Comparison of bi-directional relaying protocols

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***Abstract*— In a bi-directional relay channel, two nodes wish to exchange independent messages over a shared wireless channel with the help of a relay. In this paper, we derive achievable rate regions for four new half-duplex protocols and compare these to four existing half-duplex protocols and outer bounds. In time, our protocols consist of either two or three phases. In the two phase protocols, both users simultaneously transmit during the first phase and the relay alone transmits during the second phase, while in the three phase protocol the two users sequentially transmit followed by a transmission from the relay. The relay may forward information in one of four manners; we outline existing Amplify and Forward (AF) and Decode and Forward (DF) relaying schemes and introduce novel Compress and Forward (CF), and Mixed Forward schemes. We derive achievable rate regions for the CF and Mixed relaying schemes for the two and three phase protocols. Finally, we provide a comprehensive treatment of 8 possible half-duplex bi-directional relaying protocols in Gaussian noise, obtaining their respective achievable rate regions, outer bounds, and their relative performance under different SNR and relay geometries.**

***Index Terms*— bi-directional communication, achievable rate regions, compress and forward, relaying**

I. Introduction

Bi-directional relay channels, or wireless channels in which two nodes (a and b) wish to exchange independent messages with the help of a third relay node r, are both of fundamental and practical interest. This two-way channel [1] was first considered in [9] where an achievable rate region and an outer bound for the case in which nodes operate in *full-duplex* were obtained. In this work, we consider more practically feasible *half-duplex* communication in which a node may either transmit or receive, but not both simultaneously. Our goal is to determine spectrally efficient (measured in bits per channel use) transmission schemes and outer bounds for the half-duplex bi-directional relay channel and to compare their performance in a number of scenarios. These scenarios highlight the fact that different protocols may be optimal under different channel conditions. In this paper we consider two possible bi-directional relaying protocols which differ in their number of temporal *phases*, which may be described as:

Time Division Broadcast (TDBC) protocol: consists of the three phases $\mathsf{a} \to \mathsf{r}$, $\mathsf{b} \to \mathsf{r}$ and $\mathsf{a} \leftarrow \mathsf{r} \to \mathsf{b}$. Only a single node is transmitting at any time. Therefore the non-transmitting nodes may listen in and obtain "side information" about the other nodes' transmissions, which may be used for more efficient decoding, i.e. improved rates.

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TABLE I
Comparison between two protocols

Protocol	Side information	Number of phases	Interference
MABC	not present	2	present
TDBC	present	3	not present

Multiple Access Broadcast (MABC) protocol: consists of the two phases $\mathsf{a} \to \mathsf{r} \leftarrow \mathsf{b}$ and $\mathsf{a} \leftarrow \mathsf{r} \to \mathsf{b}$. Due to the half-duplex assumption, during phase 1 both source nodes are transmitting and thus cannot obtain any "side information" regarding the other nodes' transmission. It may nonetheless be spectrally efficient since it has less phases and may take advantage of the multiple-access channel in phase 1.

For each of the MABC and TDBC protocols, the relay may process and forward the received signals differently, depending on the relaying capabilities, knowledge and complexity. The two protocols and four relaying schemes' relative merits are summarized in Tables I and II and may be described as:

Amplify and Forward (AF): the relay r constructs its symbol by simple replication of the received symbol. The AF scheme requires no computation and carries the noise incurred in the first stage(s) forward during the latter relaying stage.

Decode and Forward (DF): the relay decodes both messages from nodes a and b before re-encoding them for transmission. The DF scheme requires the full codebooks of both a and b and a large amount of computation at the relay r.

Compress and Forward (CF): the relay *compresses* the received signal, which it then re-encodes and transmits. To do so, the relay requires the channel output distribution $p(y_{\mathsf{r}})$ at the relay rather than full codebooks.

Mixed Forward: the relay uses DF from $\mathsf{a} \to \mathsf{b}$ and uses CF from $\mathsf{a} \leftarrow \mathsf{b}$. For the mixed scheme, one of the codebooks and the channel output distribution are needed at the relay.

In [5], the DF TDBC protocol where network coding in is employed at the relay is considered. The works of [8] and [7] consider the MABC protocol, where Amplify and Denoise relaying schemes are introduced. In [4], achievable rate regions and outer bounds of the MABC protocol and the TDBC protocol with the DF relaying scheme are derived. The uni-directional CF relaying in the full-duplex channel is first introduced in [2].

Here we derive general achievable regions for the CF and Mixed relaying schemes in the TDBC and MABC protocols, which we compare with the regions and outer bounds derived in [4] in Gaussian noise. We present a comprehensive overview of the bi-directional relay channel which highlights the relative performance and tradeoffs of the different schemes under different channel conditions and relay processing capabilities.

