The Multiplexing Gain of MIMO X-Channels with Partial Transmit Side-Information

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***Abstract*— In this paper, we obtain the scaling laws of the sum-rate capacity of a MIMO X-channel, a 2 independent sender, 2 independent receiver channel with messages from each transmitter to each receiver, at high signal to noise ratios (SNR). The X-channel has sparked recent interest in the context of cooperative networks and it encompasses the interference, multiple access, and broadcast channels as special cases. Here, we consider the case with partially cooperative transmitters in which asymmetric side-information (in the form of a codeword) is available at one of the transmitters. It is proved that when there are M antennas at all four nodes, the sum-rate scales like $2M \log \mathrm{SNR}$ which is in sharp contrast to $\left[\lfloor \frac{4M}{3} \rfloor, \frac{4M}{3}\right] \log \mathrm{SNR}$ for non-cooperative X-channels [14], [11]. This further proves that, in terms of sum-rate scaling at high SNR, partial side-information at one of the transmitters and full side-information at both transmitters are equivalent in the MIMO X-channel.**

I. Introduction

Cooperation in wireless networks has sparked much recent interest in the research community. Having wireless nodes cooperate at the transmitting and/or receiving end, can, for example, improve rates, diversity, or power utilization. How much gain cooperation provides depends on a number of factors, including the signal to noise ratio (SNR) and the amount of side information at the transmitters. In this work, we look at the value of partial transmitter cooperation, in terms of the scaling law of the sum-rate in the MIMO X-channel at high SNR .[1]

The multiplexing gain of the sum-rate of the MIMO X-channel has been recently studied [14], [11], [12], [15]. The MIMO X-channel is a simple 2 transmitter (2 Tx), 2 receiver (2 Rx) channel in which each Tx has a message for each Rx. Its study is of theoretical interest, as it encompasses classical channels such as the interference, the multiple access and the broadcast channels. While the classical MIMO X channel forbids cooperation between the two Tx, in this work we allow for transmitter cooperation, in the form of transmitter *side-information.*

The three channels shown in Fig. 1 illustrate three multiuser MIMO channels with different amounts of transmitter side-information. In Fig. 1(1), the MIMO X-channel is illustrated[2]. The two transmitters, Tx 1 and Tx 2 operate independently, and each wishes to send messages to each of the two non-cooperating receivers Rx 1 and Rx 2. The messages are numbered 11, 12, 21 and 22, with the convention that indices will follow the form (Tx# Rx#). We note that the MIMO interference channel is embedded in the MIMO X channel and may be obtained by eliminating the cross-over messages 12 and 21. The messages 11, 12, 21 and 22 will be simultaneously transmitted at the rates R_{11}, R_{12}, R_{21} and R_{22} respectively. The sum-rate achieved by this tuple is defined to be $R \stackrel{\triangle}{=} R_{11} + R_{12} + R_{21} + R_{22}$.

In Fig. 1(2), the MIMO cognitive X-channel is depicted. There, side information S about the messages or encodings of Tx 1 are available to Tx 2. The side information S could be one of many things: (i) message 11 (ii) message 12 (iii) both messages 11 and 12. Motivated by the Gaussian noise channel and dirty-paper coding techniques, the side information could also be a (iv) *codeword* of Tx 1. Depending on the different types and amounts of side-information S available at Tx 2, different regions, and different multiplexing gains will be possible for the cognitive X channel [6], [11]. Fig. 1(3) shows a MIMO broadcast channel in which both Txs share all their messages (full, symmetric side-information, or full cooperation) [16], [1] with $2M$ transmit antennas and two independent Rxs with M antennas each.

The multiplexing gain of the channel depicted in Fig. $1(i)$ ($i = 1, 2, 3$) is defined as the limit of the ratio of the maximal achieved sum-rate R_i in the capacity region $\mathcal{C}_i$, to the $\log$(SNR)[3] as the SNR $\to \infty$, or

$$M_i = \lim_{\mathrm{SNR}\to\infty} \frac{\max R_i(\mathrm{SNR})}{\log(\mathrm{SNR})}.$$

The multiplexing gain of the channel of Fig. 1(1) has been recently studied [11], [14], [15]. The MIMO interference channel embedded in channel (1) is known to achieve a multiplexing gain of M [12]. This channel has no cross-over messages (messages 12 and 21). Interestingly, when cross-over information is present, as in the

[1]Although the achievable rate region we derive is for general channels, the multiplexing gain results hold only for Gaussian noise channels, hence we will often omit the term Gaussian for brevity.

