Pinsker's result to the Gaussian noise and interference case, Costa [7] achieves the capacity of an interference-free channel by careful selection of X, U. That is, when the input X to the channel is Gaussian, and the auxiliary variable U is of the form $U = X + \alpha S$ for some parameter α whose optimal value is equal to the ratio of the signal power to the signal plus noise power, the interference S is completely mitigated, and the capacity is shown to be equal to that of an interference-free channel! *Dirty paper coding* is the term first used by Costa [7] to describe the aforementioned technique which completely mitigates *a-priori* known interference over an input power constrained additive white Gaussian noise channel. Many authors use the dirty-paper coding, or Gel'fand-Pinsker coding technique in channels

[2] In all results for the single user scenario, we assume all nodes $\mathcal{S}_1, \mathcal{S}_2, \mathcal{R}_1$, and $\mathcal{R}_2$ know each others' channels (full CSI assumption).

where non-causal side information is present at the transmitters. The power of this technique was recently demonstrated in the capacity region calculation of the MIMO Gaussian broadcast channel [46], where the achievable dirty-paper coding region is shown to be capacity-achieving.

We now proceed to find achievable rate, and in some channel scenarios, capacity, regions of *cognitive radio channels.* In order to build intuition, we start off by paraphrasing the work of [28], in which the capacity region of a channel of the form of Fig.. 1.2(b) with Gaussian noise is found for weak interference. The Gaussian region is particularly intuitive as we can evaluate the region and demonstrate the rate regions obtained graphically. This allows us to compare cognitive regions with the classical competitive and cooperative regions. We then extend results to the more abstratc discrete memoryless channel case, where we highlight the results of [14] and [48].

1.2.1 Gaussian noise

We define a 2×2 *cognitive radio channel* C_{COG}, also known as an interference channel with degraded message sets (IC-DMS) in [28, 48] as in Fig.1.3, to be two point-to-point channels $\mathcal{S}_1 \to \mathcal{R}_1$ and $\mathcal{S}_2 \to \mathcal{R}_2$ in which the sender $\mathcal{S}_2$ is given, in a non-causal manner (i.e., by a genie), the encoded message X_1 which the sender $\mathcal{S}_1$ will transmit. Thus, $\mathcal{S}_1$ is the primary user, and $\mathcal{S}_2$ is a secondary, cognitive user. Let X_1 and X_2 be the random variable inputs to the channel, and let Y_1 and Y_2 be the random variable outputs of the channel. The conditional probabilities of the channel C_{COG} are fully described by $P(y_1|x_1,x_2)$ and $P(y_2|x_1,x_2)$. We first consider the case when these conditional distributions relate the input and output as in Eqns. (1.3), shown in Fig.. 1.3(a)[3]

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & a_{21} \\ a_{12} & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \tag{1.2}$$

$$N_1 \sim \mathcal{N}(0,\sigma_1^2), \qquad N_2 \sim \mathcal{N}(0,\sigma_2^2) \tag{1.3}$$

An (n, K_1, K_2, ϵ) *code* for the *cognitive radio channel* consists of K_1 codewords $x_1^n(i) \in \mathcal{X}_1^n$ for $\mathcal{S}_1$, and $K_1 \cdot K_2$ codewords $x_2^n(i,j) \in \mathcal{X}_2^n$ for $\mathcal{S}_2$, $i \in \{1,2,\ldots,K_1\}$, $j \in \{1,2,\ldots,K_2\}$, which together form the *codebook*, revealed to both senders and receivers such that the average error probabilities under some decoding scheme are less than ϵ. A rate pair (R_1, R_2) is said to be *achievable* for the cognitive radio channel if there exists a sequence of $(n, 2^{\lceil nR_1\rceil}, 2^{\lceil nR_2\rceil}, \epsilon_n)$ codes such that $\epsilon_n \to 0$ as $n \to \infty$. An *achievable*

[3] notice that we have assumed the channel between $(\mathcal{S}_1, \mathcal{R}_1)$, as well as $(\mathcal{S}_2, \mathcal{R}_2)$ are all unit. This can be assumed WLOG by multiplying the entire receive chain at $\mathcal{R}_1$ by any (non-infinite) $1/a_{11}^2$, and the receive chain at $\mathcal{R}_2$ by $1/a_{22}^2$ without altering the achievable and/or capacity results.

region is a closed subset of the positive quadrant of $\mathbb{R}^2$ of achievable rate pairs. The capacity region is the closure of the set of all achievable rate pairs (R_1, R_2). We refer the interested reader to [9, 14, 28] for further details and subtleties regarding these definitions. We will drop the block length index n when contextually clear.

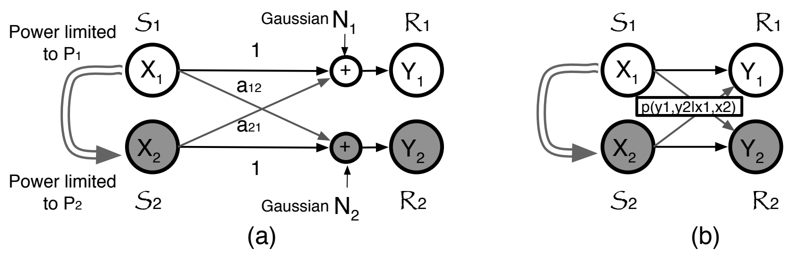

Fig. 1.3. (a) The Gaussian cognitive radio channel. (b) The discrete memoryless cognitive radio channel.

We first consider the results of [28] on the Gaussian interference channel with degraded message sets (IC-DMS or equivalently the Gaussian cognitive radio channel). The authors of this work are particularly interested in determining the maximal rate at which the secondary cognitive user may transmit such that the primary user's rate remains unchanged (that is, the primary user's rate continues to be the same as if there were no interference). However, the authors not only obtain this single point in the capacity region, but rather the entire for weak interference ($a_{21} < 1$). They require the primary receiver to employ a single-user decoder, which would be the case if no cognitive user were present. In essence, these two conditions, which they term *co-existence conditions,* require the cognitive user to remain transparent to the primary user. Of particular interest is that in the proof of the capacity region, the co-existence conditions are relaxed (allowing for joint codebook design between primary and secondary users), and the authors show that the capacity achieving coding/decoding scheme in fact satisfy these *co-existence conditions*, that is, that the primary user decoder behaves as a single user decoder.