TABLE II
COMPARISON BETWEEN FOUR RELAYING SCHEMES

Relaying	Complexity	Noise at relay	Relay needs
AF	very low	carried plus noise in rx	nothing
DF	high	perfectly eliminated	full codebooks
CF	low	carried plus distortion	$p(y_\mathsf{r})$
Mixed	moderate	partially carried	one codebook, $p(y_\mathsf{r})$

II. PRELIMINARIES

A. Notation and Definitions

We consider two terminal nodes a and b, and one relay node r. Terminal node a (resp. b) has its own message, w_a (resp. w_b), that it wishes to send to the opposite terminal node, node b (resp. a). The relay node r may assist in the bi-directional endeavor. This paper will determine achievable rate regions for bi-directional relaying protocols over half-duplex, discrete memoryless Gaussian channels.

We use R_i to denote the transmitted data rate of message w_i. We denote by $\Delta_\ell \geq 0$ the relative time duration of the ℓ^{th} phase, where $\sum_\ell \Delta_\ell = 1$. During phase ℓ we use $X_i^{(\ell)}$ to denote the input distribution and $Y_i^{(\ell)}$ to denote the distribution of the received signal of node i. Because of the half-duplex constraint, not all nodes transmit/receive during all phases and we use the symbol $\varnothing$ to denote that there is no input or no output at a particular node during a particular phase. The half-duplex constraint forces either $X_i^{(\ell)} = \varnothing$ or $Y_i^{(\ell)} = \varnothing$ for all ℓ phases. We will be constructing Compress and Forward schemes in which received signals are compressed or quantized before being re-transmitted. We let $\hat{Y}_i^{(\ell)}$ denote the quantized channel output during phase ℓ of node i. In this work, Q will denote a discrete time-sharing random variable with distribution $p(q)$.

For convenience, we drop the notation $\varnothing$ from the mutual information terms when a node is not transmitting or receiving. For example, $I(X_\mathsf{a}^{(1)}; Y_\mathsf{r}^{(1)}) = I(X_\mathsf{a}^{(1)}; Y_\mathsf{r}^{(1)} | X_\mathsf{b}^{(1)} = X_\mathsf{r}^{(1)} = \varnothing)$ in the TDBC protocol.

B. Previous results

We use the following outer bounds and achievable regions of decode and forward protocols, derived in [4] for comparison purposes in Sections IV and V.

Theorem 1: (Outer bound) The capacity region of the half-duplex bi-directional relay channel with the MABC protocol is outer bounded by

$$R_\mathsf{a} \leq \min\{\Delta_1 I(X_\mathsf{a}^{(1)}; Y_\mathsf{r}^{(1)} | X_\mathsf{b}^{(1)}, Q), \Delta_2 I(X_\mathsf{r}^{(2)}; Y_\mathsf{b}^{(2)} | Q)\} \quad (1)$$

$$R_\mathsf{b} \leq \min\{\Delta_1 I(X_\mathsf{b}^{(1)}; Y_\mathsf{r}^{(1)} | X_\mathsf{a}^{(1)}, Q), \Delta_2 I(X_\mathsf{r}^{(2)}; Y_\mathsf{a}^{(2)} | Q)\} \quad (2)$$

over all joint distributions $p(q)p^{(1)}(x_\mathsf{a}|q)p^{(1)}(x_\mathsf{b}|q)p^{(2)}(x_\mathsf{r}|q)$ with $|\mathcal{Q}| \leq 4$. □

An achievable rate region of the DF MABC protocol, an outer bound of the TDBC protocol and an achievable rate region of the DF TDBC protocol are provided in Theorem 2, 4, and 3 in [4], respectively.

III. ACHIEVABLE RATE REGIONS FOR COMPRESS AND FORWARD AND MIXED PROTOCOLS

Remark : Due to space constraints, the proofs for the Theorems 2, 3, 4, and 5 are provided in [3].

A. MABC Protocol

Theorem 2: An achievable region of the half-duplex bi-directional relay channel with the compress and forward MABC protocol is the union of

$$R_\mathsf{a} \leq \Delta_1 I(X_\mathsf{a}^{(1)}; \hat{Y}_\mathsf{r}^{(1)} | X_\mathsf{b}^{(1)}, Q) \quad (3)$$

$$R_\mathsf{b} \leq \Delta_1 I(X_\mathsf{b}^{(1)}; \hat{Y}_\mathsf{r}^{(1)} | X_\mathsf{a}^{(1)}, Q) \quad (4)$$

subject to

$$\Delta_1 I(Y_\mathsf{r}^{(1)}; \hat{Y}_\mathsf{r}^{(1)} | X_\mathsf{b}^{(1)}, Q) \leq \Delta_2 I(X_\mathsf{r}^{(2)}; Y_\mathsf{b}^{(2)} | Q) \quad (5)$$