[2]Throughout this work the term MIMO will denote that there are M antennas at each of the four nodes Tx 1, Tx 2, Rx 1 and Rx 2. In order to simplify results we will sometimes set $M = 1$, in which case we drop the word MIMO.

[3]Note that the usual factor $\frac{1}{2}$ is omitted in any rate expressions, but rather the number of times the sum-rate looks like $\log(SNR)$ is the multiplexing gain.

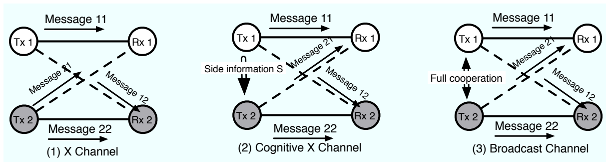


Fig. 1. Three 2×2 MIMO X-channels whose multiplexing gains are contrasted here. All Rxs decode independently. (1) MIMO X-channel with one message from each Tx to each Rx and no side-information. (2) MIMO cognitive X-channel with asymmetric side-information S: Tx 2 could be given (i) message 11 (ii) message 12 (iii) both message 11 and 12 or (iv) one of Tx 1's codewords. Tx 1 knows none of Tx 2's messages. (3) MIMO Broadcast channel: $2M$ transmitting antennas delivering messages to two independent M antenna Rxs.

MIMO X-channel of (1), the multiplexing gain lies in the range $[\lfloor \frac{4}{3}M \rfloor, \frac{4M}{3}]$ [11], [14], possibly improving upon the MIMO interference channel. The MIMO broadcast channel of Fig. 1(3) is known to have a multiplexing gain of $2M$, equal to the number of transmit, and receive antennas. In the broadcast channel, full, symmetric Tx side-information is used, that is, both Tx 1 and Tx 2 know *all* messages of the other. In this work, we show that such full transmit side-information is not strictly necessary to achieve the same sum-rate scaling of $2M$. We show that if the asymmetric side-information S is either cases (iii) or (iv) then the MIMO cognitive X-channel also achieves a sum-rate scaling of $2M$ as SNR $\to \infty$. The cases (i) and (ii) are considered in [6], [11].

In this work, we assume *non-causal* side-information, which may be either symmetric or asymmetric, but requires that all messages are known fully before transmission starts. This is in sharp contrast to the work [10], in which similar 2 Tx, 2 Rx channels with causal side-information are considered. In [10] it is shown that even if both the Txs and Rxs may cooperate using noisy links, in a causal fashion (although full duplex transmission is permitted) the multiplexing gain is always limited to 1 when $M = 1$. This is in sharp contrast to when non-causal side-information is present, resulting in a multiplexing gain of 2.

This paper is structured as follows. In section II we define the MIMO cognitive X channel and demonstrate an achievable rate region for case (iv), which we use to show that the multiplexing gain of the sum-rate is 2M. In section III we explore the effect of cross-over information on the multiplexing gain: we define and demonstrate that the multiplexing gain of the cognitive interference channel (where no cross-over messages are present) is 1, in contrast to the multiplexing gain of 2 seen in the cognitive X channel. These results allow us to compare the achievable rate regions of the cognitive and cognitive X channels in section IV at various SNRs. We conclude in section V. Due to space limitations, most of the proofs are deferred to [6].

II. The MIMO cognitive X channel

The MIMO cognitive X channel is shown in Fig. 2(a). The word cognitive stems from the work [5] in which an interference channel with asymmetric side-information between Txs is called a *cognitive radio channel* (also known as an interference channel with degraded message sets in [13], [17]). Like the cognitive radio channel, non-causal asymmetric side-information is assumed at one of the transmitters. We note that the channel under consideration could be motivated by situations other than cognitive radios and secondary spectrum sharing. In our figures, we denote side-information using a *double* arrow.

