Outer Bounds for the Interference Channel with a Cognitive Relay

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***Abstract*—In this paper, we first present an outer bound for a general interference channel with a cognitive relay, i.e., a relay that has *non-causal knowledge* of both independent messages transmitted in the interference channel. This outer bound reduces to the capacity region of the deterministic broadcast channel and of the deterministic cognitive interference channel the through nulling of certain channel inputs. It does not, however, reduce to that of certain deterministic interference channels for which capacity is known. As such, we subsequently tighten the bound for channels whose outputs satisfy an "invertibility" condition. This second outer bound now reduces to the capacity of the special class of deterministic interference channels for which capacity is known. The second outer bound is further tightened for the high-SNR deterministic approximation of the Gaussian channel by exploiting the special structure of the interference. We provide an example that suggests that this third bound is tight in at least some parameter regimes for the high-SNR deterministic approximation of the Gaussian channel. Another example shows that the third bound is capacity in the special case where there are no direct links between the non-cognitive transmitters.**

***Index Terms*—cognitive channel, interference channel, broadcast channel, relay channel, deterministic channel, high-SNR deterministic approximation of Gaussian channels.**

I. Introduction

The interference channel with a cognitive relay (IFC-CR) is a channel model of contemporary interest as it encompasses several multi-user and cognitive channel models. The IFC-CR consists of a classical two-user interference channel in which the two independent messages, each known at the corresponding source node, are also *non-causally known* at a third transmitter node, which we term the *cognitive relay* and that serves only to aid the two source nodes in their transmissions. This five-node channel generalizes a number of known channels including the broadcast (BC), the interference (IFC), and the cognitive interference channel (C-IFC).

Past work. The IFC with a relay was first introduced in [1] and [2], where the message knowledge at the relay was obtained causally and non-causally, respectively. In this work we focus on the *non-causal* version of the problem [2], sometimes also referred to as the "broadcast channel with cognitive relays" [3], and thus we will not review the large body of work related to the causal (i.e., non-cognitive) relay model.

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In [2], an achievable rate region that combines dirty-paper coding, beamforming and interference reduction techniques is derived for the Gaussian SISO IFC-CR. In [4], the achievable region of [2] is further improved upon and a sum-rate outer bound is proposed; it is shown that it is possible to achieve the degrees of freedom of a two-user interference-free channel for a large range of channel parameters. In [3], an achievable rate region that contains all previously known achievable rate regions is proposed. To the best of the authors' knowledge, outer bounds for the general (i.e., not Gaussian as in [4]) IFC-CR have not yet been considered.

Contributions. In this paper, we:
1) derive an outer bound for a general IFC-CR;
2) note that the derived outer bound reduces to the capacity region of deterministic BCs [5], of deterministic C-IFCs [6], but not to that of the class of deterministic IFCs studied in [7];
3) tighten it for deterministic IFC-CRs whose outputs satisfy an "invertibility" condition as in [7];
4) tighten it even further for the high-SNR linear deterministic approximation of the Gaussian IFC-CR (just referred to as *high-SNR channel* for short in the following), by generalizing the approach of [7] to exploit the special interference structure;
5) illustrate the achievability of this last outer bound for some parameters of the high-SNR channel.

Organization. The rest of the paper is organized as follows: Section II formally defines the channel model; Section III presents our outer bound, and shows how it may be tightened for certain deterministic IFC-CRs and for the high-SNR channel; Section IV shows achievability of the tightened outer bound for certain parameters of the high-SNR channel; and Section V concludes the paper.

II. Channel model, notation and definitions

We consider the two-user IFC-CR depicted in Fig. 1, in which the transmission of the two independent messages $W_i \in \{1, 2, ..., 2^{NR_i}\}$, $i \in \{1, 2\}$, is aided by a single *cognitive* relay, whose input to the channel has subscript c. The relay is *non-causally cognizant of both messages*. We assume classical definitions for achievable rates, and capacity inner and outer bound regions [8]. The notation $P^N_{Y_1,Y_2|X_1,X_2,X_c}$ represents the N-fold memoryless extension of the channel $P_{Y_1,Y_2|X_1,X_2,X_c}$, which describes the relationship between the channel inputs X_1, X_2, X_c and the channel outputs Y_1, Y_2.

