Cognitive Decomposition of Wireless Networks

(*Invited Paper*)

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***Abstract*— In this paper, we provide a framework for a fundamental study of the communication limits of networks of cognitive devices. It is shown that all communication in a network of cognitive and non-cognitive devices can be cast into competitive, cognitive and cooperative behaviors. An achievable rate region for the *cognitive radio channnel* (which captures the most fundamental form of cognition – vertical spectrum sharing), is presented.**

I. INTRODUCTION

Cognitive radios have received much attention in recent years for two main reasons: their flexibility, and the potential gains in spectral efficiency. Their versatile nature is exemplified by their ability to rapidly upgrade, change their transmission protocols and schemes, listen to the spectrum as well as quickly adapt to different spectral policies. This promises great gains in spectral reuse, but leaves open the question of how to efficiently and practically deploy cognitive radios. However, an even more fundamental question must first be answered: what are the theoretical gains to be made in a network employing cognitive and non-cognitive radio devices?

To date, a number of organizations have proposed methods which exploit cognitive radios to obtain higher spectral efficiency [5, 6, 10, 11]. Many of these involve the concept of spectrum sharing, or secondary spectrum licensing. These shared methods lie in contrast to current network operation, where one licensee has *exclusive access* to a designated portion of the frequency spectrum. Under this model, much of the licensed spectrum remains unused. To alleviate this, proposals which involve cognitive radios sensing these gaps in the spectrum and opportunistically employing unused *spectral holes* have recently emerged. This sharing of the spectrum can fall into two main categories [5, 10]:

- *Horizontal sharing:* All networks and users have equal rights to the spectrum, and protocols that allow for peaceful and efficient coexistence must be developed. Horizontal sharing may be without coordination, as is the case for Bluetooth and 802.11, or with coordination.
- *Vertical sharing:* Networks and users do not have equal rights to the spectrum. In its simplest form, this means primary users receive full access to the spectrum, and secondary users may access the spectrum opportunistically as long as they cause no interference to the primary users. This can be done by having the secondary users sense the wireless medium and either transmit at a low enough level so that they stay below the *interference temperature* of the primary receivers [7], or transmit during sensed *spectral holes*.

Although the *spectral hole filling* concept for cognitive radio is heuristically pleasing, it provides no fundamental insight into how much gain can be achieved in a heterogeneous network of cognitive and non-cognitive devices. We wish to study the *fundamental limits of communication in cognitive networks.* To approach this problem from a global perspective, we start with an arbitrary network and demonstrate that it can be decomposed into a *cognitive graph*. We will argue why cognitive radios motivate the introduction of a new type of cooperation in communication networks. In short, cognitive radios allow for asymmetric cooperation between transmitting nodes or clusters. This will essentially provide an alternate to spectral hole filling for interference mitigation. We then demonstrate an achievable rate region for the essential building block of the cognitive graph: the *cognitive radio channel* defined as a two sender, two receiver interference channel with asymmetric and non-causal (or *a-priori*) transmitter cooperation.

II. NETWORK DECOMPOSITION

A. Network Model

We consider an arbitrary network of wireless devices, which may be cognitive, denoted as (C), or non-cognitive (NC) radios. At any given point in time, certain transmitting nodes (T) have information which they wish to transmit to certain receiving nodes (R). Nodes that do not have any information of their own to transmit are denoted as "extra" nodes (E). We assume that nodes are not able to simultaneously transmit and receive, i.e., they must obey the half-duplex constraint. This is a reasonable assumption given current technology. Thus, a node is classified as either a (T), (R) or (E) node, but never more than one, and as either cognitive (C) or non-cognitive (NC).

If all devices simultaneously transmit, the network may suffer from interference. However, we wish to exploit the nature of cognitive radios to reduce this interference. The key to doing so is transmitter cooperation, which could lead to interference mitigation. At each point in time, depending on the device capabilities, as well as the geometry and channel gains between the various nodes, certain cognitive nodes may be able to hear and/or obtain the messages to be transmitted by other nodes. In reality, these messages would need to be obtained in real time, and could exploit the geometric gains between cooperating transmitters relative to receivers in a, for example, 2 phase protocol [4]. However, as a first step, we idealize the concept of message knowledge: whenever a (T) or (E) node is cognitive and in principle able to hear and decode the message of another transmitting node,