When speaking of a cognitive X channel one must explicitly specify the side information S. This side information could be one of a number of possibilities: (i) message 11, (ii) message 12, (iii), messages 11 and 12, or (iv) a codeword. In [11] the multiplexing gain of the cognitive MIMO X channel when the side information is a single message (cases (i) and (ii)) was recently shown to be in the range $[\lfloor \frac{3M}{2} \rfloor, \frac{3M}{2} \rfloor]$, where the lower bound was shown to be achieved through zero-forcing techniques alone. When both messages 11 and 12 are known at Tx 2, the channel contains the 2 Tx antenna Gaussian MIMO broadcast channel, which is known to contain 2 degrees of freedom (which can be obtained by setting messages 21 and 22 to have constant rates, and by using zero-forcing). In this work *we consider case (iv), where the single codeword M_{11} (encoding message 11) from Tx 1 is given to Tx 2.* We are interested not only in the multiplexing gain in case (iv), but also in the resulting achievable rate region; specifically what rates the cognitive user Tx 2 can support as SNR$\to \infty$.

Our physical channel is an interference channel [2] with direct channel coefficients of 1 and cross-over coefficients of α_{12} and α_{21}. The M-dimensional transmitted random variables $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$ are received as the signals $Y_1 \in \mathcal{Y}_1$ and $Y_2 \in \mathcal{Y}_2$ in the sets according to the conditional distributions $p(y_1|x_1, x_2)$ and $p(y_2|x_1, x_2)$. Unlike the interference channel which has two messages: 11 and 22, in the MIMO X-channel there are four messages: 11, 12, 21 and 22, which are encoded as M_{11}, M_{12}, M_{21} and M_{22} respectively, and further transmitted as X_1 and X_2. We consider an additive white Gaussian noise channel,

$$Y_1 = X_1 + \alpha_{21} X_2 + N_1 \quad (1)$$

$$Y_2 = \alpha_{12} X_1 + X_2 + N_2, \quad (2)$$

where $N_1 \sim \mathcal{N}(0, N_1)$ and $N_2 \sim \mathcal{N}(0, N_2)$ are independent and we assume individual average transmit power constraints of P_1 (P_2) on X_1 (X_2 resp.).

Standard definitions of achievable rates and regions are employed [4], [5] . Although our achievable rate region will be defined for finite alphabet sets, in order to determine an achievable region for the Gaussian noise channel, specific forms of the random variables described in Thm. 1 are assumed. As in [3], [7], [9], Thm. 1 can readily be extended to memoryless channels with discrete time and continuous alphabets by finely quantizing the input, output, and interference variables (Gaussian in this case).

We now outline an achievable rate region for the Gaussian MIMO cognitive X-channel with the codeword M_{11} as asymmetric side information (case (iv)), which will be used to demonstrate a sum-rate scaling law of $2M$. From now on we present results for case (iv). The capacity region of the Gaussian MIMO broadcast channel [16] is achieved using Costa's dirty-paper coding techniques [3]. In the MIMO cognitive X-channel, at Tx 1, the encodings M_{11} and M_{12} may be jointly generated, for example using a dirty-paper like coding scheme. That is, one message may treat the other as non-causally known interference and code so as to mitigate it. At Tx 2, not only may the encodings M_{21} and M_{22} be jointly designed, but they may additionally use the codeword M_{11} as *a-priori* known interference. Thus, Tx 2 could encode M_{22} so as to potentially mitigate the interference Y_2 will experience from M_{11} as well as M_{21}. Let R_{11} be the rate from $M_{11} \to Y_1$, R_{12} from $M_{12} \to Y_2$, R_{21} from $M_{21} \to Y_1$ and R_{22} from $M_{22} \to Y_2$.

