Achievable Rates in Cognitive Radio Channels

Natasha Devroye
Div. of Eng. & Appl. Sciences
Harvard University
Cambridge, MA 02138
e-mail: ndevroye@deas.harvard.edu

Patrick Mitran
Div. of Eng. & Appl. Sciences
Harvard University
Cambridge, MA 02138
e-mail: mitran@deas.harvard.edu

Vahid Tarokh
Div. of Eng. & Appl. Sciences
Harvard University
Cambridge, MA 02138
e-mail: vahid@deas.harvard.edu

Abstract — ***Cognitive radio*** **promises a low cost, highly flexible alternative to the classic single frequency band, single protocol wireless device. By sensing and adapting to its environment, such a device is able to fill voids in the wireless spectrum and dramatically increase spectral efficiency. In this paper, the** ***cognitive radio channel*** **is defined as an n-transmitter, m-receiver interference channel in which sender i obtains the messages senders 1 through $i-1$ plan to transmit. The two sender, two receiver case is considered. In this scenario, one user, a cognitive radio, obtains (genie assisted, or causally) knowledge of the data to be transmitted by the other user. The cognitive radio may then simultaneously transmit over the same channel, as opposed to waiting for an idle channel as in a traditional cognitive radio channel protocol. Dirty-paper coding and ideas from achievable region constructions for the interference channel are used, and an achievable region for the** ***cognitive radio channel*** **is computed. It is shown that in the Gaussian case, the described achievable region approaches the upper bounds provided by the 2×2 Gaussian MIMO broadcast channel, and an interference-free channel.**

I. Motivation

Recently, there has been an explosion of interest in cognitive and software radios. *Software Defined Radios* (SDR) [12] are devices used to communicate over the wireless medium equipped with either a general purpose processor or programmable silicon as hardware base, and enhanced by a flexible software architecture. These low-cost devices are able to operate in many frequency bands under multiple transmission protocols and employ a variety of modulation and coding schemes. *Cognitive radios* [13], are software defined radios capable of sensing their environment and making real-time decisions, without any user intervention. This allows them to change their modulation schemes or protocols so as to adapt to the sensed environment.

Apart from their low cost and flexibility, another benefit of SDR technology is spectral efficiency. Currently, FCC measurements [6], indicate that at any time roughly 10% of the unlicensed frequency spectrum is actively in use (leaving 90% unused). In current cognitive radio protocol proposals, the device listens to the wireless channel and determines, either in time or frequency, which part of the spectrum is unused [10]. It then adapts its signal to fill this void in the spectrum space, increasing the spectral efficiency. Thus, a device transmits over a certain time or frequency only when no other user does. In this paper, the cognitive radio behavior is generalized to allow two users to simultaneously transmit over the same time or frequency. Under our scheme, a cognitive radio will listen to the channel and, if sensed idle, proceed with the traditional cognitive radio channel model, that is, transmit during the voids. On the other hand, if another sender is sensed, the radio may decide to proceed with simultaneous transmission.

We intend to study the theoretic limits of such communications. Specifically, we will prove achievability, in the information theoretic sense, of a certain set of rates at which two senders (cognitive radios, denoted as $\mathcal{S}_1$ and $\mathcal{S}_2$) can transmit simultaneously over a common channel to two independent receivers $\mathcal{R}_1$, $\mathcal{R}_2$ when $\mathcal{S}_2$ is aware of the message to be sent by $\mathcal{S}_1$. Our methods borrow ideas from Costa's dirty paper coding [2], the interference channel [1], the Gaussian MIMO broadcast channel [19], and the achievable region of the interference channel described by Han and Kobayashi [9]. The results are also related, conceptually, to other communication systems in which user cooperation is employed in order to enhance the capacity. These schemes can be traced back to telegraphy, and have recently been considered in the *collaborative communications* of [16], the spatial diversity enhancing schemes obtained through user cooperation described in [17, 18], and many others such as [11, 15].