The main results of [28] stated in their Theorems 3.1 and 4.1 are summarized in the following single theorem. Here the primary user is expected power limited to P_1, the secondary user is expected power limited to P_2, and the noises at the two receivers are Gaussian of zero mean and variance N_1 and N_2 respectively.

Theorem 1. *The capacity region of the IC-DMS defined in* (1.2) *is given by the union, over all* $\alpha \in [0, 1]$*, of the rate regions*

$$0 \leq R_1 \leq \frac{1}{2}\log_2\left(1+\frac{(\sqrt{P_1}+a_{21}\sqrt{\alpha P_2})^2}{1+a_{21}^2(1-\alpha)P_2}\right)$$
$$0 \leq R_2 \leq \frac{1}{2}\log_2\left(1+(1-\alpha)P_2\right)$$

In particular, the maximal rate R_2 *(or capacity) at which a cognitive user may transmit such that the primary user's rate* R_1 *remains as in the interference-free regime (*$R_1 = \frac{1}{2}\log_2(1+P_1/N)$*) is given by*

$$R_2 = \frac{1}{2}\log_2\left(1+(1-a^*)P_2\right)$$

as long as $a_{21} < 1$*,and* a^* *is*

$$a^* = \left(\frac{\sqrt{P_1}\left(\sqrt{1+a_{21}^2P_2(1+P_1)}-1\right)}{a_{21}\sqrt{P_2}(1+P_1)}\right)^{\frac{1}{2}}.$$

Both these results are obtained using a Gaussian encoder at both the primary and cognitive transmitters. For more precise definitions of achievability in this channel, we refer to [28]. We paraphrase their achievability results here. The primary user generates its 2^{nR_1} codewords, X_1^n (block length n), by drawing the coordinates i.i.d. according to $\mathcal{N}(0,P_1)$, where we recall P_1 is the expected noise power constraint. Then, since the cognitive radio knows the message the primary user, it can form the primary user's encoding X_1^n, and performs superposition coding as:

$$X_2^n = \hat{X}_2^n + \sqrt{\frac{\alpha P_2}{P_1}}X_1^n,$$

where $\alpha \in [0,1]$. The codeword $\hat{X}_2^n$ encodes one of the 2^{nR_2} messages, and is generated by performing Costa precoding [7] (dirty-paper coding). Costa showed that to optimize the rate achieved by this dirty-paper coding, one selects $\hat{X}_2^n$ statistically independently from X_1^n, and thus i.i.d. Gaussian. Encoding is done using a standard information theoretic binning technique, which treats the message X_1^n as non-causally known interference. In order to satisfy the average power constraint of P_2 on the components of X_2^n, $\hat{X}_2^n$ must be $\mathcal{N}(0,(1-\alpha)P_2)$. The parameter α allows for a trade-off at the cognitive transmitter between aiding the primary transmitter (α close to 1) or transmitting, using a dirty paper coding technique, its own message (α close to 0). A converse, resulting in the capacity region of the cognitive radio channel under weak interference, is given in [28] and is based on the conditional entropy power inequality, and results from [46].

We illustrate this region for three different values of the channel parameters a_{12}, a_{21}, and compare it to the Gaussian MIMO broadcast channel region (in which the two transmitters may cooperate), the achievable rate region for the interference channel region obtained in [21] (the largest known to date for the Gaussian noise case) where the two transmitters must operate independently, and the time-sharing region where the two transmitters take turns using the channel, thus creating two independent channels and avoiding any interference. The capacity region for the Gaussian MIMO broadcast channel with two single antenna receivers and one transmitter with two antennas subject to per antenna power constraints of P_1 and P_2 respectively, is given by Eqn. (1.4), which may be obtained from the general formulation in [4, 46].

$$\text{Convex hull } \{(R_1, R_2) :$$

$$\begin{array}{l} R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^T+Q_1}{H_1B_2H_1^T+Q_1}\right) \\ R_2 \le \frac{1}{2}\log_2\left(\frac{H_2B_2H_2^T+Q_2}{Q_2}\right) \\ R_2 \le \frac{1}{2}\log_2\left(1+\frac{P_2}{Q_2}\right) \end{array} \bigcup \begin{array}{l} R_1 \le \frac{1}{2}\log_2\left(\frac{H_1B_1H_1^t+Q_1}{Q_1}\right) \\ R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_1+B_2)H_2^T+Q_2}{H_2B_1H_2^T+Q_2}\right) \\ R_2 \le \frac{1}{2}\log_2\left(1+\frac{P_2}{Q_2}\right) \end{array}$$

$$\text{for any 2x2 matrices } B_1, B_2 \text{ such that}$$

$$\begin{gathered} B_1 \succeq 0, \quad B_2 \succeq 0 \\ B_1 + B_2 \preceq \begin{pmatrix} P_1 & c \\ c & P_2 \end{pmatrix} \\ c^2 \le P_1P_2\} \end{gathered} \tag{1.4}$$

Here $X \succeq 0$ denotes that the matrix X is positive semi-definite, and we defined $H_1 = [1\ a_{21}]$ and $H_2 = [a_{12}\ 1]$. The achievable rate region of [21] used in these figures (as the "interference channel" achievable region) assumes the same Gaussian input distribution as in [14] and is omitted for brevity. The time sharing region for the interference channel (which requires coordination between the two transmitters, but only at the time sharing, and not coding level) is given by the region of Eqn. (1.5).

$$\text{Time-share region } \triangleq \left\{ \bigcup_{0\le\alpha\le 1} (\alpha\frac{1}{2}\log_2\left(1+\frac{P_1}{N_1}\right), (1-\alpha)\frac{1}{2}\log_2\left(1+\frac{P_2}{N_2}\right)) \right\} \tag{1.5}$$

From the Fig. 1.4, 1.6 and 1.6 we see that both users – not only the incumbent $\mathcal{S}_2$ which has the extra message knowledge – benefit from behaving in a cognitive, rather than simple time-sharing manner. This is as expected, as the decreasing α boosts R_2 rates, while increasing α (of Theorem 1) boosts R_1 rates, and so gracefully combining the two will yield benefits to both users. Time-sharing is in essence what would be theoretically achievable in spectral-gap filling models for cognitive radio channels. That is, under the assumption that an incumbent cognitive were to perfectly sense the gaps in the spectrum, and fill them by transmitting at the capacity of the point-to-point channel between ($\mathcal{S}_2$, $\mathcal{R}_2$), the best rate region one can hope to achieve is the time-sharing rate region. Where we operate on the boundary of this region depends entirely on what percentage of the time the primary user is on the channel (or the α parameter of (1.5)). The rates achieved by the cognitive user thus depend on the primary user's channel utilization. It is important to note that in the region and coding scheme described in Theorem 1 that the cognitive user's choice of the power-sharing parameter α in essence determines where on the boundary of the cognitive IC-DMS region we operate. Thus, if such a scheme were to be implemented, one would need to ensure that cognitive radios would not select an α that would be too detrimental to the rate of the primary user. Nonetheless, theoretically, the presence of the incumbent cognitive radio $\mathcal{S}_2$ can be beneficial to $\mathcal{S}_1$, and could provide incentives for the introduction of such schemes.