$$\Delta_1 I(Y_\mathsf{r}^{(1)}; \hat{Y}_\mathsf{r}^{(1)} | X_\mathsf{a}^{(1)}, Q) \leq \Delta_2 I(X_\mathsf{r}^{(2)}; Y_\mathsf{a}^{(2)} | Q) \quad (6)$$

over all joint distributions, $p(q)p^{(1)}(x_\mathsf{a}|q)p^{(1)}(x_\mathsf{b}|q)$ $p^{(1)}(y_\mathsf{r}|x_\mathsf{a}, x_\mathsf{b})p^{(1)}(\hat{y}_\mathsf{r}|y_\mathsf{r})p^{(2)}(x_\mathsf{r}|q)$ with $|\mathcal{Q}| \leq 7$ □

Theorem 3: An achievable region of the half-duplex bi-directional relay channel with the mixed forward MABC protocol is the union of

$$R_\mathsf{a} \leq \min\{\Delta_1 I(X_\mathsf{a}^{(1)}; Y_\mathsf{r}^{(1)} | Q), \quad (7)$$
$$\Delta_2 I(X_\mathsf{r}^{(2)}; Y_\mathsf{b}^{(2)} | Q) - \Delta_1 I(Y_\mathsf{r}^{(1)}; \hat{Y}_\mathsf{r}^{(1)} | X_\mathsf{b}^{(1)}, Q)\}$$

$$R_\mathsf{b} \leq \Delta_1 I(X_\mathsf{b}^{(1)}; \hat{Y}_\mathsf{r}^{(1)} | X_\mathsf{a}^{(1)}, Q) \quad (8)$$

subject to

$$\Delta_1 I(Y_\mathsf{r}^{(1)}; \hat{Y}_\mathsf{r}^{(1)} | X_\mathsf{a}^{(1)}, Q) \leq \Delta_2 I(X_\mathsf{r}^{(2)}; Y_\mathsf{a}^{(2)} | Q) \quad (9)$$

over all joint distributions, $p(q)p^{(1)}(x_\mathsf{a}|q)p^{(1)}(x_\mathsf{b}|q)$ $p^{(1)}(y_\mathsf{r}|x_\mathsf{a}, x_\mathsf{b})p^{(1)}(\hat{y}_\mathsf{r}|y_\mathsf{r})p^{(2)}(x_\mathsf{r}|q)$ with $|\mathcal{Q}| \leq 6$. □

The mixed MABC region in Theorem 3 is outer bounded by the DF MABC region in Theorem 2 in [4]. In the mixed MABC protocol, the relay r has to be able to decode w_a correctly after phase 1 without any information about w_b. If node a can decode w_b from a compressed version of y_r and knowledge of w_a, then by the information processing inequality, node r can decode w_b from y_r and w_a. Although the mixed MABC protocol does not perform as well as the DF MABC protocol, its possible benefit lies in that it only requires the relay r to possess *one* of the codebooks of a and b. Therefore, in the event that relay r has one of the codebooks of the terminal nodes, by employing the Mixed MABC protocol one can achieve a rate region which outperforms that of the CF MABC protocol, which requires no terminal node codebooks.

B. TDBC Protocol

Theorem 4: An achievable region of the half-duplex bi-directional relay channel with the compress and forward TDBC protocol is the union of

$$R_\mathsf{a} \leq \Delta_1 I(X_\mathsf{a}^{(1)}; \hat{Y}_\mathsf{r}^{(1)}, Y_\mathsf{b}^{(1)} | Q) \quad (10)$$

$$R_\mathsf{b} \leq \Delta_2 I(X_\mathsf{b}^{(2)}; \hat{Y}_\mathsf{r}^{(2)}, Y_\mathsf{a}^{(2)} | Q) \quad (11)$$

subject to

$$\Delta_1 I(Y_{\mathsf{r}}^{(1)};\hat{Y}_{\mathsf{r}}^{(1)}|Q)+\Delta_2 I(Y_{\mathsf{r}}^{(2)};\hat{Y}_{\mathsf{r}}^{(2)}|X_{\mathsf{b}}^{(2)},Q) \leq \Delta_3 I(X_{\mathsf{r}}^{(3)};Y_{\mathsf{b}}^{(3)}|Q) \quad (12)$$

$$\Delta_2 I(Y_{\mathsf{r}}^{(2)};\hat{Y}_{\mathsf{r}}^{(2)}|Q)+\Delta_1 I(Y_{\mathsf{r}}^{(1)};\hat{Y}_{\mathsf{r}}^{(1)}|X_{\mathsf{a}}^{(1)},Q) \leq \Delta_3 I(X_{\mathsf{r}}^{(3)};Y_{\mathsf{a}}^{(3)}|Q) \quad (13)$$