The paper is structured as follows: Section II defines the *genie-aided cognitive radio channel* and *genie-aided modified cognitive radio channel* as interference channels in which one sender is non-causally given the other sender's message. Section II also proves the main result: achievability of a certain rate region. The significance of our result is shown in Section III, where numerical methods are used to compute an achievable region in the additive white Gaussian noise case. Here, it is clear that our region not only extends that of [9], but that in the case of large power mismatches between the two senders, as would be expected in a rich fading environment, the achievable region described here approaches the upper bounds given by the 2×2 Gaussian MIMO broadcast channel [19], and an interference-free channel. Of note is that in [5], results are extended to the more realistic case in which $\mathcal{S}_2$ causally obtains the message of $\mathcal{S}_1$. In Section IV, we summarize the main contributions of this paper: the definition of a *cognitive radio channel*, the proof ideas and the significance of a certain achievable rate region for this channel.

II. Genie-aided Cognitive Radio Channel Definition

Define a *genie-aided cognitive radio channel* C_{COG} to be an interference channel in which $\mathcal{S}_2$ is given, in a non-causal manner (i.e., by a genie), the message x_1^n which $\mathcal{S}_1$ will transmit, as illustrated in Fig. 1 below. $\mathcal{S}_2$ can then exploit the knowledge of $\mathcal{S}_1$'s message, and potentially improve the transmission

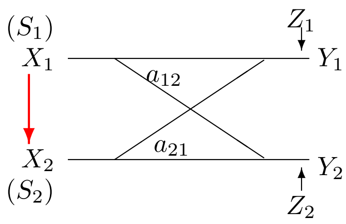


Fig. 1: The genie-aided cognitive radio channel with inputs X_1, X_2, outputs Y_1, Y_2, additive noise Z_1, Z_2 and multiplicative interference coefficients a_{12}, a_{21}. S_1's input X_1 is given to S_2, but not the other way around.

rate. It can do so using a dirty paper coding technique [2] and an achievable region construction for the interference channel [9]. Intuitively, the achievable region in [9] should lie entirely within our achievable region, since our senders are permitted to at least partially cooperate. An upper bound for our region in the Gaussian case is provided by the 2×2 MIMO broadcast channel whose capacity has recently been calculated in [19]. In [19], dirty paper coding techniques are shown to be optimal for non-degraded vector broadcast channels. Our channel model resembles that of [19], with one important difference. In the scheme of [19] it is presumed that both senders can cooperate in order to precode the transmitted signal. In our scheme, the relation between the two senders is asymmetric. The rate of $\mathcal{S}_2$ is also bounded by the rate achievable in an interference-free channel, with $a_{12} = 0$.

An (n, K_1, K_2, λ) *code* for the *genie-aided cognitive radio channel* consists of K_1 codewords $x_1^n(i) \in \mathcal{X}_1^n$ for $\mathcal{S}_1$ and $K_1 \times K_2$ codewords $x_2^n(i,j) \in \mathcal{X}_2^n$ for $\mathcal{S}_2$ which together form the *codebook*, revealed to both senders and receivers, such that the average error probabilities under some decoding scheme are less than λ. A rate pair (R_1, R_2) is said to be *achievable* for the genie-aided cognitive radio channel if there exists a sequence of $(n, 2^{nR_1}, 2^{nR_2}, \epsilon_n)$ codes such that $\epsilon_n \to 0$ as $n \to \infty$. An *achievable region* is a closed subset of the positive quadrant of $\mathbb{R}^2$ of achievable rate pairs.

The interference channel capacity, in the most general case, is still an open problem. This is the case for the *genie-aided cognitive radio channel* as well. In [9], an achievable region of the interference channel is found by first considering a modified interference channel and then establishing a correspondence between the achievable rates of the modified and the original channel models. A similar modification is made in the next subsection.

The Modified Genie-aided Cognitive Radio Channel C_{COG}^m

As in [9], we introduce a modified genie-aided cognitive radio channel, C_{COG}^m, (m for modified) and demonstrate an achievable region for C_{COG}^m. Then, a relation between an achievable rate for C_{COG}^m and an achievable rate for C_{COG} is used to establish an achievable region for C_{COG}. Define the modified genie-aided cognitive radio channel C_{COG}^m as in Fig. 2.