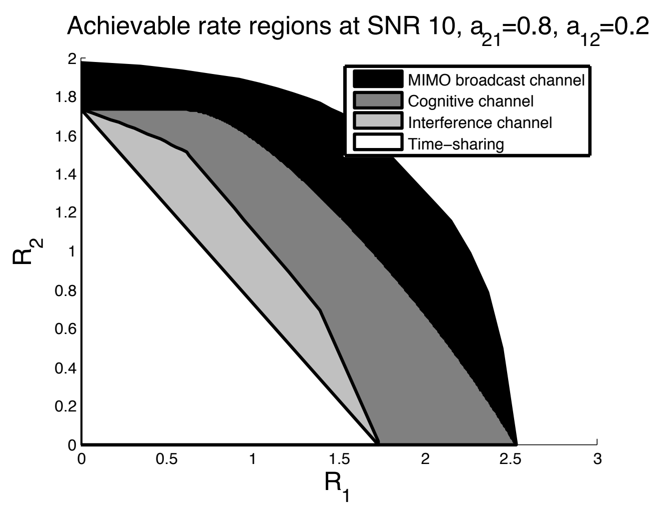

Fig. 1.4. Capacity region of the Gaussian 2×1 MIMO two receiver broadcast channel (outer), cognitive channel (middle), achievable region of the interference channel (second smallest) and time-sharing (innermost) region for Gaussian noise powers $N_1 = N_2 = 1$, power constraints $P_1 = P_2 = 10$ at the two transmitters, and channel parameters $a_{12} = 0.8$, $a_{21} = 0.2$.

1.2.2 Discrete Memoryless Channel

In the previous subsection, we outlined the capacity region for the Gaussian cognitive radio channel (or IC-DMS) in the weak interference ($a_{21} < 1$) regime. Gaussian noise channels have in general been more well studied and understood than general discrete channel models (where channel inputs and outputs

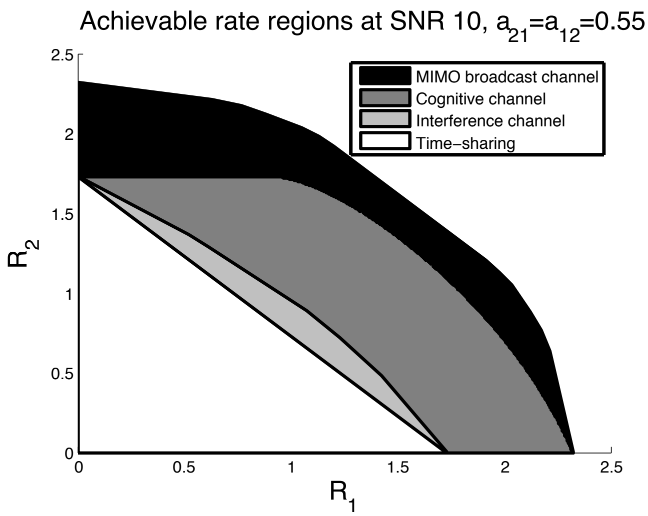

Fig. 1.5. Capacity region of the Gaussian 2×1 MIMO two receiver broadcast channel (outer), cognitive channel (middle), achievable region of the interference channel (second smallest) and time-sharing (innermost) region for Gaussian noise powers $N_1 = N_2 = 1$, power constraints $P_1 = P_2 = 10$ at the two transmitters, and channel parameters $a_{12} = 0.55$, $a_{21} = 0.55$.

take on a set of discrete values) and lend themselves well to intuition and numerical calculation. In this section we outline the current state-of-the-art for discrete cognitive channels, highlighting results from both [48] and [12, 14].

We first look at the achievable rate region defined in [14] and extended in [12]. The channel model is shown in Fig.1.3(b) where the inputs X_1, X_2 to the channel and the outputs Y_1, Y_2 of the channel are now discrete, and the conditional distribution $p(y_1, y_2|x_1, x_2)$ which characterizes the channel is discrete rather than Gaussian. Since this channel resembles an interference channel with non-causal transmit side information at one of the encoders, the authors of [14] derive an achievable rate region by combining the best to date known achievable rate region for the interference channel, that of [21], with the results for non-causal transmit side information, Gel'fand-Pinsker coding [19]. We briefly outline the achievable rate region of [12, 14].

In [21], an achievable region for the interference channel is found by first considering a modified problem and then establishing a correspondence between the achievable rates of the modified and the original channel models. We proceed in the same fashion. The channel C_{COG}^m, defined as in Fig.1.7 introduces many new auxiliary random variables, whose purposes can be made intuitively clear by relating them to auxiliary random variables in previously studied channels. They are defined and described in Table 1.1. Standard def-

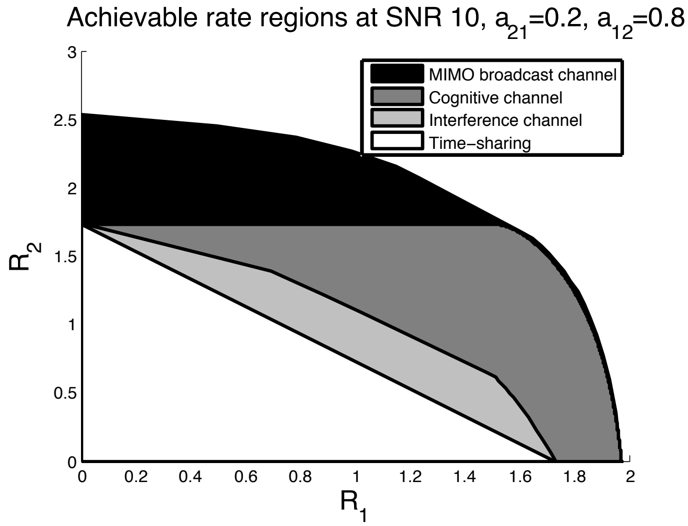

Fig. 1.6. Capacity region of the Gaussian 2×1 MIMO two receiver broadcast channel (outer), cognitive channel (middle), achievable region of the interference channel (second smallest) and time-sharing (innermost) region for Gaussian noise powers $N_1 = N_2 = 1$, power constraints $P_1 = P_2 = 10$ at the two transmitters, and channel parameters $a_{12} = 0.2$, $a_{21} = 0.8$.