over all joint distributions, $p(q)p^{(1)}(x_{\mathsf{a}}|q)p^{(1)}(y_{\mathsf{r}}|x_{\mathsf{a}})$ $p^{(1)}(\hat{y}_{\mathsf{r}}|y_{\mathsf{r}})p^{(2)}(x_{\mathsf{b}}|q)p^{(2)}(y_{\mathsf{r}}|x_{\mathsf{b}})p^{(2)}(\hat{y}_{\mathsf{r}}|y_{\mathsf{r}})p^{(3)}(x_{\mathsf{r}}|q)$ with $|\mathcal{Q}| \leq 7$. □

When the h_{a} and h_{b} links are of different strength, a scheme in which one link uses CF and the other uses DF may provide a larger rate region than if both links use CF. In the next theorem, we provide a rate region for a TDBC scenario in which the forward link uses DF and the reverse link uses CF.

Theorem 5: An achievable region for the half-duplex bi-directional relay channel with a mixed TDBC protocol is the union of

$$R_{\mathsf{a}} \leq \min\big\{\Delta_1 I(X_{\mathsf{a}}^{(1)};Y_{\mathsf{r}}^{(1)}|Q), \Delta_1 I(X_{\mathsf{a}}^{(1)};Y_{\mathsf{b}}^{(1)}|Q)+ \Delta_3 I(X_{\mathsf{r}}^{(3)};Y_{\mathsf{b}}^{(3)}|Q) - \Delta_2 I(Y_{\mathsf{r}}^{(2)};\hat{Y}_{\mathsf{r}}^{(2)}|X_{\mathsf{b}}^{(2)},Q)\big\} \quad (14)$$

$$R_{\mathsf{b}} \leq \Delta_2 I(X_{\mathsf{b}}^{(2)};\hat{Y}_{\mathsf{r}}^{(2)},Y_{\mathsf{a}}^{(2)}|Q) \quad (15)$$

subject to

$$\Delta_2 I(Y_{\mathsf{r}}^{(2)};\hat{Y}_{\mathsf{r}}^{(2)}|Q) \leq \Delta_3 I(X_{\mathsf{r}}^{(3)};Y_{\mathsf{a}}^{(3)}|Q) \quad (16)$$

over all joint distributions, $p(q)p^{(1)}(x_{\mathsf{a}}|q)p^{(2)}(x_{\mathsf{b}}|q)p^{(2)}(y_{\mathsf{r}}|x_{\mathsf{b}})$ $p^{(2)}(\hat{y}_{\mathsf{r}}|y_{\mathsf{r}})p^{(3)}(x_{\mathsf{r}}|q)$ with $|\mathcal{Q}| \leq 6$. □

IV. Gaussian Case

We apply the previous results to the Gaussian channel. Since strong typicality does not apply to continuous random variables, the achievable regions from the theorems in the previous section do not directly apply to continuous domains. However, for the Gaussian input distributions which we will assume in the following, the Markov lemma of [6], which generalizes the Markov lemma to the continuous domains, ensures that the achievable regions in the previous section hold in the Gaussian case.

The corresponding Gaussian channel model is:

$$Y_{\mathsf{a}}[m] = h_{\mathsf{ra}}X_{\mathsf{r}}[m] + h_{\mathsf{ba}}X_{\mathsf{b}}[m] + Z_{\mathsf{a}}[m] \quad (17)$$

$$Y_{\mathsf{b}}[m] = h_{\mathsf{rb}}X_{\mathsf{r}}[m] + h_{\mathsf{ab}}X_{\mathsf{a}}[m] + Z_{\mathsf{b}}[m] \quad (18)$$

$$Y_{\mathsf{r}}[m] = h_{\mathsf{ar}}X_{\mathsf{a}}[m] + h_{\mathsf{br}}X_{\mathsf{b}}[m] + Z_{\mathsf{r}}[m] \quad (19)$$

where $X_{\mathsf{a}}[m]$, $X_{\mathsf{b}}[m]$ and $X_{\mathsf{r}}[m]$ follow the input distributions $X_i^{(\ell)} \sim \mathcal{N}(0,P_i)$, where $m \in [n\sum_{j=0}^{i-1}\Delta_j+1, n\sum_{j=0}^{i}\Delta_j]$ and $\mathcal{N}(\mu,\sigma^2)$ denotes a Gaussian variable with mean μ and variance σ^2, and ℓ corresponds to the appropriate phase. If node i is in the receiving mode, the input alphabet is $\varnothing$ rather than an element of $\mathbb{C}$. That is, $X_i[m] = \varnothing$ means that the input symbol does not exist in the above mathematical channel model. For example, in the first phase of the TDBC protocol, $X_{\mathsf{r}}[m] = X_{\mathsf{b}}[m] = \varnothing$. Hence, the corresponding channel model is :