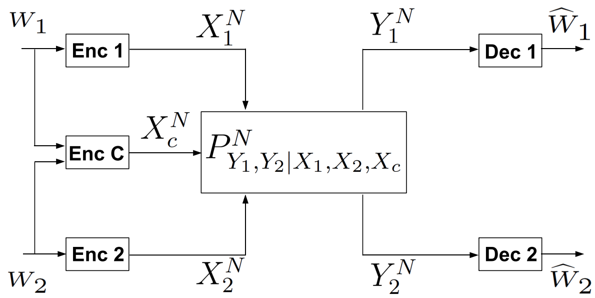

Fig. 1. The interference channel with a cognitive relay (IFC-CR).

The IFC-CR contains three well-studied multi-user channels as special cases:
a) Interference channel (IFC): if $X_c = \emptyset$;
b) Broadcast channel (BC): if $X_1 = X_2 = \emptyset$; and
c) Cognitive channel (C-IFC): if $X_1 = \emptyset$ or $X_2 = \emptyset$.

The largest known achievable rate region for the IFC-CR presented in [3] combines ideas from the achievable rate regions of these three special channel models it subsumes. Outer bounds only exist for the sum-rate of the Gaussian SISO IFC-CR [4]. In the next section we derive an outer bound for the whole capacity region of a general IFC-CR.

III. Outer bounds for the IFC-CR

We first derive an outer bound valid for all memoryless IFC-CRs. We then tighten this bound by developing further inequalities for a class of deterministic channels and for the high-SNR channel in the spirit of [7]. Finally, we evaluate our tightened bound for the high-SNR channel.

A. General IFC-CR outer bounds

Theorem III.1. *If (R_1, R_2) lies in the capacity region of the IFC-CR, then the following must hold for any $\widetilde{Y}_1$ and $\widetilde{Y}_2$ having the same marginal distributions as Y_1 and Y_2, respectively, but otherwise arbitrarily correlated:*

$$R_1 \le I(Y_1; X_1, X_c|Q, X_2), \tag{1a}$$
$$R_2 \le I(Y_2; X_2, X_c|Q, X_1), \tag{1b}$$
$$R_1 + R_2 \le I(Y_2; X_1, X_2, X_c|Q) + I(Y_1; X_1, X_c|Q, \widetilde{Y_2}, X_2), \tag{1c}$$
$$R_1 + R_2 \le I(Y_1; X_1, X_2, X_c|Q) + I(Y_2; X_2, X_c|Q, \widetilde{Y_1}, X_1), \tag{1d}$$

for some input distribution

$$P_{Q,X_1,X_2,X_c} = P_Q P_{X_1|Q} P_{X_2|Q} P_{X_c|X_1,X_2,Q}.$$

Proof: We only outline the proof here for sake of space. The complete proof of this and all the other theorems in the paper may be found in [9]. The outer bound may be thought of as the intersection of two C-IFC outer bounds [6] obtained by non-causally providing one of the transmitters with the message of the other transmitter. For the sum-rates, since the receivers cannot cooperate, the capacity cannot depend on the correlation among the output signals, as first observed in [10] for BCs. By giving a receiver as side-information a signal that has the same marginal distribution as the other user's output, but that is otherwise arbitrarily correlated with its own output, we obtain the two sum-rate bounds. The same idea was used in [6] for the C-IFC and in [11] for cooperative IFCs. ■

Remark 1: Th. III.1 reduces to the capacity region of a deterministic BC when $X_1 = X_2 = \emptyset$ and to the capacity of a deterministic C-IFC when either $X_2 = \emptyset$ or $X_1 = \emptyset$. However, Th. III.1 does not reduce to the capacity region of the class of deterministic IFCs studied in [7] when $X_c = \emptyset$. In the following we thus develop additional rate bounds to cover this latter case. □