Let $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$ be the random variable inputs to the channel. Let $Y_1 \in \mathcal{Y}_1$and $Y_2 \in \mathcal{Y}_2$ be the random variable outputs of the channel. The conditional probabilities of the discrete memoryless C_{COG}^m are the same as those of the discrete memoryless C_{COG} and are fully described by $p(y_1|x_1,x_2)$ and $p(y_2|x_1,x_2)$ for all values x_1, x_2, y_1 and y_2.

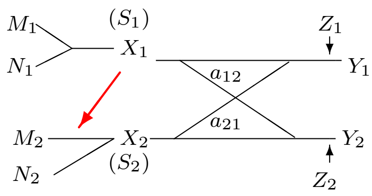


Fig. 2: The modified cognitive radio channel with auxiliary random variables M_1, M_2, N_1, N_2, inputs X_1, X_2, additive noise Z_1, Z_2, outputs Y_1, Y_2 and multiplicative interference coefficients a_{12}, a_{21}.

The modified genie-aided cognitive radio channel introduces two pairs of auxiliary random variables: (M_1, N_1) and (M_2, N_2). The random variables $M_1 \in \mathcal{M}_1$ and $M_2 \in \mathcal{M}_2$ represent, as in [9], the private information to be sent from $\mathcal{S}_1 \to \mathcal{R}_1$ and $\mathcal{S}_2 \to \mathcal{R}_2$ respectively. In contrast, the random variables $N_1 \in \mathcal{N}_1$ and $N_2 \in \mathcal{N}_2$ represent the public information to be sent from $\mathcal{S}_1 \to (\mathcal{R}_1, \mathcal{R}_2)$ and $\mathcal{S}_2 \to (\mathcal{R}_1, \mathcal{R}_2)$ respectively. The function of these M_1, N_1, M_2, N_2 is as in [9]: to decompose or define *explicitly* the information to be transmitted between various input and output pairs.

In this work, M_2 and N_2 also serve a dual purpose: these auxiliary random variables are analogous to the auxiliary random variables of Gel'fand and Pinsker [8] or Cover and Chiang [3]. They serve as fictitious inputs to the channel, so that after $\mathcal{S}_2$ is informed of the message of $\mathcal{S}_1$ non-causally (or equivalently, is given x_1^n), the channel still looks or behaves like a Discrete Memoryless Channel (DMC) from (M_1, N_1, M_2, N_2) to (Y_1, Y_2). As in [3, 8], there is a penalty in using this approach which will be reflected by a reduction in achievable rates (compared to the fictitious DMC from (M_1, N_1, M_2, N_2) to (Y_1, Y_2)) for the links which use the non-causal information.

Similar to the definition of a *code* in the cognitive radio channel case, define an $(n, K_{11}, K_{12}, K_{21}, K_{22}, \lambda)$ code for the modified genie-aided cognitive radio channel as a set of $K_{11} \times K_{12}$ codewords $x_1^n(i,j) \in \mathcal{X}_1^n$ for $\mathcal{S}_1$ and $K_{11} \times K_{12} \times K_{21} \times K_{22}$ codewords $x_2^n(i,j,k,l) \in \mathcal{X}_2^n$ for $\mathcal{S}_2$ such that the average probability of decoding error is less than λ. Call a quadruple $(R_{11}, R_{12}, R_{21}, R_{22})$ achievable if there exists a sequence of $(n, 2^{nR_{11}}, 2^{nR_{12}}, 2^{nR_{21}}, 2^{nR_{22}}, \epsilon_n)$ codes such that $\epsilon_n \to 0$ as $n \to \infty$. An achievable region of a modified genie-aided cognitive radio channel is the closure of a subset of the positive region of $\mathbb{R}^4$ of achievable rate quadruples.