initions of achievable rates and regions are employed [9, 13] and omitted for brevity. Then an achievable region for the discrete cognitive radio channel is given by:

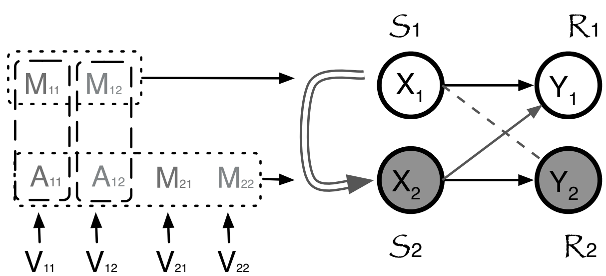

Fig. 1.7. The modified cognitive radio channel with auxiliary random variables M_{11}, M_{12} and M_{21}, M_{22}, inputs X_1 and X_2, and outputs Y_1 and Y_2. The auxiliary random variable A_{11}, A_{12} associated with $\mathcal{S}_2$, aids in the transmission of M_{11} and M_{12} respectively. The vectors V_{11}, V_{12}, V_{21} and V_{22} denote the effective random variables encoding the transmission of the private and public messages.

Table 1.1. Description of random variables and rates in Theorem 2.

(Random) variable names	(Random) variable descriptions
M_{11}, M_{22}	Private information from $\mathcal{S}_1 \to \mathcal{R}_1$ and $\mathcal{S}_2 \to \mathcal{R}_2$ resp.
M_{12}, M_{21}	Public information from $\mathcal{S}_1 \to (\mathcal{R}_1, \mathcal{R}_2)$ and $\mathcal{S}_2 \to (\mathcal{R}_1, \mathcal{R}_2)$ resp.
R_{11}, R_{22}	Rate between $\mathcal{S}_1 \to \mathcal{R}_1$ and $\mathcal{S}_2 \to \mathcal{R}_2$ resp.
R_{12}, R_{21}	Rate between $\mathcal{S}_1 \to (\mathcal{R}_1, \mathcal{R}_2)$ and $\mathcal{S}_2 \to (\mathcal{R}_1, \mathcal{R}_2)$ resp.
A_{11}, A_{12}	Variables at $\mathcal{S}_2$ that aid in transmitting M_{11}, M_{12} resp.
$V_{11} = (M_{11}, A_{11}), V_{12} = (M_{12}, A_{12})$	Vector helping transmit the private/public (resp.) information of $\mathcal{S}_1$
$V_{21} = M_{21}, V_{22} = M_{22}$	Public and private message of $\mathcal{S}_2$. Also the auxiliary random variables for Gel'fand-Pinsker coding
W	Time-sharing random variable, independent of messages

Theorem 2. *Let* $Z \triangleq (Y_1,Y_2,X_1,X_2,V_{11},V_{12},V_{21},V_{22},W)$*, be as shown in Fig.1.7. Let* $\mathcal{P}$ *be the set of distributions on* Z *that can be decomposed into the form*

$$\begin{aligned}
&P(w) \times [P(m_{11}|w)P(m_{12}|w)P(x_1|m_{11}, m_{12}, w)] \\
&\times [P(a_{11}|m_{11}, w)P(a_{12}|m_{12}, w)] \\
&\times [P(m_{21}|v_{11}, v_{12}, w)P(m_{22}|v_{11}, v_{12}, w)] \\
&\times [P(x_2|m_{21}, m_{22}, a_{11}, a_{12}, w)]\, P(y_1|x_1, x_2)P(y_2|x_1, x_2),
\end{aligned} \tag{1.6}$$

where $P(y_1|x_1, x_2)$ *and* $P(y_2|x_1, x_2)$ *are fixed by the channel. Let* $T_1 \triangleq \{11, 12, 21\}$ *and* $T_2 \triangleq \{12, 21, 22\}$*. For any* $Z \in \mathcal{P}$*, let* $S(Z)$ *be the set of all rate tuples* $(R_{11}, R_{12}, R_{21}, R_{22})$ *(as defined in Table 1) of non-negative real numbers such that there exist non-negative reals* $L_{11}, L_{12}, L_{21}, L_{22}$ *satisfying:*

$$\bigcap_{T \subset \{11,12\}} \left(\sum_{t \in T} R_t \right) \leq I(X_1; \mathbf{M}_T | \mathbf{M}_{\overline{T}}) \tag{1.7}$$

$$R_{11} = L_{11} \tag{1.8}$$

$$R_{12} = L_{12} \tag{1.9}$$

$$R_{21} \leq L_{21} - I(V_{21}; V_{11}, V_{12}) \tag{1.10}$$

$$R_{22} \leq L_{22} - I(V_{22}; V_{11}, V_{12}) \tag{1.11}$$

$$\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) \leq I(Y_1, \mathbf{V}_{\overline{T}}; \mathbf{V}_T | W) + f(\mathbf{V}_T | W) \tag{1.12}$$

$$\bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2} \right) \leq I(Y_2, \mathbf{V}_{\overline{T}}; \mathbf{V}_T | W) + f(\mathbf{V}_T | W), \tag{1.13}$$

where $f(\mathbf{v}_T)$ *denotes the divergence between the joint distribution of the random variables* $\mathbf{V}_T$ *in* (1.6) *and their product distribution (where all components are independent).* $\overline{T}$ *denotes the complement of the subset* T *with respect to* T_1 *in* (1.12)*, with respect to* T_2 *in* (1.13)*, and* $\mathbf{V}_T$ *denotes the vector of* V_i *such that* $i \in T$*. Let* S *be the closure of* $\cup_{Z \in \mathcal{P}} S(Z)$*. Then any pair* $(R_{11} + R_{12}, R_{21} + R_{22})$ *for which* $(R_{11}, R_{12}, R_{21}, R_{22}) \in S$ *is achievable for* C_{COG}*.*