B. Further bounds for a class of IFC-CRs

Consider, in the spirit of [7], IFC-CRs whose outputs satisfy:

$$\begin{aligned} &Y_1 = f_1(X_1, X_c, V_{12}),\ V_{12} = g_2(X_2, Z_1) : \\ &\quad H(Y_1|X_1, X_c) = H(V_{12}|X_1, X_c) = H(V_{12}|X_c), \end{aligned} \tag{2a}$$
$$\begin{aligned} &Y_2 = f_1(X_2, X_c, V_{21}),\ V_{21} = g_1(X_1, Z_2) : \\ &\quad H(Y_2|X_2, X_c) = H(V_{21}|X_2, X_c) = H(V_{21}|X_c), \end{aligned} \tag{2b}$$

where the functions f_1, f_2, g_1 and g_2 are deterministic, and Z_1 and Z_2 are "noise" random variables (RVs) independent of the inputs. Notice the invertibility conditions in (2a) and (2b) (and recall that $X_1 = X_1(W_1)$ is independent of $X_2 = X_2(W_2)$). We tighten the outer bound of Th.III.1 as follows:

Theorem III.2. *If (R_1, R_2) lies in the capacity region of the IFC-CR, then the following must hold:*

$$R_1 \le \text{(1a)},\ R_2 \le \text{(1b)},\ R_1 + R_2 \le \min\{\text{(1c)}, \text{(1d)}\}, \tag{3a}$$
$$\begin{aligned} N(R_1 + R_2) \le\ & I(V_{21}^N; X_c^N) + H(Y_1^N|\widetilde{V}_{21}^N) - H(\widetilde{V}_{21}^N|X_1^N) \\ & + I(V_{12}^N; X_c^N) + H(Y_2^N|\widetilde{V}_{12}^N) - H(\widetilde{V}_{12}^N|X_2^N), \end{aligned} \tag{3b}$$
$$\begin{aligned} N(2R_1 + R_2) \le\ & -H(\widetilde{V}_{21}^N|X_1^N) - 2H(V_{12}^N|X_2^N) \\ & + H(Y_1^N) + H(Y_1^N|\widetilde{V}_{21}^N, X_2^N) + H(Y_2^N|\widetilde{V}_{12}^N) \\ & + I(V_{12}^N; X_c^N) + I(V_{21}^N; X_c^N), \end{aligned} \tag{3c}$$
$$\begin{aligned} N(R_1 + 2R_2) \le\ & -H(\widetilde{V}_{12}^N|X_2^N) - 2H(V_{21}^N|X_1^N) \\ & + H(Y_2^N) + H(Y_2^N|\widetilde{V}_{12}^N, X_1^N) + H(Y_1^N|\widetilde{V}_{21}^N) \\ & + I(V_{21}^N; X_c^N) + I(V_{12}^N; X_c^N), \end{aligned} \tag{3d}$$

where (3a) holds under the hypothesis of Th.III.1, and where $\widetilde{V}_{21}, \widetilde{V}_{12}$ are conditionally independent copies of V_{12} and V_{21}, that is, distributed jointly with (Q, X_1, X_2, X_c) with $P_{\widetilde{V}_{21},\widetilde{V}_{12}|Q,X_1,X_2,X_c} = P_{\widetilde{V}_{21}|Q,X_1} P_{\widetilde{V}_{12}|Q,X_2}$.

Proof: The proof may be found in [9]. ■

Remark 2: The single-letterization of the outer bound in Theorem III.2 is not straightforward: the term $I(V_{ij}^N = g_i(X_j^N, Z_i); X_c^N)$ cannot be single-letterized using standard arguments since X_j and X_c can have any joint distribution. For discrete alphabets, this term can be upper bounded as $I(V_{ij}^N; X_c^N) \le N \min\{H(V_{ij}), H(X_c)\}$. □

Remark 3: When $X_c = \emptyset$, the outer bound in Th. III.2 reduces to that of the deterministic IFCs considered in [7]. □