Theorem 1: Let $Z \triangleq (Y_1,\ Y_2,\ X_1,\ X_2,\ M_{11},\ M_{12},\ M_{21},\ M_{22})$, and let $\mathcal{P}$ be the set of distributions on Z that can be decomposed into the form

$$\begin{array}{c} p(m_{11}|m_{12})p(m_{12})p(m_{21})p(m_{22}|m_{11},m_{21}) \\ p(x_1|m_{11},m_{12})p(x_2|m_{11},m_{21},m_{22}) \\ p(y_1|x_1,x_2)p(y_2|x_1,x_2), \end{array} \tag{3}$$

where we additionally require $p(m_{12},m_{22}) = p(m_{12})p(m_{22})$. For any $Z \in \mathcal{P}$, let $S(Z)$ be the set of all tuples $(R_{11},R_{12},R_{21},R_{22})$ of non-negative real numbers such that:

$$\begin{aligned} R_{11} &\le I(M_{11};Y_1|M_{21}) - I(M_{11};M_{12}) \\ R_{21} &\le I(M_{21};Y_1|M_{11}) \\ R_{11}+R_{21} &\le I(M_{11},M_{21};Y_1) - I(M_{11};M_{12}) \end{aligned}$$

$$\begin{aligned} R_{12} &\le I(M_{12};Y_2|M_{22}) \\ R_{22} &\le I(M_{22};Y_2|M_{12}) - I(M_{22};M_{11},M_{21}) \\ R_{12}+R_{22} &\le I(M_{12},M_{22};Y_2) - I(M_{22};M_{11},M_{21}) \end{aligned}$$

Any element in the closure of $\cup_{Z\in\mathcal{P}} S(Z)$ is achievable.

Proof: The codebook generation, encoding, decoding schemes and formal probability of error analysis are deferred to the manuscript in preparation [6]. Heuristically, notice that the channel from $(M_{11},M_{21}) \to Y_1$ is a multiple access which employs dirty paper coding [3], reducing the rate R_{11} by $I(M_{11};M_{12})$ (like in Gel'fand-Pinsker [8] coding). Similarly, for the MAC $(M_{12},M_{22}) \to Y_2$ the encodings M_{12} and M_{22} are independent (this is true in particular in the Gaussian case of interest in the next subsection, and so we simplify our theorem by ensuring the condition $p(m_{12},m_{22}) = p(m_{12})p(m_{22})$) so that the regular MAC equations hold. The rate R_{22} is subject to a penalty of $I(M_{22};M_{11},M_{21})$ in order to guarantee finding an n-sequence m_{22} in the desired bin that is jointly typical with any *given* (m_{11},m_{21}). ■

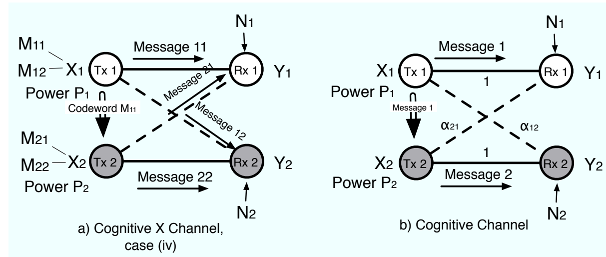


Fig. 2. Additive Gaussian noise interference channels with cross-over parameters α_{12}, α_{21}, transmitted encodings X_1, X_2 with expected transmit power limitations P_1 and P_2, and received signals Y_1 and Y_2. (a) Cognitive X channel: four messages encoded as M_{11}, M_{12}, M_{21}, M_{22}. M_{11} is the partial and asymmetric message knowledge at X_2. (b) Cognitive channel:two messages encoded as M_{11}, M_{22}. X_1 is the asymmetric side-information known at X_2.

A. MIMO Cognitive X-channel multiplexing gain is $2M$

We use the above achievable rate region to show that the sum-rate of the MIMO X-channel with *partial asymmetric side-information* has a multiplexing gain $2M$.

Corollary 2: Consider the MIMO additive Gaussian X-channel with partial asymmetric side-information described in eqns. (1), (2) and Fig. 2(a) with $P_1 = P_2 = P$. Then

$$\lim_{P\to\infty} \frac{\max R_{11}+R_{12}+R_{21}+R_{22}}{\log P} = 2M, \tag{4}$$

where the $\max$ is taken over all $(R_{11},R_{12},R_{21},R_{22}) \in \mathcal{C}_{X-cog}$, where $\mathcal{C}_{X-cog}$ is the capacity region of the MIMO cognitive X-channel.