Proof outline: The main intuition is as follows: the equations in (1.7) ensure that when $\mathcal{S}_2$ is presented with X_1 by the genie, the auxiliary variables M_{11} and M_{12} can be recovered. Eqs. (1.12) and (1.13) correspond to the equations for two overlapping MAC channels seen between the effective random variables $\mathbf{V}_{T_1} \to \mathcal{R}_1$, and $\mathbf{V}_{T_2} \to \mathcal{R}_2$. Eqs. (1.10) and (1.11) are necessary for the Gel'fand-Pinsker [19] coding scheme to work ($I(V_{21}; V_{11}, V_{12})$ and $I(V_{22}; V_{11}, V_{12})$ are the penalties for using non-causal side information. The $f(\mathbf{V}_T)$ terms correspond to the highly unlikely events of certain variables being correctly decoded despite others being in error. Intuitively, the sender $\mathcal{S}_2$ could aid in transmitting the message of $\mathcal{S}_1$ (the A_{11}, A_{12} random variables) or it could dirty paper code against the interference it will see (the M_{21}, M_{22} variables). The theorem smoothly interpolates between these two options. Details may be found in [12, 14].

The work [48] also considers the discrete memoryless IC-DMS (or discrete cognitive radio channel), and looks at the Gaussian IC-DMS as a special case. The authors in this work are motivated by a sensor network in which one sensor has better sensing capabilities than another. The one with the better channel is thus able to detect two sensed events, while another is only able to detect one. This problem then reduces to the interference channel with degraded message sets (where the message of one user is a subset of the other user's message). The authors define three types of *weak interference* (as opposed to the *very strong* and *strong* interference typically seen in the interference channel literature [5]), an achievable rate region, outer bounds, and conditions under which these outer bounds are tight. They then look at a Gaussian noise example in which their region is tight, and for which the result is as described in the capacity region of [28]. We summarize some of their main results in the single following theorem. It provides an inner and an outer bound on the discrete IC-DMS, which turns out to be the capacity region for the types of interference specified.

Theorem 3. Inner bound: *Let $\mathcal{R}_{in}$ be the set of all rate pairs (R_1, R_2) (same as in the cognitive radio channel) such that*

$$R_1 \leq I(V, X_1; Y_1)$$
$$R_2 \leq I(U; Y_2) - I(U; V, X_1)$$

for the probability distribution $p(x_1, x_2, u, v, y_1, y_2)$ that factors as

$$p(v, x_1)p(u|v, x_1)p(x_2|u)p(y_1, y_2|x_1, x_2).$$

Then $\mathcal{R}_{in}$ is an achievable rate region for the IC-DMS where transmitter $\mathcal{S}_2$ knows both messages and transmitter $\mathcal{S}_1$ only knows one.
Outer bound: *Define $\mathcal{R}_o$ to be the set of all rate pairs (R_1, R_2) such that*

$$R_1 \leq I(V, X_1; Y_1)$$
$$R_2 \leq I(X_1; Y_2|X_1)$$
$$R_1 + R_2 \leq I(V, X_1; Y_1) + I(X_2; Y_2|V, X_1),$$

for the probability distribution $p(x_1, x_2, v, y_1, y_2)$ that factors as

$$p(v, x_1)p(x_2|v)p(y_1, y_2|x_1, x_2).$$

Then R_o is an outer bound for the capacity of the IC-DMS.
Capacity conditions: *If there exists a probability transition matrix $q_1(y_2|x_2, y_1)$ such that*

$$p(y_2|x_1, x_2) = \sum_{y_1} p(y_1|x_1, x_2)q_1(y_2|x_2, y_1),$$

or if there exists a probability transition matrix $q_2(y_1|x_1, y_2)$ such that

$$p(y_1|x_1, x_2) = \sum_{y_2} p(y_2|x_1, x_2)q_2(y_1|x_1, y_2),$$

then the set of all rate pairs (R_1, R_2) such that

$$R_1 \leq I(V, X_1; Y_1) \tag{1.14}$$
$$R_2 \leq I(X_2; Y_2|V, X_1) \tag{1.15}$$

for the probability distribution $p(x_1, x_2, y_1, y_2)$ that factors as

$$p(v, x_1)p(x_2|v)p(y_1, y_2|x_1, x_2),$$

is the capacity region of the IC-DMS.

Since the channel of [48] is the same as the cognitive radio channel [12,14], direct comparisons between their resepective bounds may be made. Whereas the outer bounds are equivalent, due to the fact that the inner bounds for the discrete memoryless channel involve non-trivial unions over all distributions of a certain form, it is unclear a priori which region is more general. However, the authors demonstrate that all *Gaussian* weak interference channels satisfy the *capacity conditions* of the theorem, and thus the region of (1.14)-(1.15) is the capacity region. This capacity region in the Gaussian noise case is shown to be explicitly equal to that of [28], and, numerically, to that of of [12], specialized to the Gaussian noise case.

1.2.3 Further Results

Numerous other works have considered fundamental limits of systems involving cognitive radios. The work [24] considers again the cognitive radio channel, referred to as the *interference channel with unidirectional cooperation*. There, one set of conditions for which the capacity region of the channel coincides with that of the channel in which both messages are required at both receivers is derived. Notice that in the cognitive radio channel this added condition, of being able to decode both messages at both receivers, is not assumed. This is related to the work [23] on the compound multiple access channel with common information, in which the capacity region for another set of *strong interference*-type conditions is computed. Notice that whereas [48] considers *weak interference* conditions, [24] considers *strong interference conditions.* Their results on the cognitive radio channel capacity read as follows:

Theorem 4. *For an interference channel with unidirectional cooperation satisfying*

$$I(X_2;Y_2|X_1) \leq I(X_2;Y_1|X_1)$$
$$I(X_1,X_2;Y_1) \leq I(X_1,X_2;Y_2)$$

for all joint distributions on X_1 and X_2, the capacity region $\mathcal{C}$ is given by

$$\mathcal{C} = \bigcup \left\{ (R_1,R_2) : \begin{array}{c} R_2 \leq I(X_2;Y_2|X_1) \\ R_1 + R_2 \leq I(X_1,X_2;Y_1) \end{array} \right\},$$

where the union is over joint distributions $p(x_1,x_2,y_1,y_2)$.