C. Outer bound for the high-SNR IFC-CR

The outer bound of Th. III.2 may be further tightened for the high-SNR IFC-CR. This channel, as developed in [7], models a Gaussian noise channel as the receive SNRs grow to infinity. The high-SNR channel is a deterministic binary linear channel with outputs:

$$Y_u = \boldsymbol{S}^{m-n_{u1}} X_1 \ \oplus\ \boldsymbol{S}^{m-n_{uc}} X_c \ \oplus\ \boldsymbol{S}^{m-n_{u2}} X_2, \tag{4}$$

for $u \in \{1,2\}$, where the inputs are binary vectors of length $m \triangleq \max\{n_{11}, n_{12}, n_{21}, n_{22}, n_{1c}, n_{2c}\}$, $\boldsymbol{S}$ is a shift matrix of dimensions $m \times m$, and $\oplus$ denotes the binary XOR operation. The high-SNR channel belongs to the class of deterministic IFC-CRs whose outputs are described by:

$$Y_1 = f_1(X_1, \ell_1(V_{1c}, V_{12})),\ V_{12} = g_2(X_2),\ V_{1c} = h_1(X_c),$$
$$Y_2 = f_2(X_2, \ell_2(V_{2c}, V_{21})),\ V_{21} = g_1(X_1),\ V_{2c} = h_2(X_c),$$

for some deterministic functions $f_1, f_2, \ell_1, \ell_2, g_1, g_2, h_1, h_2$ and subject to the invertibility conditions in (2a) and (2b).

The capacity achieving strategy for the high-SNR channel has provided insights on capacity approaching strategies for the corresponding Gaussian channel, and has allowed the determination of capacity to within a constant gap for the IFC [12] and C-IFC [13]. We hope that a similar result may be derived for the Gaussian IFC-CR using achievable schemes inspired by the high-SNR approximation. For the high-SNR channel, we tighten the rate bounds in (3) by replacing the term $I(V_{21}^N; X_c^N)$ (resp. $I(V_{12}^N; X_c^N)$) with $I(V_{21}^N; V_{2c}^N)$ (resp. $I(V_{12}^N; V_{1c}^N)$). This "substitution" of X_c by its functions $V_{1c} = h_1(X_c)$ and $V_{2c} = h_2(X_c)$ is not possible in general since it is not generally known how the input X_c affects the channel outputs.

Remark 3: This step of tightening the bound highlights the stumbling block in deriving outer bounds for general IFC and BCs: in general we do not know the *exact* form of the interfering signal(s) at a given receiver for any possible input distribution. Assuming that the channel is deterministic and in a certain way "invertible", allows one to exactly determine the interference. Notice also that in the tightened bound, "conditioning" on the interference generated by X_j at the output Y_i, given by V_{ij} (rather than on X_j itself), implies that the interference has been removed without necessarily decoding the message corresponding to X_j. □

Evaluation of the tightened bound yields:

Theorem III.3. *If (R_1, R_2) lies in the capacity region of the high-SNR IFC-CR, then*