As mentioned in [9], the introduction of a time-sharing random variable W is thought to strictly extend the achievable region obtained using a convex hull operation. Thus, let $W \in \mathcal{W}$ be a time-sharing random variable. The paper's main theorems (1, 2 and 3) are outlined next.

Theorem 1 *Let $Z := (Y_1, Y_2, X_1, X_2, M_1, N_1, M_2, N_2, W)$, and let $\mathcal{P}$ be the set of distributions on Z that can be decomposed into the form*

$$p(w)p(m_1|w)p(n_1|w)p(x_1|m_1,n_1,w) \tag{1}$$

$$p(m_2|x_1,w)p(n_2|x_1,w)p(x_2|m_2,n_2,w)$$

$$\times\, p(y_1|x_1,x_2)p(y_2|x_1,x_2). \tag{2}$$

For any $Z \in \mathcal{P}$, let $S(Z)$ be the set of all quadruples $(R_{11}, R_{12}, R_{21}, R_{22})$ of non-negative real numbers such that *there exist non-negative real* (L_{21}, L_{22}) *satisfying:*

$$R_{21} \leq L_{21} - I(N_2; X_1|W) \quad (3)$$
$$R_{22} \leq L_{22} - I(M_2; X_1|W) \quad (4)$$

$$R_{11} \leq I(Y_1, N_1, N_2; M_1|W) \quad (5)$$
$$R_{12} \leq I(Y_1, M_1, N_2; N_1|W) \quad (6)$$
$$L_{21} \leq I(Y_1, M_1, N_1; N_2|W) \quad (7)$$
$$R_{11} + R_{12} \leq I(Y_1, N_2; M_1, N_1|W) \quad (8)$$
$$R_{11} + L_{21} \leq I(Y_1, N_1; M_1, N_2|W) \quad (9)$$
$$R_{12} + L_{21} \leq I(Y_1, M_1; N_1, N_2|W) \quad (10)$$
$$R_{11} + R_{12} + L_{21} \leq I(Y_1; M_1, N_1, N_2|W) \quad (11)$$

$$L_{22} \leq I(Y_2, N_1, N_2; M_2|W) \quad (12)$$
$$R_{12} \leq I(Y_2, N_2, M_2; N_1|W) \quad (13)$$
$$L_{21} \leq I(Y_2, N_1, M_2; N_2|W) \quad (14)$$
$$L_{22} + L_{21} \leq I(Y_2, N_1; M_2, N_2|W) \quad (15)$$
$$L_{22} + R_{12} \leq I(Y_2, N_2; M_2, N_1|W) \quad (16)$$
$$R_{12} + L_{21} \leq I(Y_2, M_2; N_1, N_2|W) \quad (17)$$
$$L_{22} + R_{21} + L_{12} \leq I(Y_2; M_2, N_1, N_2|W). \quad (18)$$

Let S *be the closure of* $\cup_{Z \in \mathcal{P}} S(Z)$*. Then any element of* S *is achievable for the modified genie-aided cognitive radio channel* C^m_{COG}*.*

Proof: The full proof is given in [5]. Due to space constraints, an outline of the proof of achievability is given here. It is sufficient to show the achievability of the interior elements of $S(Z)$ for each $Z \in \mathcal{P}$. So, fix $Z = (Y_1, Y_2, X_1, X_2, M_1, N_1, M_2, N_2, W)$ and take any $(R_{11}, R_{12}, R_{21}, R_{22})$ and (L_{21}, L_{22}) satisfying the constraints of the theorem. The standard notation and notions of strong ϵ-typicality, strong joint typicality, and strongly typical sets of [4] will be used.

