Capacity bounds on multi-pair two-way communication with a base-station aided by a relay

Sang Joon Kim, Besma Smida, and Natasha Devroye

***Abstract*—The multi-pair bi-directional relay network under consideration consists of one base-station, multiple (say m) terminal nodes and one relay, all of which are half-duplex, in which, contrary to prior work, each node has a direct link with every other node. Each of the m terminal nodes exchanges messages with the base-station in a bi-directional fashion, leading to $2m$ total messages to be communicated with the (possible) help of the relay. Our contributions are: 1) the introduction of three new temporal protocols which fully exploit the two-way nature of the data, over-heard side-information through network coding, random binning, and compress-and-forward terminal node cooperation, 2) derivations of achievable rate regions and 3) cut-set based outer bounds for the multi-pair network, and 4) a numerical evaluation of the derived regions in Gaussian noise which illustrate the performance of the proposed protocols.**

***Index Terms*—bi-directional relaying, decode and forward, multi-pair, binning**

I. Introduction

The simplest bi-directional *relay* network consists of a pair of *terminal nodes* that wish to exchange messages through the use of a single relay. While the capacity of this channel is still unknown in general, it has been of great recent interest (see references in [1] and [2]) due to its relevance in future wireless networks. The single relay, single pair bi-directional relay channel has been extended in a number of ways: 1) the consideration of a single bi-directional link using *multiple relays* [3]–[7], and 2) the consideration of *multiple bi-directional links* sharing a single, common relay [8]–[11].

The relay network considered in this paper falls into the second category and consists of a base station (node 0) which wishes to communicate simultaneously in a bi-directional fashion with multiple terminal nodes (node 1, $\cdots$, node m) with the help of one relay node (node r). Due to limitations of current technology, all nodes are assumed to be half-duplex and thus cannot transmit and receive simultaneously. This network topology is motivated by recent pushes to extend the coverage, reliability and/or data rates of wireless networks. For example, in a cellular scenario, a relay station is able to enhance the connectivity between a base station and terminals at its cell boundary. The relays may be connected to the base station using a wireless link rather than a wired one, resulting in savings to the operators' backhaul costs. Another motivating example is satellite communication: satellites can be used to relay signals from one ground station to multiple vehicular terminals on or close to the earth's surface. In this work, we determine bounds on the capacity regions - which may serve as guides and benchmarks in the design of - such multi-pair two-way communication networks aided by a single relay node.

Sang Joon Kim was with the School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138. Email: sangkim@post.harvard.edu. Besma Smida is with the Department of Electrical and Computer Engineering, Purdue University Calumet, Hammond, IN 46323. E-mail: Besma.Smida@calumet.purdue.edu. Natasha Devroye is with the Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, IL 60607. Email: devroye@ece.uic.edu.

A. Related work

In [8] a network in which K half-duplex source/destination pairs wish to exchange messages in a bi-directional fashion through a single multi-antenna relay is investigated from a diversity-multiplexing gain perspective. The authors of [9] consider a similar channel model and propose the use of a CDMA strategy to support multiple level QoS to different users. In [10] multiple bi-directional pairs communicate over a shared relay in the absence of a direct link between end nodes, where, in Gaussian noise, a carefully constructed superposition scheme of random and lattice codes was used. Finally, in [11], an arbitrary number of clusters (nodes within a cluster all wish to exchange messages) of arbitrary numbers of full-duplex nodes are assumed to communicate simultaneously through the use of a single relay in AWGN. In all four examples of multi-pair bi-directional communication with a single relay, no direct link between the terminal nodes is assumed to exist, simplifying the analysis as the tradeoff between relayed and directly communication information is avoided; no "over-heard" side information, or terminal node cooperation is considered.

B. Our contributions

We consider one base station, multiple terminal nodes and one relay, which operate in half-duplex mode and have direct links to each other, as shown in Fig. 1. The desired bi-directional links may be deduced from the included messages $W_{i,j}$ from node i destined to node j, and $\tilde{W}_{i,j}$ the estimate at node j of the message $W_{i.j}$. The base-station is denoted as node with index 0. Three elements of the formulated problem are markedly different from prior information theoretic work in this area:

1) the assumption that one end of the bi-directional links is a single base-station rather than independent nodes.
2) fully connected network - our nodes can all hear each other. This allows for the possibility of causal cooperation between nodes as well as direct transmission between the base-station and the nodes, using the relay only when beneficial.
3) in contrast to [10], [11], our nodes are half-duplex.