$$R_1 \le \max\{n_{11}, n_{1c}\} \tag{5a}$$
$$R_2 \le \max\{n_{22}, n_{2c}\} \tag{5b}$$
$$\begin{aligned} R_1 + R_2 \le\ & 1_{\{n_{11}-n_{1c} \neq n_{21}-n_{2c}\}} \left([n_{11} - \max\{n_{12}, n_{1c}\}]^+ \right. \\ & \left. + \max\{n_{22} + n_{1c}, n_{2c} + n_{12}\}\right) \\ & + 1_{\{n_{11}-n_{1c} = n_{21}-n_{2c}\}} \left(\max\{n_{22}, n_{21}, n_{2c}\} \right. \\ & \left. + [n_{11} - n_{21}]^+\right) \end{aligned} \tag{5c}$$
$$\begin{aligned} R_1 + R_2 \le\ & 1_{\{n_{22}-n_{2c} \neq n_{12}-n_{1c}\}} \left([n_{22} - \max\{n_{21}, n_{2c}\}]^+ \right. \\ & \left. + \max\{n_{11} + n_{2c}, n_{1c} + n_{21}\}\right) \\ & + 1_{\{n_{22}-n_{2c} = n_{12}-n_{1c}\}} \left(\max\{n_{11}, n_{12}, n_{1c}\} \right. \\ & \left. + [n_{22} - n_{12}]^+\right) \end{aligned} \tag{5d}$$
$$\begin{aligned} R_1 + R_2 \le\ & \max\{n_{11} - n_{21}, n_{12}, n_{1c}\} + \min\{n_{1c}, n_{12}\} \\ & + \max\{n_{22} - n_{12}, n_{21}, n_{2c}\} + \min\{n_{2c}, n_{21}\} \end{aligned} \tag{5e}$$
$$\begin{aligned} 2R_1 + R_2 \le\ & \max\{n_{11}, n_{12}, n_{1c}\} \\ & + \max\{n_{11} - n_{21}, n_{12}, n_{1c}\} + \min\{n_{1c}, n_{12}\} \\ & + \max\{n_{22} - n_{12}, n_{21}, n_{2c}\} + \min\{n_{2c}, n_{21}\} \end{aligned} \tag{5f}$$
$$\begin{aligned} R_1 + 2R_2 \le\ & \max\{n_{22}, n_{21}, n_{2c}\} \\ & + \max\{n_{11} - n_{21}, n_{12}, n_{1c}\} + \min\{n_{1c}, n_{12}\} \\ & + \max\{n_{22} - n_{12}, n_{21}, n_{2c}\} + \min\{n_{2c}, n_{21}\}. \end{aligned} \tag{5g}$$

Proof: The rate-bounds in the tightened version of Th. III.2 can be upper bounded by using the fact that the uniform distribution maximizes the entropy of a discrete RV [9] to obtain the rate region in (5). The multi-letter mutual information terms can be single-letterized as described in Remark 2. ■

IV. Achieving the outer bound in Th. III.3

While it remains to be shown that the outer bound of Th.III.3 is tight for the general high-SNR channel, in this section we demonstrate by example that it is achievable for certain channel parameters. We consider two examples: Example I: the *strong signal, mixed cognition and weak interference regime at both decoders* given by $n_{11} > n_{1c} > n_{12}$ and $n_{22} > n_{2c} > n_{21}$; and Example II: the *no-interference regime for both decoders* given by $n_{12} = n_{21} = 0$.

A. Example I

Corollary IV.1. *In the case of strong signal, mixed cognition and weak interference at both decoders, the capacity is*

$$R_1 \le n_{11}, \quad R_2 \le n_{22}.$$

Proof: It can be shown that the outer bound of Th.III.3 reduces to the region in Corollary IV.1 when $n_{11} > n_{1c} > n_{12}$ and $n_{22} > n_{2c} > n_{21}$ [9]. The formal proof of the achievability of the point $(R_1, R_2) = (n_{11}, n_{22})$ is provided in [9]. We provide a sketch of the proof aided by the graphical representation of the achievable scheme in Fig. 2. Our aim is to highlight the innovative cooperation strategy implemented by the cognitive relay compared to the capacity achieving strategies of the high-SNR IFC [12] and of the high-SNR C-IFC [14].

Extensions of the IFC and C-IFC. In Fig. 2, the left section represents the three channel inputs X_1, X_c, X_2 and the right section represents the channel outputs Y_1 and Y_2. Each output is the modulo-2 sum of the three (down-shifted) inputs. The (blue, $45°$ hatched) blocks in the upper-left section are the bits sent by user 1; the (red, $-45°$ hatched) blocks in the lower-left section are the bits sent by user 2. The down-shifted version of blue and red blocks appear on the right section. When the cognitive relay is absent, our channel model reduces to the high-SNR IFC of [12]. In this channel cooperation is not possible and the transmission of one encoder produces interference at the non intended receiver. Receiver 1 observes $n_{11} - n_{12}$ of the (blue, $45°$ hatched) bits from encoder 1 above the n_{12} (red, $-45°$ hatched) bits from encoder 2. Decoder 1 has no knowledge of the interference produced by encoder 2 and thus is able to decode only the most significant $n_{11} - n_{12}$ (blue, $45°$ hatched) bits. Similarly, receiver 2 only decodes the most significant $n_{22} - n_{21}$ (red,$-45°$ hatched) bits received above the interference. Without cognitive relay is possible to achieve only $(R_1, R_2) = (n_{11} - n_{12}, n_{22} - n_{21})$.