$$Y_{\mathsf{b}}[m] = h_{\mathsf{ab}}X_{\mathsf{a}}[m] + Z_{\mathsf{b}}[m] \quad (20)$$

$$Y_{\mathsf{r}}[m] = h_{\mathsf{ar}}X_{\mathsf{a}}[m] + Z_{\mathsf{r}}[m]. \quad (21)$$

h_{ij} is the effective channel gain between transmitter i and receiver j, which is modeled as a complex number. We assume that the channel is reciprocal such that $h_{ij} = h_{ji}$ and each node is fully aware of h_{ar}, h_{br} and h_{ab} (i.e. we have full CSI). The noise at all receivers is of unit power, additive, white Gaussian, complex and circularly symmetric. For convenience of analysis, we also define the function $C(x) := \log_2(1+x)$.

For the analysis of the Compress and Forward scheme, we assume $\hat{Y}_{\mathsf{r}}^{(\ell)}$ are zero mean Gaussians and define $P_y^{(\ell)} := E[(Y_{\mathsf{r}}^{(\ell)})^2]$, $P_{\hat{y}}^{(\ell)} := E[(\hat{Y}_{\mathsf{r}}^{(\ell)})^2]$ and $\sigma_y^{(\ell)} := E[\hat{Y}_{\mathsf{r}}^{(\ell)}Y_{\mathsf{r}}^{(\ell)}]$. Then the relation between the received Y_{r} and the compressed $\hat{Y}_{\mathsf{r}}$ are given by the following equivalent channel model:

$$\hat{Y}_{\mathsf{r}}[m] = h_{\mathsf{r\hat{r}}}Y_{\mathsf{r}}[m] + Z_{\mathsf{\hat{r}}}[m] \quad (22)$$

where $h_{\mathsf{r\hat{r}}} = \frac{\sigma_y^{(\ell)}}{P_y^{(\ell)}}$ and $Z_{\mathsf{\hat{r}}} \sim \mathcal{N}(0, P_{\hat{y}}^{(\ell)} - \frac{(\sigma_y^{(\ell)})^2}{P_y^{(\ell)}})$.

We consider four different relaying schemes (i.e. ways in which the relay processes and forwards the received signal) for each MABC and TDBC bi-directional protocol: *Amplify and Forward (AF)*, *Decode and Forward (DF)*, *Compress and Forward (CF)*, and *Mixed Forward (Mixed)*. In addition to achievable rate regions, we apply outer bounds of the MABC and TDBC protocols to the Gaussian channel.

For example, an achievable rate region of the AF MABC protocol is given by:

$$R_{\mathsf{a}} \leq \frac{1}{2}C\left(\frac{|h_{\mathsf{ar}}|^2|h_{\mathsf{br}}|^2P_{\mathsf{a}}P_{\mathsf{r}}}{|h_{\mathsf{ar}}|^2P_{\mathsf{a}}+|h_{\mathsf{br}}|^2P_{\mathsf{b}}+|h_{\mathsf{br}}|^2P_{\mathsf{r}}+1}\right) \quad (23)$$

$$R_{\mathsf{b}} \leq \frac{1}{2}C\left(\frac{|h_{\mathsf{ar}}|^2|h_{\mathsf{br}}|^2P_{\mathsf{b}}P_{\mathsf{r}}}{|h_{\mathsf{ar}}|^2P_{\mathsf{a}}+|h_{\mathsf{br}}|^2P_{\mathsf{b}}+|h_{\mathsf{ar}}|^2P_{\mathsf{r}}+1}\right). \quad (24)$$

For the other cases, we can easily find data rate regions in the Gaussian case to apply the previous Theorems to the Gaussian channel.

V. Achievable regions in the Gaussian channel

A. Achievable rate region comparisons

We compare the achievable rate regions and outer bounds of the 10 aforementioned protocols for symmetric source to relay channel gains at transmit SNRs of $0, 20$ and 50dB and $h_{\mathsf{ar}} = h_{\mathsf{br}} = 1$ (Figs. 1, 2, 3). Rate regions for difference channel gains may be found in [3].

In the low SNR regime, the DF MABC protocol dominates the other protocols. The MABC protocol in general outperforms the TDBC protocol as the benefits of side information and reduced interference are relatively small in this regime. The DF scheme outperforms the other schemes since the relatively large amount of noise in the first phase (and the second phase in the TDBC protocol) can be eliminated in the DF scheme, which cannot be done using the other schemes. In contrast, the DF TDBC protocol dominates the other protocols at high SNR since the direct link is strong enough to convey information in this regime.