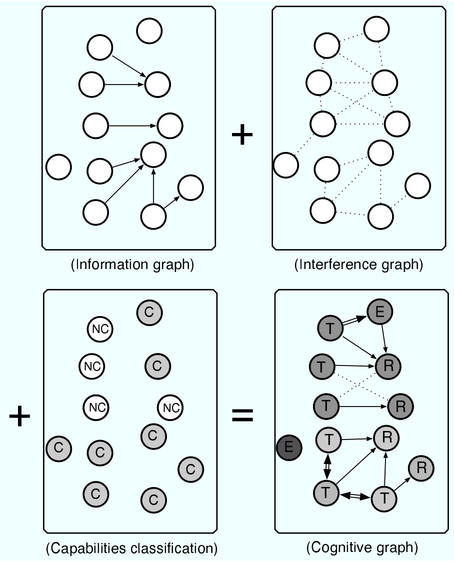

Fig. 1. The information and interference graphs, together with the capabilities classification yield the cognitive decomposition graph.

we assume it has full *a-priori* knowledge. We call this the *genie assumption*, as these messages could have been given to the appropriate transmitters by a genie. Notice that we explicitly allow for asymmetric message knowledge, and that this message knowledge is between potentially transmitting nodes only. We ignore cognitive receiving nodes for now. In this paper, all transmitter cooperation occurs under the genie assumption. Protocols which remove this assumption are discussed in [4].

We now demonstrate that given a snapshot of a network and three pieces of information: an *information graph*, an *interference graph* and a *capabilities classification* as in Fig. 1, transmission scenarios in which there is some form of transmitter cooperation are captured in a *cognitive graph*: a set of disjoint non-interfering *groups* of nodes, each of which consists of a set of *clusters* behaving in an inter/intra cluster competitive, cognitive, or cooperative manner.

The information graph: This directed graph captures which nodes have independent information to be sent to which receivers at a given moment in time.

The interference graph: This undirected graph captures the interference in a network. If two nodes can hear each other, and thus potentially interfere with each other, then an edge exists between them. Notice that for a (T) node to be able to transmit to an (R) node, an edge in the interference graph should appear between them.

The capabilities classification: This partition of the nodes then labels them as cognitive (C) or non-cognitive (NC). A node is (C) when it is able and willing to sense and adapt to its environment. Note that an (NC) node could model either a wireless device that does not have cognitive capabilities, or could alternately model devices that do not require cognition to communicate. For example, in vertical spectrum sharing, the (possibly paying) primary users are guaranteed spectrum access; secondary users must avoid interfering with these primary users, so primary user cognition may not be necessary for transmission. While receivers can be (C) or (NC) in our formulation, this has no impact, as we do not allow for receiver cooperation in our current model.

Cognitive graph: From the information graph, interference graph, and capabilities classification, we can form a cognitive graph in the following steps:

1) Label all nodes as either (T) if they wish to transmit, (R) if they plan to receive, and (E) if they have no information of their own to transmit. This information may be obtained from the information graph.
2) For each node (T) that wishes to transmit, create a transmission arc (solid) between it and any (R) nodes it wishes to transmit to, provided they share an edge in the interference graph.
3) For each pair of nodes (T) and (R) connected by an edge in the interference graph but not by an arc in the information graph, create an interference edge (dotted) in the cognitive graph.
4) For each cognitive node (E) or (T) that shares an edge with another (E) or (T) node in the interference graph, join the second (E) or (T) node to the first (E) or (T) node by a cognitive arc (double).
5) For each (E) or (T) node that has cognitive genie-aided information of another (T) node in the cognitive graph, create a transmission arc (solid) between the first (E) or (T) node and the receiver of the second (T) node if these share an edge in the interference graph.

Once the cognitive graph is complete, the solid arcs indicate desired information paths from (T) / (E) to (R), the solid double arcs indicate a priori message knowledge (possibly asymmetric) and the dotted edges between (T) and (R) nodes indicate interference.

B. Cognitive Graph Decomposition

The cognitive graph gives us information on the interference seen, and the transmitter cooperation that is possible. We assume all (T), (R) and (E) nodes have full channel knowledge. This assumption is used to simplify and idealize the problem, and will provide an upper bound to any real world scenario.