The case of causal (rather than non-causal) message knowledge is considered in [14], where various protocols for which $\mathcal{S}_2$ causally obtains the message of $\mathcal{S}_1$ are presented. In [12] the simple 2×2 cognitive radio channel is extended to the multiple access scenario. That is, the authors consider a channel in which a cognitive $(q \to 1)$ multiple access channel has non-causal knowledge of the messages of the primary $(p \to 1)$ multiple access channel. In a similar vein, the work [33] considers a multiple access channel with channel state known non-causally at one one of the encoders. Inner and outer bounds are obtained for this channel, with the interesting conclusion that when channel state information is present at only one of the encoders, full interference mitigation (achieved in Costa's dirty-paper coding) is not possible. This is in sharp contrast to the complete state (or interference) mitigation that is possible in both dirty paper coding for a point-to-point channel [7] as well as generalized dirty paper coding in a multiple access channel in which the state is known at both encoders [30]. This channel differs of course from that of [12] as only a single multiple access channel is considered.

Although up until now we have emphasized the gains to be made when a primary user and a secondary cognitive user simultaneously transmit, a simpler strategy by which cognitive radios may improve spectral efficiency is by sensing and filling in *spectral gaps.* The achievable rate region for this spectral gap filling region, when a single frequency band is shared over time, is given by the timesharing regions of Fig.1.4 1.5, 1.6. The work in [25] and [43] addresses issues involved in the opportunistic sensing of and communication over such spectral holes. Notice that the time-sharing regions correspond to *temporal* holes rather than frequency holes. In [25, 43] capacity inner and outer bounds for a cognitive transmitter-receiver pair acting as secondary users in a network of primary users are derived. According to the authors, the capacity is limited by the *distributed* and *dynamic* nature of the spectral activity which these cognitive radios wish to exploit. The authors use the term *distributed* to denote the different views of local spectral activity at the cognitive transmitter and receiver. In addition to the spectrum availability being location-dependent, it will also vary with time, depending on the data that must be sent at different moments. The authors use the term *dynamic* to indicate the temporal variation of the spectral activity of the primary users. We refer the interested reader to [25, 43] for further details.

Cognitive radios could also serve as relays in a network. In this section, we have highlighted results for when a cognitive radio has its own information to transmit. However, when no such information is at hand, it could act as a relay and aid a primary user in transmitting its message. For this type of communication, information theoretic limits can be found in the literature pertaining to the classical *relay channel.* The relay channel, which in its simplest and most classical form is a three-terminal channel with one source, one relay (without its own information to transmit) and one destination, were introduced by van der Meulen [44], and various variations of the problem were later studied by others [2], [10]. The current state of the art is well summarized in [35]. Three major issues are ignored in the classical relay channel framework: the half-duplex constraint of most practical wireless systems, the compound nature, and the non-degraded nature of most wireless channels. Some of these issues are addressed in the *collaborative communications* framework of [38].

We have so far looked at information theoretic limits of channels with cognitive radios under the assumption that all nodes has full CSI. In this scenario, cognitive radios were beneficial in terms of overall achievable rates due to their ability to mitigate the primary user's interference (to the cognitive receiver), or alternatively to strengthen it. The ability to mitigate interference is, qualitatively, dependent on having full CSI at the transmitters. In the next section, we explore the benefits of cognitive radios in fading channel, under varying assumptions of the CSI.

1.3 Fundamental Limits of Cognitive Radio Channels: Imperfect CSI and Fading Channels

The information-theoretic study of fading channels is by no means a new field. The traditional notion of capacity is extended to include concepts such as the *ergodic capacity*, the *distribution of capacity* (leading to "capacity-versus-outage" results), *delay-limited capacity*, and *compound channels*, to mention a few popular measures. The large number of different perspectives on fading channels are a consequence not only of the different underlying fading models assumed, but also the different amounts, and types (noisy, perfect, distribution only) of channel state information (CSI) available to the transmitter(s) and/or receiver(s). For example, when CSI, modeled as a time-varying random process, is known perfectly to all transmitters and receivers, but varies over time in an ergodic fashion, then the transmitter may continually adjust its rate to the current channel conditions. The *ergodic capacity (region)* is thus defined and obtained, roughly speaking, by taking the expectation with respect to the channel parameter distribution, of the capacity (region). On the other hand, if the transmitter has no CSI, and one demands error-free transmission at all times, then the achievable rates are significantly reduced, roughly to the rates sustainable by the "worst" channel conditions. However, if, for a certain rate R one allows for channel outages with probability ϵ (during which reliable communication at the rate R is impossible) then we can define the outage capacity C_ϵ as the maximal rate which can be achieved such that the probability of outage is less than ϵ. Because of the breadth and wealth of information on this subject matter, we refer interested readers to [3] for detailed references on information-theoretic and communications aspects of fading channels. Here, we review only some of the very recent results on cognitive radio channels when the channel coefficients are fading and partially unknown, or possibly unknown, to all or some of the nodes. These include the compound Gel'fand-Pinkser channel, carbon copying onto dirty paper (an analogy related to Costa's dirty paper coding), as well as Gel'fand-Pinkser coding with uncertainty in the phase of the non-causally known interference.

1.3.1 The Compound Gel'fand-Pinsker Channel

Most of the rate regions considered thus far have employed non-causal side information at the cognitive transmitter to mitigate primary user interference at the secondary receiver. The gains exploited by the cognitive transmitter extended directly from the results of Gel'fand-Pinsker [19] on channels with non-causal side information at the transmitter. These results assume all channel parameters are perfectly known a-priori by the transmitters and receivers. Some more realistic models of fading channels assume CSI, or fading coefficients (h is the input-output relation $Y = hX + N$) are available to the receiver but not the transmitter. The work [37] generalizes the results of Gel'fand-Pinsker to the case where the channel is parameterized continuously by β (belonging to a compact set $\mathcal{C}$) unknown to the transmitter, but assumed known at the receiver. For communication over a channel without side-information, such channels are often called compound channels[4] [11, 47].