When the cognitive relay is present, it can pre-cancel the interference experienced by one decoder, as in the high-SNR C-IFC [14]. Let the input of the cognitive relay (mid-section on the left side) be non-zero only in the blue shaded block.

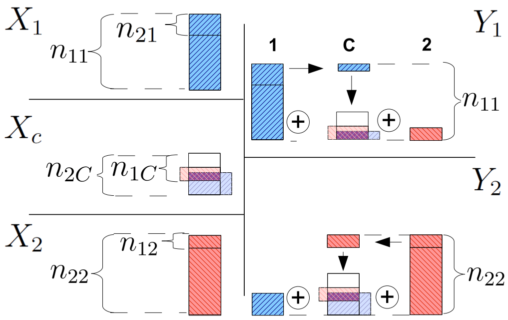

Fig. 2. Capacity achieving scheme for Example I.

By placing in the blue-shaded block the same n_{21} (blue, $45°$ hatched) bits that interfere at decoder 2, the relay pre-cancels the interference at this user's receiver. The achievable rates in this case are $(R_1, R_2) = (n_{11} - n_{12}, n_{22})$. In a similar manner, the cognitive relay can pre-cancel the interference generated by user 2 at receiver 1 by using the red-shaded block (mid-section on the left side in Fig. 2). With this strategy, we pre-cancel the interference at a single decoder only and we improve the rates with respect to the case where the cognitive relay is absent; however, we are unable to achieve $(R_1, R_2) = (n_{11}, n_{22})$.

A unique scheme for the IFC-CR. To achieve the outer bound $(R_1, R_2) = (n_{11}, n_{22})$, we must be able to pre-cancel the interference at both decoders simultaneously. To do so, let the cognitive transmitter send the sum of the inputs that grant the pre-cancelation of the interference at a single decoder, i.e., the XOR of the (blue, $45°$ hatched) and of the (red,$-45°$ hatched) blocks in Fig. 2. With this input at the cognitive relay, Y_1 is the XOR of the signal from transmitter 1 and a shifted version of the interference at decoder 2 (purple, cross hatched block). Decoder 1 is able to decode this set of bits since $n_{11} > n_{1c}$ and remove it from Y_1. Transmitter 2 operates in a similar manner by decoding a shifted version of the interference at receiver 1 and adding it to Y_2 to obtain the message transmitted by encoder 2. This shows that the rate $(R_1, R_2) = (n_{11}, n_{22})$ is achievable. ■

The cognitive relay effectively trades an unknown interference term with a known one that each receiver is able to decode. This strategy generalizes to the case when the pre-coding by the cognitive relay against the interference at one decoder may be decoded by the other. We are currently investigating the applicability of this idea in a more general setting.

B. Example II

In this example we show that the outer bound of Th.III.3 is tight in the absence of interfering links: $n_{12} = n_{21} = 0$.

Corollary IV.2. *The capacity of the high-SNR channel without interfering links is*

$$R_1 \le \max\{n_{11}, n_{1c}\} \quad R_2 \le \max\{n_{22}, n_{2c}\}$$
$$R_1 + R_2 \le \max\{n_{22}, n_{2c}\} + \max\{n_{11}, n_{1c} - n_{2c}\}$$
$$R_1 + R_2 \le \max\{n_{11}, n_{1c}\} + \max\{n_{22}, n_{2c} - n_{1c}\}.$$

Remark 4: The region in Corollary IV.2 is a case where Th.III.1 and Th.III.2 coincide. In this case, if in addition either $n_{11} = 0$ or $n_{22} = 0$, the region reduces to the capacity region of the high-SNR C-IFC determined in [6]. □

Proof: It can be easily seen that the outer bound of Th.III.3 reduces to the region in Corollary IV.2 when $n_{11} > n_{1c} > n_{12}$ and $n_{22} > n_{2c} > n_{21}$.