In order to fully describe all transmitter cooperation strategies in a wireless network employing cognitive radios as described by the cognitive graph, the following notions are needed. A *group* is a set of connected nodes (ignoring the direction of arcs). It is easy to see that a cognitive graph may be partitioned into groups, and that, by construction, these groups will not interfere with each other. They may independently encode their messages and simultaneously transmit with no interference. Thus, it is of interest to calculate the capacity region of each group. Within a group, we may further divide the nodes into *clusters*. A *cluster* is defined as a set of nodes connected only through solid arcs to a single receiver. We assume all receivers are independent and unable to cooperate. Thus, there exists one cluster per receiver.

Intra-Cluster behavior: within a single cluster, we may partition transmitter cooperation into three classes:

- *Competitive:* all (T) within a cluster encode their messages independently. They compete for the channel. If there are no arcs between any of the (T) and (E) nodes within a cluster, that cluster behaves competitively.
- *Cooperative:* all the (T) / (E) in a cluster know the messages of all the other (T) in that cluster a priori. These require bi-directional cognitive (double) arcs between all (T) nodes of that cluster. A cluster consisting of a single transmitter is said to be cooperative.
- *Cognitive:* all clusters that are not competitive or cooperative, i.e., some but not all of the (T) / (E) in a cluster know the messages to be transmitted by other (T) in the cluster *a-priori* (solid double arcs). This is an asymmetric form of cooperation, which may allow the user with the message knowledge to mitigate interference, or aid in the transmission of the *a-priori* known messages.

Inter-cluster behavior: when two (or more) clusters within one group are connected through undesired interference (dotted) edges or share (T) / (E) nodes, we can speak of inter-cluster behavior.

- *Competitive:* when all (T) / (E) nodes of one cluster are independent of all (T) / (E) nodes of another cluster, the *clusters* compete for the channel during simultaneous transmission. Note that competitive inter-cluster behavior does not imply anything about the competitive, cooperative, or cognitive behavior of nodes within one cluster. The clusters will be linked through interference (dotted) edges.
- *Cooperative:* all the (T) / (E) nodes in one cluster know the messages of a second cluster and vice-versa. Clusters under consideration know each others' messages and so the clusters can cooperate, at the cluster level, to transmit their messages, potentially reducing interference.
- *Cognitive:* encompasses all clusters that do not behave competitively or cooperatively, that is, when a subset of the (T) nodes in one cluster knows the messages to be transmitted by a subset of the (T) nodes of the other clusters, we call this inter-cluster cognitive behavior. The cluster with the message knowledge may be able to at least partially mitigate some interference from the other cluster(s).

Note that if nodes $(X) \Leftrightarrow (Y)$ and $(Y) \Leftrightarrow (Z)$ (where $\Leftrightarrow$ indicates two-way cognition, or cooperation) then one may suppose $(X) \Leftrightarrow (Z)$. This only makes sense if there is no overhead to cognition and all message knowledge is assumed to be non-causal and instantaneous. This transitivity property may break down once messages must be causally obtained, and our model does not enforce such transitivity of cognition. We have the following theorem, which follows directly from the construction and definitions above.

Theorem 1: At a point in time, if given information and interference graphs as well as a capabilities classification, we may construct a cognitive graph which identifies the non-interfering *groups*, and the interfering *clusters* within each group. All forms of user cooperation within a cluster is described as *competitive, cognitive*, or *cooperative* behaviors. Furthermore, between clusters in the same group, we may have *competitive, cognitive,* or *cooperative* behavior.

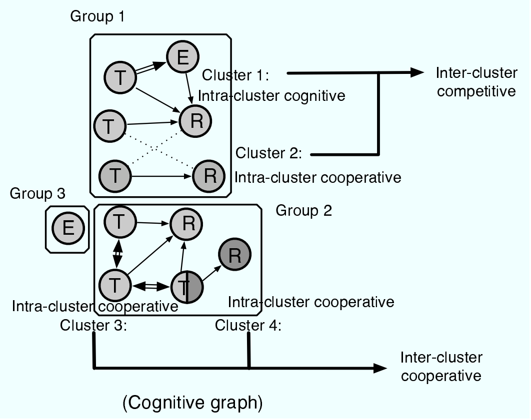

Fig. 2. The resulting groups, clusters, and their behaviors.

We demonstrate this decomposition by example and construct the cognitive graph from the given information, interference and capabilities graphs, and indicate the groups, clusters, and their inter and intra-cluster behaviors in Fig. 2.