For the traditional discrete compound channel with input X, output Y and conditional probability mass function (PMF) $P^{\beta}_{Y|X}$ where β denotes the unknown parameter at the transmitter, it is well known that the capacity, [47], is given by

$$C = \sup_{P_X} \inf_{\beta} I^{\beta}(X;Y), \tag{1.16}$$

where $I^{\beta}(X;Y) \triangleq I(P_X, P^{\beta}_{Y|X})$ denotes the mutual information between X and Y given the realization of the channel parameter is β. The main result of [37] in the finite alphabet case for compound channels with side-information at the transmitter is given in Theorem 5.

Theorem 5. *The capacity C of the discrete memoryless compound channel with side information at the transmitter is bounded by $C_L \leq C \leq C_U$ where*

$$C_L = \sup_{P_{U|X,S,W}, P_{X|S,W}, P_W} \left[\inf_{\beta \in \mathcal{C}} \left[I^{\beta}(U;Y|W) - I(U;S|W) \right] \right], \tag{1.17}$$

$$C_U = \sup_{\{P^{\beta}_{U|X,S,W}\}_{\beta}, P_{X|S,W}, P_W} \left[\inf_{\beta \in \mathcal{C}} \left[I^{\beta}(U;Y|W) - I(U;S|W) \right] \right], \tag{1.18}$$

and the suprema are over all finite alphabet auxiliary random variables U and finite alphabet time-sharing random variables W and $\{P^{\beta}_{U|X,S,W}\}_{\beta}$ denotes any family of distributions (i.e., in C_U a distribution $P_{U|X,S,W}$ is chosen for each β before the minimization over β is performed) .

Since the joint distribution $P_{X,S,W}$ in C_U does not depend on β, the upper bound C_U is in general tighter than if a genie had revealed β to the transmitter. When $\mathcal{C}$ is a singleton (the degenerate case), the bounds reduce to the well known Gel'fand-Pinsker result.

The authors of [37] then apply this result to the cognitive radio scenario, and consider the problem of encoding a message V with knowledge

[4] In [47], the channel is said to be compound provided that β is unknown to either the transmitter, the receiver or both. Here, we are interested in the scenario where β is known to the receiver but not the transmitter.

of a Gaussian interfering signal S of power Q. The encoder output X is also power constrained to $P = Q$ and the signal received at the decoder is $Y = \beta_1 X + \beta_2 S + Z$ where Z is independent Gaussian noise and the compound parameter is $\beta := (\beta_1, \beta_2)$.

Similar to Costa's scheme, they suggest $U = X + \alpha S$, where α is now chosen as a function of the second order statistics of β_1 and β_2 as

$$\alpha = \frac{\mu_1^* \mu_2 SNR}{(|\mu_1|^2 + \sigma_1^2) SNR + 1}, \tag{1.19}$$

where μ_i and σ_i^2 are the mean and variance of β_i. This choice for Ricean fading channels, where β_1 and β_2 have K-factors K_1 and K_2 respectively satisfies:

1. If $K_1, K_2 \to \infty$, then the scheme is identical to Costa's with $\alpha = P/(P + N)$ and the interference is perfectly mitigated.
2. If either $K_1 \to 0$ or $K_2 \to 0$, the scheme treats the interferer as noise.
3. The performance does not depend on the phase difference between μ_1 and μ_2 as this choice of α rotates the mean channels so that their phases are aligned.

The authors numerically found that the achievable rates for given outage probabilities are good over a wide range of K-factors, by comparing the rates to the outage capacity of an interference-free scenario and a scenario where the interference is treated as noise.

1.3.2 Carbon copying onto Dirty Paper

In a similar vein, the work [29] focuses on a generalization of the Gel'fand-Pinkser problem in which a single transmitter wishes to transmit a common message to multiple receivers (which they call the multicast channel). They consider memoryless channels in Gaussian and noiseless binary special cases. Each receiver experiences different interference which is non-causally known at the transmitter. This problem is equivalent to that of dirty-paper coding (between a single transmitter and a single receiver) where only imperfect knowledge of the state (interference) is available to the transmitter. The collection of possible (random) interferences may be thought of as the interferences at different receivers, and the problem reduces to finding the maximal rate at which we can communicate the single message to all users simultaneously.

Of particular interest to the cognitive radio channel with unknown fading is the Gaussian case considered in [29]. They restrict their attention to sending a common message to two users with different interferences, which they assume to be i.i.d. Gaussian. This corresponds to a single input-single output channel where the interfering sequence can take on one of two possible values. Although this would not be the case in a situation in which the interfering sequence S is known but the channel fading coefficient h is not (Gaussian noise input-output relation $Y = X + hS + N$), the results and techniques are

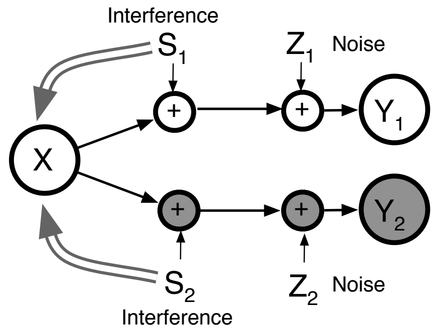

Fig. 1.8. The two user Gaussian multicast channel model of [29] with additive interference. The encoder knows the two independent interfering sequences S_1, S_2 and wishes to transmit a single, common message to each of the two receivers. The channel is subject to additive white Gaussian noise Z_1, Z_2.

of great interest nonetheless. Their main results are upper and lower bounds on the channel capacity in this two user case, paraphrased in Theorem 6.

Theorem 6. *The Gaussian multicast channel with Gaussian noise of power $N = 1$, input power constraint P, independent interference sequences of zero mean and power Q is bounded according to*

$$R_- \leq C \leq R_+,$$

where[5]

$$R++ = \begin{cases} \frac{1}{4}\log_2(1+P) + \frac{1}{4}\log_2\left(\frac{P+Q+1+2\sqrt{PQ}}{Q}\right) & Q \geq 4 \\ \frac{1}{4}\log_2\left(\frac{1+P}{Q/4+1}\right) + \frac{1}{4}\log_2\left(\frac{P+Q+1+2\sqrt{PQ}}{Q/4+1}\right) & Q < 4 \end{cases}$$

and

$$R_- \begin{cases} \frac{1}{2}\log_2\left(1+\frac{P}{Q/2+1}\right) & Q/2 < 1 \\ \frac{1}{2}\log_2\left(\frac{P+Q/2+1}{Q}\right) + \frac{1}{2}\log_2\left(\frac{Q}{2}\right) & 1 \leq Q/2 \leq P+1 \\ \frac{1}{4}\log_2(1+P) & Q/2 \geq P+1 \end{cases}$$