We divide the achievability proof into three subcases. The achievability proofs of the first two cases below are available in [9]. The remaining achievability proof is presented graphically using the block representation introduced in Section IV-A. We note that all achievability proofs operate over a *single* channel use. All proofs are by inspection rather than through the systematic and judicious choice of RVs in a general achievable rate region such as that of [3]–a topic left for future work.

- **Capacity for weak cognition at both decoders:** when $n_{11} \ge n_{1c}$ and $n_{22} \ge n_{2c}$ the cognitive links n_{1c} and n_{2c} convey fewer clean bits than both direct links n_{11} and n_{22} respectively, and the outer bound reduces to $R_1 \le n_{11}, R_2 \le n_{22}$, achieved by keeping the cognitive relay silent.
- **Capacity for strong cognition at both decoders:** when $n_{11} < n_{1c}$, $n_{22} < n_{2c}$ the cognitive links n_{1c} and n_{2c} convey more clean bits than both direct links n_{11} and n_{22} respectively, and the outer bound simplifies to

$$R_1 \le n_{1c}, \quad R_2 \le n_{2c}$$
$$R_1 + R_2 \le \min\{\max\{n_{2c} + n_{11}, n_{1c}\}, \max\{n_{22} + n_{1c}, n_{2c}\}\}.$$

In [9] the corner points are achieved by having the two primary users send all the available clean bits along the respective direct links, and the cognitive relay utilizes its most significant bits to send bits for one user above the its direct link, attaining the single rate bound, and may use (parts of) its least significant bits to convey clean bits to the other user without creating interference with the direct transmissions.

- **Capacity for strong cognition for one decoder and weak cognition at the other:** when $n_{11} \ge n_{1c}$, $n_{22} < n_{2c}$ the cognitive link n_{1c} conveys less clean bits to decoder 1 than the direct link n_{11}; the reverse is true for n_{2c} and n_{22}. The condition $n_{11} < n_{1c}$, $n_{22} \ge n_{2c}$ is obtained by switching the role of the users. In this case, the outer bound becomes:

$$R_1 \le n_{11}, \quad R_2 \le n_{2c}$$
$$R_1 + R_2 \le n_{11} + \max\{n_{22}, n_{2c} - n_{1c}\}.$$

We again try to achieve the two corner points, but in this case each requires a different achievability scheme. We denote the binary vector of R_{iP} bits for user i as $\boldsymbol{b}_i^{R_{iP}}$. Similarly $(\boldsymbol{b}_i)_k^j$ indicates the bits between position k and j of $\boldsymbol{b}_i$. We use $\mathbf{0}^j$ to indicate a vector of length j of all zeros.

Corner point 1: To achieve the corner point where the rate bound for R_1 meets the sum rate outer bound:
i) Transmitter 1 sends n_{11} bits to receiver 1 as

$$X_1 = [\boldsymbol{b}_1^{n_{11}} \ \mathbf{0}^{m-n_{11}}],$$

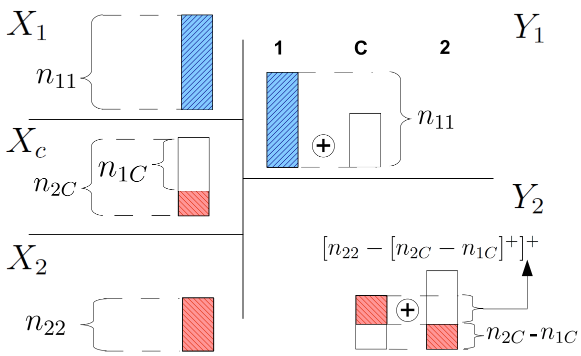

Fig. 3. The case of strong cognition for one decoder and weak cognition at the other, achievability scheme for corner point 1.