In the TDBC protocol, the CF scheme does not outperform the DF scheme since it lacks the multiple access gain seen by the DF and has reduced efficiency due to the channel's half-duplex nature. However, under the MABC protocol, the CF scheme outperforms the DF scheme in the high SNR regime.

This is because the interfering transmissions of the terminal nodes affect the DF MABC scheme due to the multiple-access nature but not the CF scheme (as it must not decode the signals).

The achievable rate region of the Mixed TDBC protocol lies between the CF TDBC protocol and the DF TDBC protocol. In the TDBC protocol, $\max_{R_\mathsf{b}} R_\mathsf{a}^{MIX} = \max_{R_\mathsf{b}} R_\mathsf{a}^{DF}$ and $\max_{R_\mathsf{a}} R_\mathsf{b}^{MIX} = \max_{R_\mathsf{a}} R_\mathsf{b}^{CF}$, where R_a^{MIX} (resp. R_b^{MIX}) is the data rate of node a (resp. b) in the Mixed scheme. Here the max is taken over all rates in the achievable rate regions. The $\max R_\mathsf{a}^{MIX}$ (resp. $\max R_\mathsf{b}^{MIX}$) is achieved by taking $\Delta_2 = 0$ (resp. $\Delta_1 = 0$). The rates R_a^{DF} and R_b^{CF} are similarly defined and $\max R_\mathsf{a}^{DF}$ and $\max R_\mathsf{b}^{CF}$ are achieved in an analogous manner. Therefore, the point $(\max R_\mathsf{a}^{DF}, 0)$ lies in both the Mixed scheme and the DF scheme while the point $(0, \max R_\mathsf{b}^{CF})$ lies in both the Mixed scheme and the CF scheme. While in the TDBC scheme a particular rate may be set to 0 by indirectly setting the appropriate interval Δ_i to 0, in the MABC protocol this is not possible. In the MABC protocol, even when $R_\mathsf{a} = 0$ or $R_\mathsf{b} = 0$, the transmit power (P_a or P_b) remains constant in the first phase and does not decrease to 0, acting as additional noise for the opposite transmission. Therefore, $\max_{R_\mathsf{b}} R_\mathsf{a}^{MIX} \leq \max_{R_\mathsf{b}} R_\mathsf{a}^{DF}$. This interference seen in MABC protocols is especially pronounced in the high SNR regime, where the gap between the intercept points of the Mixed scheme and the DF scheme is seen to grow as the interference increases (with SNR). We note that this effect is due to our assumption that transmitters blast at full power regardless of the rate of transmission. If we were to allow for optimization of the transmission power, larger achievable rate regions for the Mixed MABC protocol could result.

In Figs. 1, 2 and 3, the AF scheme is always outer bounded by the CF scheme. However, this is not true for all channel conditions [3].

In the low SNR regime, the achievable region of the DF MABC protocol and the outer bound of the MABC protocol are tight, while in the high SNR regime, the achievable region of the CF MABC protocol is tight. For the TDBC protocol, there is a very small gap between the achievable region of the DF TDBC protocol and the outer bound of the TDBC protocol since interference is not an issue for the TDBC protocol and hence decoding is thus, intuitively, near optimal.

B. Maximum Sum Data Rate

In this subsection we plot the maximum sum-rate $R_\mathsf{a} + R_\mathsf{b}$ as a function of the transmit SNR for the symmetric ($h_\mathsf{a} = h_\mathsf{b} = 1$) and two asymmetric cases ($h_\mathsf{a} = 0.5, h_\mathsf{b} = 2$ and vice versa). As expected, different schemes are optimal for different SNR values. The sum-rate is basically proportional to the SNR in dB scale since the sum-rate is roughly the logarithm of the SNR. In Fig. 4 around 12 dB and in Fig. 5, 6 around 30 dB, the relative performance of the CF MABC protocol and the DF MABC protocol changes. At lower SNRs, the DF MABC protocol is better, while at higher SNRs, the CF MABC protocol is better. The AF MABC protocol is always worse than the CF MABC protocol in the symmetric case (Fig. 4). However, in the asymmetric cases, this is no longer true.

In the TDBC protocol, the sum-rate of the mixed TDBC protocol lies between the DF scheme and the CF scheme. An interesting point is that in the asymmetric cases (Fig. 5, 6), the slope for the DF MABC scheme changes at around 25 dB. To explain this, we note that in the low SNR regime, interference from the weaker channel is relatively small compared to the noise in the channel. However, as the transmit SNR increases, the interference becomes large relatively larger and the last (sum-rate) inequality in Theorem 2 in [4] then significantly affects and limits the maximal sum-rate.