Proof: We sketch the proof for $M = 1$, and defer details, as well as the more involved proof for general M to [6]. Roughly speaking, the general M case is proven by evaluating the same mutual information terms of Thm. 1, as done for $M = 1$, with the added complications (such as matrix inversions) that arise from considering vectors rather than scalars.

First, note that the multiplexing gain of the MIMO broadcast channel, whose capacity region outer bounds ours, with 2 transmit antennas and single receive antennas at the Rxs is 2. We will in fact prove that 2 is achievable using the scheme of Thm. 1. To do so, we specify forms for the variables, and then optimize the dirty paper coding parameters, similar to Costa's technique [3]. The Gaussian distributions we assume on all variables are of the form

$$\begin{array}{ll} M_{11} = U_{11} + \gamma_1 U_{12} & U_{11} \sim \mathcal{N}(0,P_{11}) \\ M_{12} = U_{12}, & U_{12} \sim \mathcal{N}(0,P_{12}) \\ M_{21} = U_{21}, & U_{21} \sim \mathcal{N}(0,P_{21}) \\ M_{22} = U_{22} + \gamma_2(U_{21} + a_{12}U_{11}) & U_{22} \sim \mathcal{N}(0,P_{22}) \end{array}$$

$$X_1 = U_{11} + U_{12}$$
$$X_2 = U_{21} + U_{22} + \sqrt{\tfrac{(1-\beta)P_2}{P_{11}}} U_{11}$$
$$Y_1 = \left(1 + a_{21}\sqrt{\tfrac{(1-\beta)P_2}{P_{11}}}\right) U_{11} + U_{12} + a_{21}(U_{21} + U_{22}) + N_1$$
$$Y_2 = \left(a_{12} + \sqrt{\tfrac{(1-\beta)P_2}{P_{11}}}\right) U_{11} + a_{12}U_{12} + (U_{21} + U_{22}) + N_2.$$

where $P_1 = P_{11} + P_{12}$ and $\beta P_2 = P_{21} + P_{22}$.

Here the variables $U_{11}, U_{12}, U_{21}, U_{22}$ are all independent, encoding the four messages to be transmitted. The β parameter divides power at Tx 2: βP_2 is used in transmitting its own messages, m_{21} and m_{22}, while the remainder of the power, $(1-\beta)P_2$ is used to reinforce the message encoded as m_{11} of Tx 1. The rates $R_1 = R_{11} + R_{21}$ and $R_2 = R_{12} + R_{22}$ to each Rx are calculated separately and each is maximized with respect to the relevant dirty-paper coding parameter (γ_1 for Tx 1, and γ_2 for Tx 2). The bounds of Thm. 1 may be evaluated by combining the appropriate determinants of sub-matrices of the overall covariance matrix $E[\Theta\Theta^T]$ where $\Theta \triangleq (M_{11}, M_{21}, M_{12}, M_{22}, Y_1, Y_2)$. The six bounds of Thm. 1 are evaluated explicitly in [6]; we simply demonstrate the resulting expressions for the sum-rate to Rx 1, R_1 as well as to Rx 2, R_2 in the expressions (5) and (6) resp. In order to simplify the expressions, we have set $\theta \triangleq \sqrt{\frac{(1-\beta)P_2}{P_{11}}}$. We could search for the γ_1 and γ_2 that jointly optimize $R_1 + R_2$. However, noticing that R_1 depends only on γ_1, we heuristically select γ_1 to maximize R_1, as (8) When we substitute this γ_1 into the bounds on Rx 1's total sum-rate we obtain the bound (7). Notice that an important cancellation occurs in the denominator of (7) when the optimal γ_1 is substituted.

$$\gamma_1 = \frac{P_{11}(1 + a_{21}\theta)}{P_{11}(1 + a_{21}\theta)^2 + \alpha_{21}^2 P_{22} + N_1}, \quad \gamma_2 = \frac{P_{22}}{P_{22} + N_2} \tag{8}$$