Codebook generation: Let some distribution on Z of the form (2) be given. To generate the codebook, first let $w^n := (w^{(1)}, w^{(2)}, \ldots, w^{(n)})$ be a sequence in $\mathcal{W}^n$ chosen randomly according to $\prod_{t=1}^n p(w^{(t)})$ and known to $\mathcal{S}_1, \mathcal{S}_2, \mathcal{R}_1$ and $\mathcal{R}_2$. $\mathcal{S}_1$ generates its codebook by generating $2^{nR_{11}}$ n-sequences $m_1(i)$ i.i.d. according to $\prod_{t=1}^n p(m_1^{(t)}|w^{(t)})$ and $2^{nR_{12}}$ n-sequences $n_1(j)$ i.i.d. according to $\prod_{t=1}^n p(n_1^{(t)}|w^{(t)})$. On the other hand, $\mathcal{S}_2$ generates its codebook by generating $2^{nL_{21}}$ n-sequences $n_2(l)$ i.i.d. according to $\prod_{t=1}^n p(n_2^{(t)}|w^{(t)})$ and throws these into $2^{nR_{21}}$ bins uniformly, and analogously for the $2^{nL_{22}}$ n-sequences $m_2(k)$ that are uniformly thrown into $2^{nR_{22}}$ bins. Define the message index spaces $\mathcal{S}_{ij} := \{1, 2, \ldots, 2^{n(R_{ij})}\}$, for $i, j = 1, 2$. The aim is to send a four dimensional message $s := (s_{11}, s_{12}, s_{21}, s_{22}) \in \mathcal{S} := \mathcal{S}_{11} \times \mathcal{S}_{12} \times \mathcal{S}_{21} \times \mathcal{S}_{22}$. For $\mathcal{S}_1$ to send (s_{11}, s_{12}), it looks up the sequences $m_1^n(s_{11})$, $n_1^n(s_{12})$ and sends $x_1^n = f^n(m_1^n(s_{11}), n_1^n(s_{12})|w^n)$, for $f^n(m_1^n, n_1^n|w^n) := (f(m_1^{(1)}, n_1^{(1)}|w^{(1)}), \cdots, f(m_1^{(n)}, n_1^{(n)}|w^{(n)}))$, some function $f(\cdot, \cdot|w)$ for each $w \in \mathcal{W}$. In order for $\mathcal{S}_2$ to send s_{21} and s_{22} (recall that these are bin indices), its encoder is given the message x_1^n (equivalently $m_1^n(s_{11})$ and $n_1^n(s_{12})$ if $\mathcal{S}_2$ knows how they map to x_1^n) to be transmitted by $\mathcal{S}_1$. To send s_{21}, the encoder looks in bin s_{21} for a sequence n_2^n such that $(n_2^n, m_1^n, n_1^n, w^n)$ are jointly typical. The message s_{22} analogously yields a sequence m_2^n. The transmitted x_2^n is generated i.i.d. according to $\prod_{t=1}^n p(x_2^{(t)}|m_2^{(t)}, n_2^{(t)}, w^{(t)})$.

Decoding: We describe the strong joint typicality based decoding for $\mathcal{R}_1$ only. $\mathcal{R}_2$ decodes independently from $\mathcal{R}_1$, and decodes in an analogous fashion. The receiver $\mathcal{R}_1$ forms the set, for the given w^n and y_1^n, $S_1(y_1^n, w^n) := \{(m_1^n, n_1^n, n_2^n) : (y_1^n, m_1^n, n_1^n, n_2^n, w^n) \in A_\epsilon^n(Y_1, M_1, N_1, N_2|W)\}$. Since $\mathcal{R}_1$ will be decoding message and bin indices, let $B(m_1^n)$ and $B(n_1^n)$ be the message indices of the n-sequences m_1^n, n_1^n respectively, while $B(m_2^n)$ and $B(n_2^n)$ are bin indices of the n-sequences m_2^n, n_2^n respectively. Then if all $(m_1^n, \cdot, \cdot) \in S_1(y_1^n, w^n)$ have the same message index, we decode the message $B(m_1^n)$. Otherwise, an error is declared. Define the decoding for n_1^n, m_2^n and n_2^n analogously. For the full probability of error analysis, we refer to the Appendix of [5]. In brief, to ensure correct encoding, we must ensure a sufficient number of n-sequences are placed in each bin for $\mathcal{S}_2$ so that for any x_1^n, we can find a jointly typical sequence m_2^n or n_2^n in the desired bin. Using the method of [8], with high probability, we can find a sequence u^n in a desired bin that is jointly typical with a given x^n provided there are at least $I(U; X)$ sequences in that bin. Correct encoding is thus ensured by equations 3 and 4. The remaining equations correspond to the capacity region of the two MAC channels from $(M_1, N_1, N_2) \to Y_1$ and $(N_1, M_2, N_2) \to Y_2$, and will ensure correct decoding. Thus, if $(R_{11}, R_{12}, R_{21}, R_{22})$ and (L_{21}, L_{22}) are as in the statement of the theorem, then reliable communication is possible. ■