III. 2×2 COGNITIVE RADIO CHANNEL

To fully understand the transmission limits of a network, we must study both inter-cluster and intra-cluster cognitive behavior. The decomposition theorem highlights an important concept for future wireless and cognitive radio channels: that of asymmetric channel knowledge and cooperation. Certain asymmetric channels have been considered: for example in [13], among other results, the capacity of a channel with asymmetric cooperation between two transmitters in a multiple access is computed. In [2, 3] we introduced the *cognitive radio channel*, which captures the most basic form of asymmetric transmitter cooperation for the interference channel. The interference channel is a two independent sender, two independent receiver channel where the two messages that are simultaneously transmitted interfere with each other. Despite this channel's simplicity, its capacity in the most general case is still an open problem. We wish to study the information theoretic limits of interference channels with asymmetric transmitter cooperation, also known as *cognitive radio channels.* To this end, in this paper, we review the best known achievable region for the cognitive radio channel, that of [3], and compare it to inner and outer bounds on the region.

We define a 2×2 *genie-aided cognitive radio channel* C_{COG}, as in Fig. 3(b), to be two point to point channels $\mathcal{S}_1 \rightarrow \mathcal{R}_1$ and $\mathcal{S}_2 \rightarrow \mathcal{R}_2$ in which the sender $\mathcal{S}_2$ is given, in a non-causal manner (i.e., by a genie), the message X_1 which the sender $\mathcal{S}_1$ will transmit. Fig 3(a) demonstrates competitive behavior (independent transmitters), while Fig.3(c) demonstrates cooperative behavior. Let X_1 and X_2 be the random

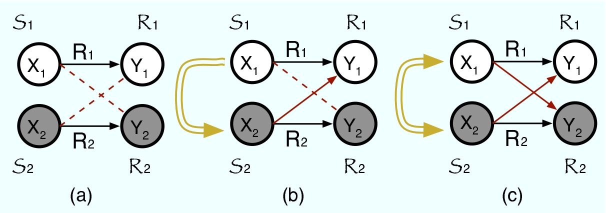

Fig. 3. Dotted edges indicate unwanted interference, solid arcs indicate desired transmission arcs, and double arcs between transmitters indicate *a-priori* message knowledge. (a) Competitive interference. (b) Genie-aided cognitive radio channel. (c) Cooperative broadcast channel.

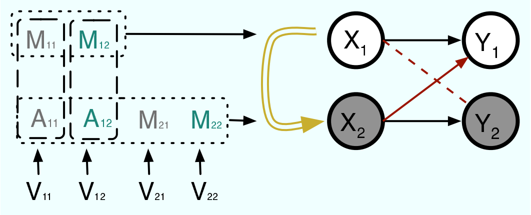

Fig. 4. The modified cognitive radio channel with auxiliary random variables M_{11}, M_{12} and M_{21}, M_{22}, inputs X_1 and X_2, and outputs Y_1 and Y_2. The auxiliary random variable A_{11}, A_{12} associated with $\mathcal{S}_2$, aids in the transmission of M_{11} and M_{12} respectively. The vectors V_{11}, V_{12}, V_{21} and V_{22} denote the effective random variables encoding the transmission of the private and public messages.

variable inputs to the channel, and let Y_1 and Y_2 be the random variable outputs of the channel. The conditional probabilities of the discrete memoryless C_{COG} are fully described by $P(y_1|x_1,x_2)$ and $P(y_2|x_1,x_2)$.

In [9], an achievable region for the interference channel is found by first considering a modified problem and then establishing a correspondence between the achievable rates of the modified and the original channel models.

The channel C^m_{COG}, defined as in Fig. 4 introduces many new auxiliary random variables, whose purposes can be made intuitively clear by relating them to auxiliary random variables in previously studied channels. They are defined and described in Table I. Standard definitions of achievable rates and regions are employed [1, 2]. Then an achievable region for the 2×2 cognitive radio channel is given by:

Theorem 2: Let $Z \overset{\triangle}{=} (Y_1, Y_2, X_1, X_2, V_{11}, V_{12}, V_{21}, V_{22}, W)$, be as shown in Fig. 4. Let $\mathcal{P}$ be the set of distributions on Z that can be decomposed into the form

$$\begin{aligned} &P(w) \times [P(m_{11}|w)P(m_{12}|w)P(x_1|m_{11}, m_{12}, w)] \\ &\times [P(a_{11}|m_{11}, w)P(a_{12}|m_{12}, w)] \\ &\quad \times [P(m_{21}|v_{11}, v_{12}, w)P(m_{22}|v_{11}, v_{12}, w)] \\ &\qquad \times [P(x_2|m_{21}, m_{22}, a_{11}, a_{12}, w)]\, P(y_1|x_1, x_2)P(y_2|x_1, x_2), \end{aligned} \tag{1}$$