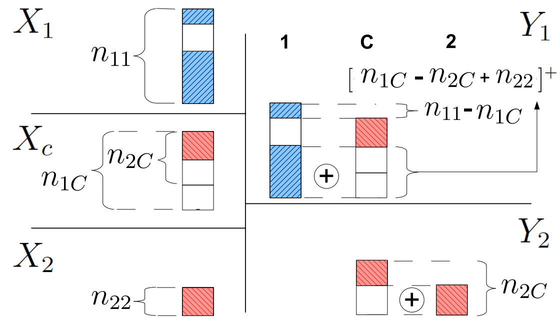

Fig. 4. The case of strong cognition for one decoder and weak cognition at the other, achievability scheme for corner point 2.

ii) The cognitive relay sends $[n_{2c} - n_{1c}]^+$ bits in the least significant bits from the cognitive relay to receiver 2 without creating interference at receiver 1 as

$$X_c = [\boldsymbol{b}_2^{[n_{2c}-n_{1c}]^+} \ \mathbf{0}^{m-[n_{2c}-n_{1c}]^+}],$$

iii) Transmitter 2 sends $[n_{22} - [n_{2c} - n_{1c}]^+]^+$ bits to be received above the bits broadcasted from the cognitive relay at receiver 2 as

$$X_2 = [(\boldsymbol{b}_2)_{[n_{2c}-n_{1c}]^+}^{[n_{2c}-n_{1c}]^+ + [n_{22}-[n_{2c}-n_{1c}]^+]^+} \ \mathbf{0}^{m-[n_{22}-[n_{2c}-n_{1c}]^+]^+}].$$

Fig. 3 graphically illustrates the scheme.

Corner point 2: To achieve the corner point where the rate bound for R_2 meets the sum rate outer bound, we use a similar strategy as for the previous corner point, but this time transmitter 1 sends additional bits above the interference coming from the cognitive relay, as:
i) Transmitter 2 sends n_{22} bits for receiver 2 through the direct link

$$X_2 = [\boldsymbol{b}_2^{n_{22}} \ \mathbf{0}^{m-n_{22}}],$$

ii) The cognitive relay sends $n_{2c} - n_{22}$ bits in the most significant bits for receiver 2 achieving the full rate $R_2 = n_{2c}$

$$X_c = [\boldsymbol{b}_2^{n_{2c}-n_{22}} \ \mathbf{0}^{m-n_{2c}+n_{22}}],$$

iii) Transmitter 1 sends $[n_{1c} - n_{2c} + n_{22}]^+$ bits below the interference from the cognitive relay at receiver 1. It also transmits $n_{11} - n_{1c}$ bits that will be received above the interference level from the cognitive relay as

$$X_1 = [\boldsymbol{b}_1^{n_{11}-n_{1c}} \ \mathbf{0}^{\min\{n_{1c}, n_{2c}-n_{22}\}} \ (\boldsymbol{b}_1)_{n_{11}-n_{1c}}^{n_{11}-n_{1c}+[n_{1c}-n_{2c}+n_{22}]^+} \ \mathbf{0}^{m-n_{11}}].$$

Fig. 4 graphically illustrates the scheme.

■

V. Conclusions

In this work we derived the first general outer bounds for the interference channel with a cognitive relay and showed the achievability of the proposed outer bound for the high-SNR deterministic approximation of the Gaussian interference channel with a cognitive relay for certain parameter regimes. The proposed outer bound is also tight for the deterministic channel models it encompasses: the deterministic broadcast channel, certain deterministic interference channels, and the deterministic cognitive interference channel. Our results leave multiple interesting open questions. We are currently investigating whether the presented high-SNR outer bound is tight in all parameter regimes. Outcomes and insights obtained for the general high-SNR capacity region will be then used to possibly determine a constant gap between an inner and our outer bound for the Gaussian channel–a result that would generalize numerous "constant gap" results including that of the interference and cognitive interference channels.

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