Direct application of Lemma 2.1 in [9] to C^m_{COG} demonstrates that if the rate quadruple $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable for C^m_{COG}, then the rate pair $(R_{11}+R_{12}, R_{21}+R_{22})$ is achievable for C_{COG}.

Another important rate pair for the genie-aided cognitive radio channel is achievable: that in which $\mathcal{S}_2$ transmits no information of its own to $\mathcal{R}_2$, and simply aids $\mathcal{S}_1$ in sending its message to $\mathcal{R}_1$. When this is the case, the rate pair $(R_1^*, 0)$ is achievable, where R_1^* is the capacity of the vector channel $(\mathcal{S}_1, \mathcal{S}_2) \to \mathcal{R}_1$.

Theorem 2 *Consider the vector channel* $(\mathcal{S}_1, \mathcal{S}_2) \to \mathcal{R}_1$ *described by the conditional probability density* $p(y_1|x_1, x_2)$ *for all* $y_1 \in \mathcal{Y}_1$, $x_1 \in \mathcal{X}_1$, $x_2 \in \mathcal{X}_2$*. The rate pair* $(R_1^*, 0)$ *is achievable, where*

$$R_1^* := \max_{p(x_1, x_2)} I(X_1, X_2; Y_1). \quad (19)$$

□

Note however, that the analogous rate pair $(0, R_1^*)$ is not achievable, since that would involve $\mathcal{S}_1$ aiding $\mathcal{S}_2$ in sending its message, which cannot happen under our assumptions; $\mathcal{S}_2$ knows $\mathcal{S}_1$'s message, but not vice versa. Finally, by the usual time-sharing arguments,

Theorem 3 *The convex hull of the points of Theorem 1 and Theorem 2 is achievable.* □

Next, an achievable region is demonstrated in the Gaussian case.

III. The Gaussian Cognitive Radio Channel

Consider the genie-aided cognitive radio channel, depicted in Fig. 3 with independent additive noise $Z_1 \sim \mathcal{N}(0, Q_1)$ and $Z_2 \sim \mathcal{N}(0, Q_2)$. In order to determine an achievable region for the modified Gaussian genie-aided cognitive radio channel, specific forms of the random variables described in Theorem 1 are assumed. As in [2, 7, 9], Theorems 1, 2 and 3 can readily be extended to memoryless channels with discrete time and continuous alphabets by finely quantizing the input, output, and interference variables (Gaussian in this case). Let W, the time-sharing random variable, be constant. Consider the

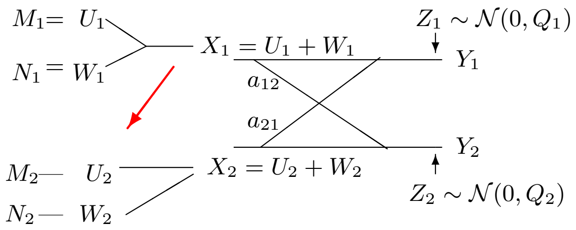


Fig. 3: The modified Gaussian genie-aided cognitive radio channel with inputs X_1, X_2, auxiliary random variables $U_1, W_1, U_2, W_2, M_1, N_1, M_2, N_2$, outputs Y_1, Y_2, additive Gaussian noise Z_1, Z_2 and multiplicative interference coefficients a_{12}, a_{21}.