Although we could maximize R_2 with respect to γ_2, we use a simpler and more heuristic approach and simply minimize the denominator of the sum-rate $R_2 = R_{12} + R_{22}$ with respect to γ_2, which yields γ_2 as in (8). It is interesting to note that this is exactly the same dirty paper coding parameter as Costa derives. It is intuitively pleasing, and although it does not strictly maximize R_2 with respect to γ_2, as we will see shortly, it performs sufficiently well in the limit of large SNR, thus performs asymptotically optimally. If we fix P_{22} (does not scale with P), set $\beta = 1$ (or $\theta = 0$) and let $P_{11} = P_{12} = P_{21}$ all scale like P, subject to $P_{11} + P_{12} = P$ and $P_{21} + P_{22} = P$, then the bound on the total sum rate to both Rxs $R_{11} + R_{12} + R_{21} + R_{22}$ scales like $2\log P$. This can be seen by noting that γ_2 remains fixed and $\gamma_1 \to 1$ as $P \to \infty$. Keeping P_{22} fixed was crucial for achieving the $\log P$ scaling in R_1. Intuitively, this is because of the asymmetric message knowledge; the interference Tx 2 causes the Tx 1 is not mitigated. Keeping P_{22} constant still allows Tx 2 to dirty paper code, or mitigate the interference caused by M_{11} and M_{21} to Rx 2's signal Y_2, while causing asymptotically (as $P_{11}, P_{12}, P_{21} \to \infty$) negligible interference to Y_1. Setting $\beta = 1$, or $\theta = 0$ is also crucial in order to ensure the sum-rate to Rx 2 given in (6) is not killed by Tx 1's transmissions. ■

III. The cognitive interference channel

In the previous section we demonstrated that the scaling law of the sum-rate of the MIMO cognitive X channel, with *partial transmitter side-information* is $2M$, which is optimal in the limit as SNR $\to \infty$. In this section we investigate whether partial asymmetric side-information is always equivalent to full symmetric transmitter side information in terms of sum-rate scaling as SNR $\to \infty$. To do so we look at another channel with partial asymmetric side information at the transmitters: the recently explored cognitive interference channel (also known as the interference channel with degraded message sets [13] or the cognitive radio channel [5]), shown in Fig. 2(b). We consider the same additive Gaussian noise channel as in (1), (2). The only difference with the cognitive X-channel is the absence of cross-over messages 12 and 21. We will see that while partial asymmetric side information in the X-channel results in the same sum-rate scaling as a fully cooperative (at the transmitters) X-channel, the opposite is true of partial asymmetric side information in the interference channel: at high SNR its sum-rate scales like the interference channel. In other words, although partial side-information may help the interference channel in a medium SNR-regime [5], [13], at high SNR, one cannot improve the scaling law of the sum-rate. The Gaussian cognitive interference channel considered here is the same channel as that of [13], where its capacity region is derived for the case $\alpha_{21} \le 1$, and sum-rate capacity is found for $\alpha_{21} > 1$. The next theorem is a direct result of [13].

Theorem 3: Consider the Gaussian interference channel where Tx 2 has non-causal knowledge of the message of Tx 1, described in eqns. (1), (2) and Fig. 2(b) with $P_1 = P_2 = P$. Then

$$\lim_{P\to\infty} \frac{\max_{(R_1,R_2)\in\mathcal{C}} R_1 + R_2}{\log P} = 1, \tag{9}$$

where R_i corresponds to the rates from the i-th source to the i-th Rx, and $\mathcal{C}$ is the capacity region of the channel. □

The proof of this result is deferred to work [6], and employs eqns. (24), (25) and Corollary 4.1 of [13].

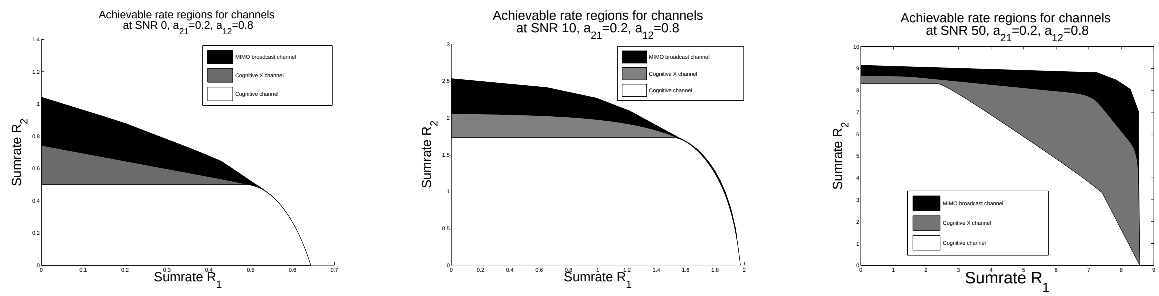
IV. Comparison of Cognitive and Cognitive X channel regions at various SNRs