where $P(y_1|x_1,x_2)$ and $P(y_2|x_1,x_2)$ are fixed by the channel. Let $T_1 \overset{\triangle}{=} \{11, 12, 21\}$ and $T_2 \overset{\triangle}{=} \{12, 21, 22\}$. For any $Z \in \mathcal{P}$, let $S(Z)$ be the set of all tuples $(R_{11}, R_{12}, R_{21}, R_{22})$ of non-negative real numbers such that there exist non-negative reals $L_{11}, L_{12}, L_{21}, L_{22}$ satisfying:

$$\bigcap_{T\subset\{11,12\}} \left(\sum_{t\in T} R_t\right) \le I(X_1; \mathbf{M}_T|\mathbf{M}_{\overline{T}}) \tag{2}$$

$$R_{11} = L_{11} \tag{3}$$

$$R_{12} = L_{12} \tag{4}$$

$$R_{21} \le L_{21} - I(V_{21}; V_{11}, V_{12}) \tag{5}$$

$$R_{22} \le L_{22} - I(V_{22}; V_{11}, V_{12}) \tag{6}$$

$$\bigcap_{T\subset T_1} \left(\sum_{t_1\in T} L_{t_1}\right) \le I(Y_1, \mathbf{V}_{\overline{T}}; \mathbf{V}_T|W) \tag{7}$$

$$\bigcap_{T\subset T_2} \left(\sum_{t_2\in T} L_{t_2}\right) \le I(Y_2, \mathbf{V}_{\overline{T}}; \mathbf{V}_T|W), \tag{8}$$

$\overline{T}$ denotes the complement of the subset T with respect to T_1 in (7), with respect to T_2 in (8), and $\mathbf{V}_T$ denotes the vector of V_i such that $i \in T$. Let S be the closure of $\cup_{Z\in\mathcal{P}} S(Z)$. Then any pair $(R_{11}+R_{12}, R_{21}+R_{22})$ for which $(R_{11}, R_{12}, R_{21}, R_{22}) \in S$ is achievable for C_{COG}. □

Proof outline: The main intuition is as follows: the equations in (2) ensure that when $\mathcal{S}_2$ is presented with X_1 by the genie, the auxiliary variables M_{11} and M_{12} can be recovered. Eqs. (7) and (8) correspond to the equations for two overlapping MAC channels seen between the effective random variables $\mathbf{V}_{T_1} \to \mathcal{R}_1$, and $\mathbf{V}_{T_2} \to \mathcal{R}_2$. Eqs. (5) and (6) are necessary for the Gel'fand-Pinsker [8] coding scheme to work ($I(V_{21}; V_{11}, V_{12})$ and $I(V_{22}; V_{11}, V_{12})$ are the penalties for using non-causal side information). Intuitively, the sender $\mathcal{S}_2$ could aid in transmitting the message of $\mathcal{S}_1$ (the A_* random variables) or it could dirty paper code against the interference it will see (the M_{2*} variables). We smoothly interpolate between these two options.

IV. ACHIEVABLE RATES FOR GAUSSIAN NOISE

Consider the 2×2 genie-aided cognitive radio channel described by the input, noise and output relations:

$$\begin{aligned} Y_1 &= X_1 + a_{21}X_2 + Z_1 \\ Y_2 &= a_{12}X_1 + X_2 + Z_2 \end{aligned}$$

where a_{12}, a_{21} are the crossover (channel) coefficients, $Z_1 \sim \mathcal{N}(0, Q_1)$ and $Z_2 \sim \mathcal{N}(0, Q_2)$ are independent AWGN terms, X_1 and X_2 are constrained to to average powers P_1 and P_2 respectively, and $\mathcal{S}_2$ is given X_1 non-causally. In order to determine an achievable region for the modified Gaussian genie-aided cognitive radio channel, specific forms of the random variables described in Theorem 2 are assumed, and are analogous to the assumptions found in [3].

The resulting achievable region, in the presence of additive white Gaussian noise for the case of identical transmitter powers ($P_1 = P_2$) and identical receiver noise powers ($Q_1 = Q_2$), is presented in Figure 5. The ratio of transmit power to receiver noise power is 7.78 dB. The cross-over parameters in the interference channel are $a_{12} = a_{21} = 0.55$.