case where, for certain $\alpha, \beta \in \mathbb{R}$ and $\lambda, \overline{\lambda}, \gamma, \overline{\gamma} \in [0,1]$, with $\lambda + \overline{\lambda} = 1$, $\gamma + \overline{\gamma} = 1$, and additional independent auxiliary random variables U_1, W_1, U_2, W_2 as in Fig. 3, the following hold:

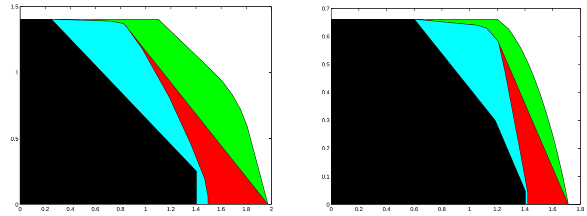
$$
\begin{aligned}
U_1 &= M_1 \quad \sim \mathcal{N}(0, \lambda P_1) \\
W_1 &= N_1 \quad \sim \mathcal{N}(0, \overline{\lambda} P_1) \\
X_1 &= U_1 + W_1 = M_1 + N_1 \quad \sim \mathcal{N}(0, P_1) \\
M_2 &= U_2 + \alpha X_1 \quad \text{where } U_2 \sim \mathcal{N}(0, \gamma P_2) \\
N_2 &= W_2 + \beta X_1 \quad \text{where } W_2 \sim \mathcal{N}(0, \overline{\gamma} P_2) \\
X_2 &= U_2 + W_2 \quad \sim \mathcal{N}(0, P_2).
\end{aligned}
$$

The achievable regions thus obtained for the Gaussian genie-aided cognitive radio channel are plotted in Fig. 4. The innermost region (black) corresponds to the achievable region of [9], and is obtained by setting $\alpha = \beta = 0$. As expected, because of the extra information at the encoder and the partial use of a dirty-paper coding technique, our achievable region in Theorem 1, the second to smallest region (cyan) in Fig. 4, extends that of [9]. Our overall achievable region, that of Theorem 3, further extends that of Theorem 1, as seen by the second largest (red) region in Fig. 4. Simulations were carried out until further simulations extended the regions negligibly.

An upper bound on our achievable rate region is provided by the 2×2 Gaussian MIMO broadcast channel, whose capacity was recently computed in [19]. Here, the two senders can fully cooperate (fully symmetric system). We calculate this region for input covariance constraint matrix of the form $S = \begin{pmatrix} P_1 & c \\ c & P_2 \end{pmatrix}$, for some $-\sqrt{P_1 P_2} \leq c \leq \sqrt{P_1 P_2}$ (which ensures S is positive semi-definite), and which mimics the power constraints P_1 and P_2 on each individual sender (asymmetric problem). The largest region in Fig. 4 is the intersection of the 2×2 Gaussian MIMO broadcast channel capacity region with the bound on $\mathcal{S}_2$'s rate $R_2 \leq \frac{1}{2}\log(1 + P_2/Q_2)$ provided by the interference-free channel in which $a_{12} = 0$.

IV. Conclusion

Although interest in cognitive radio technology has exploded recently, theoretical knowledge concerning its limits is still being acquired. In this paper, we contribute to this



(a) (b)

Fig. 4: The innermost polyhedron (black) is the achievable region of [9]. The next to smallest (cyan) is the achievable region for the genie-aided cognitive radio channel in Theorem 1. The second to largest region (red) is the achievable region of the cognitive radio channel (Theorem 3). The largest region (green) is the intersection of the capacity region of the 2×2 MIMO broadcast channel with the outer bound on R_2 of an interference-free Gaussian channel of capacity $1/2\log(1+P_2/Q_2)$. In (a) $Q_1 = Q_2 = 1$, $a_{12} = a_{21} = 0.55$, $P_1 = P_2 = 6$, in (b) $Q_1 = Q_2 = 1$, $a_{12} = a_{21} = 0.55$, $P_1 = 6$, $P_2 = 1.5$. *Note that since $\mathcal{S}_2$ knows $\mathcal{S}_1$'s message, it could aid $\mathcal{S}_1$ in sending it and boost R_1 above the interference-free channel case of $a_{21} = 0$, up to the vector channel rate of R_1^*.*