The upper bound is obtained by considering a single-interference Gaussian channel, and arguing why the achievable rate for the two-interference channel of interest cannot be higher. The lower bound is an explicit expression of the maximization of

$$R_- = \max_{\{(P_S,P_V):P_S\geq 0, P_V\geq 0, P_S+P_V\leq P\}} \frac{1}{2}\log_2\left(1+\frac{P_S}{P_V+Q/2+1}\right) + \frac{1}{4}\left(1+P_V\right),$$

[5] The upper bound can be further tightened by considering $\min(R_+,\ \log_2(1+P))$, where the second expression corresponds to the multicasting rate when the interference is absent.

which they show to be achievable through a combination of superposition coding, dirty paper coding, time-sharing, and representing the two interferences S_1, S_2 as

$$S_1^n = S^n + V^n \qquad S^n = (S_1^n + S_2^N)/2$$
$$S_2^n = S^n - V^n \qquad V^n = (S_1^n - S_2^n)/2.$$

The main idea is to split the message into two parts: the first is dirty-paper coded to mitigate the "common interference" S. The second message is then, in a time-sharing manner, first sent to the first receiver (again by dirty-paper coding against the remaining first interference) and then to the second receiver (also by dirty-paper coding against the remaining second interference). The superposition of these two parts is transmitted.

The authors then consider *robust dirty-paper coding*. That is, the channel model is the usual $Y = X + S + Z$ (Z is Gaussian noise), and the interference sequence S[6] is either $\beta_1 S_0$ or $\beta_2 S_0$, where we interpret S_0 as the interference known to the encoder, while the fading coefficient β_1, β_2 is not. The lower bound on the channel capacity is then given by the expression

$$C_\beta \geq \max_{(P_s,P_v):P_s\geq 0, P_v \geq 0, P_s+P_v\leq P} \frac{1}{2}\log_2\left(1 + \frac{P_S}{1+(\beta_1-\beta_2)^2 Q/4 + P_v}\right) + \frac{1}{4}\log_2(1+P_v).$$

Both the work [29] as well as [37] provide upper and lower bounds to dirty-paper coding when the interference signal is imperfectly known. Although these results do not give explicit rate regions for the cognitive radio channel with unknown coefficients, they provide valuable insights and techniques which could be of use to future results in this direction.

1.3.3 Gel'fand-Pinkser Coding with Unknown Phase

Continuing in the line of work of [29, 37], the authors in [20] consider the channel

$$Y = X + (|h|e^{j\theta})S + Z \tag{1.20}$$

where the interference S and the fading amplitude $|h| = 1$ are known to the transmitter, while the phase of the fading coefficient, θ is unknown. They are interested in determining whether the decrease in rate due to the unknown phase is substantial enough to warrant trying to learn the phase (i.e. through a training scheme, taking into account the required overhead). They derive upper and lower bounds on the rates achievable under these conditions, as well as an upper bound on the rate for a given outage probability.

An upper bound on the channel capacity with phase uncertainty is obtained by considering the same channel where the phase uncertainty is reduced to either 0 or π, resulting in Theorem 7

[6] We have dropped all n superscripts for clarity

Theorem 7. *The channel described by* (1.20) *with additive white Gaussian noise* $Z \sim \mathcal{N}(0,1)$*, average power constraint* P*, interference* $S \sim \mathcal{N}(0,Q)$ *known at the transmitter, has achievable rates* R *bounded by*

$$R \leq \frac{1}{2}\log_2\left(\frac{(P+Q+1)^2}{4Q}\right).$$

This bound is shown to be the same as ignoring interference at high signal to interference ratio (SIR), and to be loose at low SIR. The authors then derive achievable rates at low SIR in a method similar to [29]. By sectoring the interference uncertainty circle into a few sectors, and then time-sharing between dirty-paper coding the 'central' interference vector in each sector (treating the residual interference as noise), they are able to demonstrate that if the phase uncertainty is $\delta\phi \in [-\pi/2, \pi/2]$ (difference between the actual phase θ and the phase of the central angle used to dirty paper code against) then an achievable rate is given as in Theorem 8

Theorem 8. *For a phase uncertainty of* $\delta\phi$*, an achievable rate is given by*

$$R = \sup_{\alpha\in[0,1]} \log_2\left(\frac{P}{\epsilon(\delta\phi)}\right),$$

where

$$\epsilon(\delta\phi) = (1-\beta(\delta\phi))^2P + (\alpha^2 + \beta(\delta\phi)^2 - 2\alpha\beta(\delta\phi)\cos(\phi))Q + \beta(\delta\phi)^2$$

and

$$\beta(\theta) \triangleq \frac{P+\alpha\cos(\theta)Q}{P+Q+1}$$

.

To note is that this bound is only useful at low SIR, and it improves upon the scheme which treats interference as noise. The authors go on to propose a phase estimation scheme with help from primary transmitter pilot tones, which allows them avoid wasting energy and time by transmitting in useless sectors.

1.4 Conclusion

Recent results on information theoretic limits of wireless channels involving cognitive radios were outlined. The capacity region of the interference channel with degraded message sets, also known as the cognitive radio channel, has been found in the weak interference regime. This region demonstrates that behaving in a cognitive fashion, where primary and secondary messages are transmitted in a non-orthogonal fashion, is beneficial, in terms of achievable rates, for both the primary and secondary cognitive users. In particular, when the primary user's signal is known at the cognitive transmitter, it is possible to achieve better rates than those offered by spectral gap filling solutions. However, these results depend on both the non-causal knowledge as well as having perfect CSI at the transmitters. When this is not the case, as shown in the second part of this chapter, interference mitigation techniques may suffer in terms of rate. In channels employing cognitive radios there is therefore a tradeoff between learning the channel and interference in order to mitigate it, and avoiding interference altogether by spectral gap filling.

List of Abbreviations and Symbols

MIMO Multiple Input Multiple Output
CSI Channel State Information
SIR Signal to Interference Ratio
SNR Signal to Noise Ratio
MAC Multiple Access Channel
BC Broadcast Channel
IC-DMS Interference Channel with Degraded Message Sets
ARQ Automatic Repeat Request

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