C. Relay position

In this subsection we plot the maximum sum-rate $R_\mathsf{a} + R_\mathsf{b}$ as a function of the relay position $d_\mathsf{ar} = \zeta d_\mathsf{ab}$ $(0 < \zeta < 1)$ when the relay r is located on the line between a and b. Thus, $d_\mathsf{br} = (1-\zeta)d_\mathsf{ab}$. We apply $h_\mathsf{ab} = 0.2$ and $P_\mathsf{a} = P_\mathsf{b} = P_\mathsf{r} = 20$ dB and let $|h_{ij}|^2 = k/d_{ij}^{3.8}$ for k constant and a path-loss exponent of 3.8. We consider three constraints on the sum-rate in the three Figs. 7, 8 and 9. In the first, the sum-rate is maximized without any additional constraints. For the latter two we consider more realistic scenarios in which the sum-rate is constrained. In many communication systems, uplink and downlink rates are not equal. More specifically, it is not uncommon for the dowlink rate to be 2 to 4 times greater than that of the uplink. In Figs. 8 and 9 we plot the maximal sum-rate of the protocols under a $\sigma = R_\mathsf{a}/R_\mathsf{b}$ rate ratio restriction.

For the MABC protocol, if the relay location is biased, then the DF MABC protocol outperforms the CF MABC protocol and the AF MABC protocol outperforms the CF MABC protocol. This effect is more explicit in the constrained cases. In contrast, for the TDBC protocol, in order of increasing complexity, (and performance), the relaying schemes are AF, CF, Mixed and DF in all cases. As expected, the sum-rate for the mixed MABC protocol is much worse than those of the other protocols.

The sum-rate plot for the mixed protocol is not symmetric since it uses different forwarding for each link. In addition, the sum-rates in the constrained case where $\sigma = 2$ (Fig. 9) are asymmetric even for non-mixed protocols. The intuitive reason for this is that the rates are constrained in an asymmetric way, and hence a particular, non-midpoint distance will be optimal even for CF and DF forwarding schemes. The performance of the $\sigma = 2$ sum-rate for the DF MABC protocol is remarkably asymmetric where it peaks and almost touches the outer bound at $\zeta = 0.6$. These plots and region optimizations may be useful when determining the optimal relay position subject to particular rate constraints.

VI. Conclusion

In this paper, we derived achievable rate regions for 4 new half-duplex bi-directional relaying protocols. We specialized 8 different achievable regions and 2 outer bounds to the Gaussian case and numerically evaluated them under various channel conditions. For the MABC protocol, DF or CF is the optimal scheme, depending on the given channel and SNR regime. In the TDBC protocol, the relative performance of the forwarding schemes agrees with the amount of information

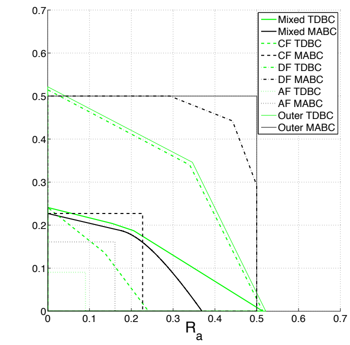

Fig. 1. Comparison of bi-directional regions with $h_{\mathsf{ar}} = h_{\mathsf{br}} = 1$, $h_{\mathsf{ab}} = 0.2$, $P_{\mathsf{a}} = P_{\mathsf{b}} = P_{\mathsf{r}} = 1$ and $N_{\mathsf{a}} = N_{\mathsf{b}} = N_{\mathsf{r}} = 1$.

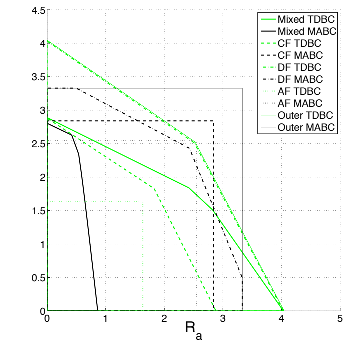

Fig. 2. Comparison of bi-directional regions with $h_{\mathsf{ar}} = h_{\mathsf{br}} = 1$, $h_{\mathsf{ab}} = 0.2$, $P_{\mathsf{a}} = P_{\mathsf{b}} = P_{\mathsf{r}} = 100$ and $N_{\mathsf{a}} = N_{\mathsf{b}} = N_{\mathsf{r}} = 1$.