emerging field by defining and proving an achievable region for a more flexible and potentially more efficient transmission model for cognitive radio channels. In contrast to the traditional cognitive radio channel model or protocol in which a sender fills voids in time/spectrum (i.e., wait for silence or unused frequencies), a second sender may transmit with an existing sender at the same time or in the same frequency band. Thus the generalized *cognitive radio channel* is modeled as an interference channel in which two senders (more generally m) communicate over a common medium to two independent, non-cooperating receivers (more generally n), and the k-th sender knows the messages of the $k-1$ preceding senders. We computed an achievable region for the *genie-aided cognitive radio channel* in which one sender is non-causally given the other's message. In this scheme, the sender with the non-causal interference knowledge uses dirty paper coding, as in [2], to cancel the interference from $\mathcal{S}_1$ to $\mathcal{S}_2$. Dirty paper coding is performed on top of the information-separating technique first proposed by Han and Kobayashi in [9], which yields, in most cases [14], the largest to date known achievable region for the interference channel. Simulations in a Gaussian noise case show that the region achieved approaches the 2×2 MIMO channel upper bound, as well as the ideal upper bound on R_2 provided by an interference-free channel. We described a coding technique and provided theoretical answers to some of the questions in the emerging field of cognitive radios.

References

[1] A. Carleial, "Interference channels," *IEEE Trans. Inform. Theory*, vol. IT-24, no. 1, pp. 60–70, Jan. 1978.

[2] M. Costa, "Writing on dirty paper," *IEEE Trans. Inform. Theory*, vol. IT-29, pp. 439–441, May 1983.

[3] T. Cover and M. Chiang, "Duality between channel capacity and rate distortion," *IEEE Trans. Inform. Theory*, vol. 48, no. 6, 2002.

[4] T. Cover and J. Thomas, *Elements of Information Theory.* New York: John Wiley & Sons, 1991.

[5] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," Submitted to *IEEE Trans. Inform. Theory*, Nov. 2004.

[6] F. C. C. S. P. T. Force, "Fcc report of the spectrum efficiency working group," FCC, Tech. Rep., 2002.

[7] R. G. Gallagher, *Information Theory and Reliable Communication.* New York: Wiley, 1968, ch. 7.

[8] S. Gel'fand and M. Pinsker, "Coding for channels with random parameters," *Probl. Contr. and Inform. Theory*, vol. 9, no. 1, pp. 19–31, 1980.

[9] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inform. Theory*, vol. IT-27, no. 1, pp. 49–60, 1981.

[10] W. D. Horne, "Adaptive spectrum access: Using the full spectrum space." [Online]. Available: http://tprc.org/papers/2003/225/Adaptive_Spectrum_Horne.pdf

[11] T. Hunter, A. Hedayat, M. Janani, and A. Nostratinia, "Coded cooperation with space-time transmission and iterative decoding," in *WNCG Wireless Networking Symposium*, Oct. 2003.

[12] J.Mitola, "The software radio architecture," *IEEE Commun. Mag.*, vol. 33, no. 5, pp. 26–38, May 1995.

[13] ——, "Cognitive radio," Ph.D. dissertation, Royal Institute of Technology (KTH), 2000.

[14] G. Kramer, "Outer bounds on the capacity of Gaussian interference channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 3, Mar. 2004.

[15] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[16] P. Mitran, H. Ochiai, and V. Tarokh, "Space-time diversity enhancements using collaborative communication," To appear in *IEEE Trans. Inform. Theory*, 2005.

[17] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[18] ——, "User cooperation diversity–part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.

[19] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian MIMO broadcast channel," Submitted to *IEEE Trans. Inform. Theory*, July 2004.