In this section, we numerically evaluate the capacity region of the cognitive channel of Fig. 2(b) and [13] and compare it with the achievable region of the cognitive X channel described in Thm. 1 and Fig. 2(a) under our choice of variables, as well as the MIMO broadcast channel with 2 Tx antennas and 2 single antenna receivers. In doing so, we highlight the dependence of the rate region,

$$R_{11}+R_{21} \leq \frac{1}{2}\log_2\left(\frac{(P_{11})(P_{11}(1+a_{21}\theta)^2+P_{12}+\alpha_{21}^2(P_{21}+P_{22})+N_1)}{\gamma_1^2 P_{12}(P_{11}(1+a_{21}\theta)^2+\alpha_{21}^2 P_{22}+N_1)-2\gamma_1 P_{11}P_{12}(1+a_{21}\theta)+P_{11}(P_{12}+\alpha_{21}^2 P_{22}+N_1)}\right) \quad (5)$$

$$R_{12}+R_{22} \leq \frac{1}{2}\log_2\left(\frac{(P_{11}(a_{12}+\theta)^2+a_{12}^2 P_{12}+P_{21}+P_{22}+N_2)(\gamma_1^2 P_{12}(P_{22}+\gamma_2^2(a_{12}+\theta)^2 P_{11})+P_{11}P_{22})}{(P_{11}+\gamma_1^2 P_{12})(\gamma_2^2((P_{22}+N_2)(P_{21}+a_{12}^2)+P_{11}P_{21}\theta^2)+\gamma_2(-2P_{22}(P_{11}(a_{12}+\theta)^2+P_{21}))+P_{22}(P_{11}(a_{12}+\theta)^2+P_{21}+N_2))}\right) \quad (6)$$

$$R_{11}+R_{21} \leq \frac{1}{2}\log_2\left(\frac{(P_{11}(1+a_{21}\theta)^2+P_{12}+\alpha_{21}^2(P_{21}+P_{22})+N_1)(P_{11}(1+a_{21}\theta)^2+\alpha_{21}^2 P_{22}+N_1)}{(\alpha_{21}^2 P_{22}+N_1)(P_{11}(1+a_{21}\theta)^2+P_{12}+\alpha_{21}^2 P_{22}+N_1)}\right) \quad (7)$$



(a) SNR = 0dB | (b) SNR=10dB | (c) SNR=50dB

Fig. 3. Comparison of the cognitive interference and the cognitive X channels at various SNRs.

and in particular the sum-rate scaling, on the SNR. At 0 and 10dB receive SNR, the two regions almost coincide for large R_1. At high SNR (50dB) the sum-rate scaling increases, and the gap between the sum-rate achieved by the cognitive interference and the cognitive X-channels widens, confirming the sum-rate scaling laws of 1 and 2 respectively. Fig. 3 contrasts the achievable rate regions for cross-over parameters $\alpha_{12}=0.8,\ \alpha_{21}=0.2$ at the SNRs 0, 10 and 50 dB. Notice that the broadcast channel always forms an outer bound, but that the achievable rate region appears to be tight for large R_1.

V. Conclusion

In this paper we have derived an achievable rate region for the MIMO cognitive X channel with a dirty-paper encoded codeword as asymmetric side information. We used this to show that shown that the multiplexing gain of the sum-rate of is $2M$, achieving the optimal sum-rate scaling.This could lead one to think that asymmetric side-information, rather than symmetric side-information (of full cooperation) between Txs always yields the optimal sum-rate scaling. However, we then showed that this is not the case: in the interference channel with asymmetric side-information, where no cross-over information is permitted, the scaling of the sum-rate for the single antenna channel is 1, rather than the optimal 2. Of interest for future research are the derivation of achievable rate regions for cases the cognitive X channel with side information (i), (ii), and (iii), which would compliment the multiplexing gains derived in [11] for these cases.

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