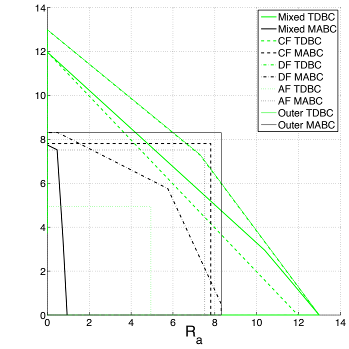

Fig. 3. Comparison of bi-directional regions with $h_{\mathsf{ar}} = h_{\mathsf{br}} = 1$, $h_{\mathsf{ab}} = 0.2$, $P_{\mathsf{a}} = P_{\mathsf{b}} = P_{\mathsf{r}} = 10^5$ and $N_{\mathsf{a}} = N_{\mathsf{b}} = N_{\mathsf{r}} = 1$.

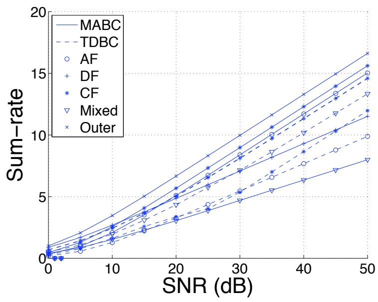

Fig. 4. Maximum sum-rate of the 8 bi-directional protocols and 2 outer bounds at different SNR. Here $h_{\mathsf{ar}} = h_{\mathsf{br}} = 1$ and $h_{\mathsf{ab}} = 0.2$

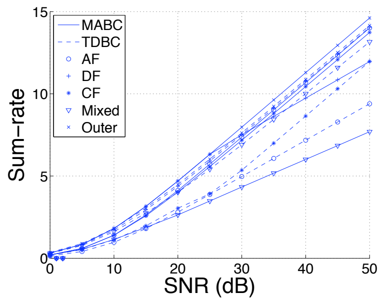

Fig. 5. Maximum sum-rate of the 8 bi-directional protocols and 2 outer bounds at different SNR. Here $h_{\mathsf{ar}} = 0.5$, $h_{\mathsf{br}} = 2$ and $h_{\mathsf{ab}} = 0.2$

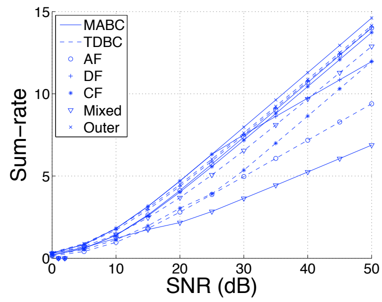

Fig. 6. Maximum sum-rate of the 8 bi-directional protocols and 2 outer bounds at different SNR. Here $h_{\mathsf{ar}} = 2$, $h_{\mathsf{br}} = 0.5$ and $h_{\mathsf{ab}} = 0.2$

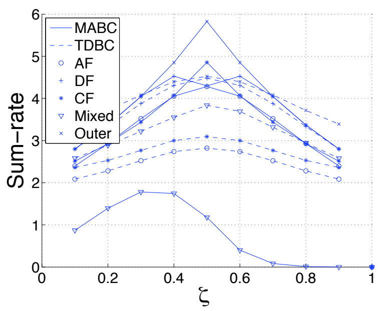

Fig. 7. Maximum sum-rate of the 8 bi-directional protocols at different relay position. Here $h_{\mathsf{ab}} = 0.2$ and no rate constraints.

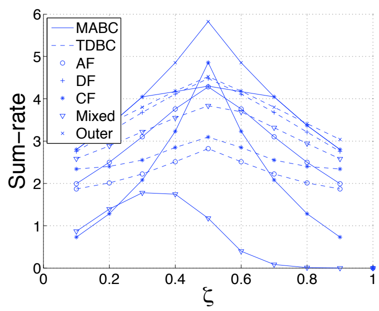

Fig. 8. Maximum sum-rate of the 8 bi-directional protocols at different relay position. Here $h_{\mathsf{ab}} = 0.2$ and $\sigma = 1$ ($R_{\mathsf{a}} = R_{\mathsf{b}}$).

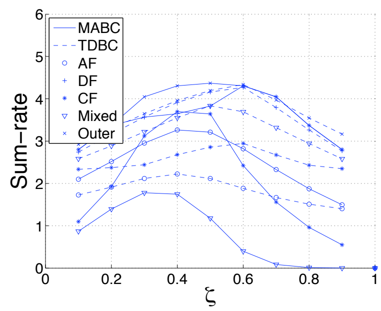

Fig. 9. Maximum sum-rate of the 8 bi-directional protocols at different relay position. Here $h_{\mathsf{ab}} = 0.2$ and $\sigma = 2$ ($R_{\mathsf{a}} = 2R_{\mathsf{b}}$).

and complexity available at the relay, that is, in order of increasing complexity, (and performance), the protocols are AF, CF, Mixed and DF. In general, the MABC protocol outperforms the TDBC protocol in the low SNR regime, while the reverse is true in the high SNR regime.

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