In the figure, we see 4 regions. The time-sharing region (1) displays the result of pure time sharing of the wireless channel between users X_1 and X_2. Points in this region are

TABLE I

DESCRIPTION OF RANDOM VARIABLES IN THEOREM 2.

(Random) variable names	(Random) variable descriptions
M_{11}, M_{22}	Private info from $\mathcal{S}_1 \rightarrow \mathcal{R}_1$ and $\mathcal{S}_2 \rightarrow \mathcal{R}_2$ resp.
M_{12}, M_{21}	Public info from $\mathcal{S}_1 \rightarrow (\mathcal{R}_1, \mathcal{R}_2)$ and $\mathcal{S}_2 \rightarrow (\mathcal{R}_1, \mathcal{R}_2)$ resp.
A_{11}, A_{12}	Variables at $\mathcal{S}_2$ that aid in transmitting M_{11}, M_{12} resp.
$V_{11} = (M_{11}, A_{11}), V_{12} = (M_{12}, A_{12})$	Vector helping transmit the private/public (resp.) info of $\mathcal{S}_1$
$V_{21} = M_{21}, V_{22} = M_{22}$	Public, private message of $\mathcal{S}_2$.
	Also the auxiliary random variables for Gel'fand-Pinsker coding
W	Time-sharing random variable, independent of messages

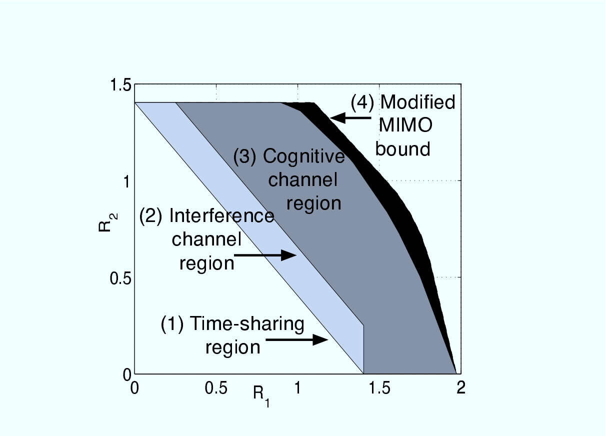

Fig. 5. Rate regions (R_1, R_2) for 2×2 wireless channels.

obtained by letting X_1 transmit for a fraction of the time, during which X_2 refrains, and vice versa. The interference channel region (2) corresponds to the best known achievable region [9] of the classical information theoretic interference channel. In this region, both senders encode independently, and there is no message *a-priori* knowledge by either transmitter of the other's message.

The cognitive channel region (3) is the achievable region described here and in [3]. In this case X_2 received the message of X_1 non-causally from a genie, and X_2 uses a coding scheme which combines interference mitigation with relaying the message of X_1. We see that both users – not only the incumbent X_2 which has the extra message knowledge – benefit from using this scheme. This is as expected, as the selfish strategy boosts R_2 rates, while the selfless one boosts R_1 rates, and so gracefully combining the two will yield benefits to both users. Thus, the presence of the incumbent cognitive radio X_2 can be beneficial to X_1, a point which is of practical significance. This could provide yet another incentive for the introduction of such schemes.

The modified MIMO bound region (4) is an outer bound on the capacity of this channel: the 2x2 Multiple Input Multiple Output Gaussian Broadcast Channel capacity region [12], where we have restricted the form of the transmit covariance matrix to be of the form $\begin{pmatrix} P_1 & c \\ c & P_2 \end{pmatrix}$, to more closely resemble our constraints, intersected with the capacity bound on $R_2 \leq I(Y_2; X_2|X_1)$ for the channel for $X_2 \rightarrow Y_2$ in the absence of interference from X_1.

V. CONCLUSION

In this paper, we investigated fundamental limits of communication in a wireless network of cognitive and non-cognitive devices. Given such a network's information graph, interference graph and capabilities classification, we constructed a cognitive graph. This is partitioned into disjoint non-interfering *groups*, each of which consists of potentially overlapping *clusters*. Within each cluster (intra-cluster) and between clusters (inter-cluster) different types of behaviors exist (competitive, cognitive, and cooperative) that embody the entire range of possible transmitter strategies. We then considered one of the most fundamental forms of cognitive behavior in which one transmitter knows, *a-priori*, the message another transmitter is to send. We computed an achievable rate region and illustrated it for the Gaussian case.

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