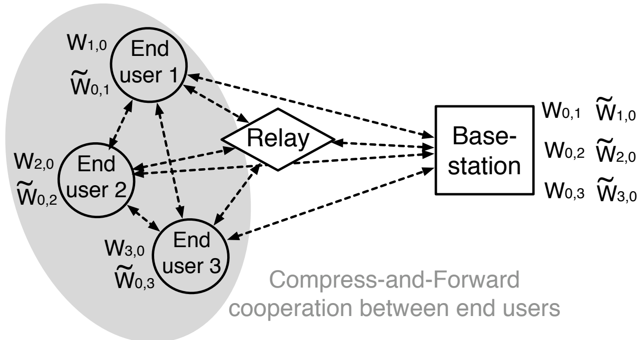

Fig. 1. Our physical channel model consists of multiple independent bi-directional desired communication flows (message $W_{i,0}$ and $W_{0,i}$ wish to be exchanged) between multiple terminal nodes and a single base-station. Communications may be aided using one relay node. $\tilde{W_{i,j}}$ is the estimate at node j of the message $W_{i,j}$.

Our central **contributions** are:

• We propose three temporal protocols which we call the FMABC *(Full Multiple Access Broadcast)*, PMABC *(Partial Multiple Access Broadcast)* and FTDBC *(Full Time Division Broadcast)* protocols.

• We determine inner bounds on the capacity region of the multi-pair bi-directional relay network. Key elements of the schemes employed to do so include the use of multi-user protocols in which more than one terminal may be transmitting/receiving at one time as in MAC and BC channels, *random binning* to exploit over-heard side-information when the protocol permits, and the use of a flow-by-flow *network coding* strategy which exploits the two-way nature of data flows - all of which will be detailed in Section III.

• We derive achievable rate regions where *cooperation* between end users is enabled. This is possible in protocols in which certain nodes *over-hear* other nodes' transmissions. To the best of the authors' knowledge, this is the first consideration of *cooperation between end-users* in a multi-pair bi-directional channel. In this work, we consider a Compress-and-Forward-based causal cooperation scheme [12], selected due to the rough intuition that CF outperforms for example DF-based cooperation/relaying when the helping node is close to the final destination, which is expected to be one of the scenarios of practical interest.

• We present modified cut-set-based *outer bounds* on the capacity region of this network.

We note that due to lack of space, the statements and proofs of many of these results are omitted, but may be found at [2].

II. Notations and Definitions

We consider a base station (node 0), a set of terminal nodes $\mathcal{B} := \{1, 2, \cdots, \mathsf{m}\}$ and a relay r which aids in the communication between the terminal nodes and the base station. We define $\mathcal{M} := \mathcal{B} \cup \{0\} = \{0, 1, 2, \cdots, \mathsf{m}\}$. We use $R_{i,j}$ to denote the rate of communication from node i to node j, i.e. the message between node i and node j, $W_{i,j}$, lies in the set $\mathcal{S}_{i,j} := \{0, \ldots, \lfloor 2^{nR_{i,j}} \rfloor - 1\}$. Similarly, $R_{S,T}$ is the sum of rates from set S to set T where $S, T \subseteq \mathcal{M}$ at which the messages $W_{S,T} := \{W_{i,j} | i \in S, j \in T,\ S, T \subseteq \mathcal{M}\}$ may be reliably communicated. We assume that each end user communicates with the base station bi-directionally and that no information is directly exchanged between end users: i.e. every pair of terminal nodes 0 and $i \in [1, m]$ wishes to exchange independent messages while $R_{i,j} = 0$ (or is undefined) for all $i, j \in \mathcal{B}$. Thus, there are a total of $2m$ messages in our network: m from node 0 to each node $i \in \mathcal{B}$, and m from each node $i \in \mathcal{B}$ to node 0, as shown in Fig. 1.

Communication takes place over a number of channel uses, n and rates are achieved in the classical asymptotic sense as $n \to \infty$ [1]. Node i has input alphabet $\mathcal{X}_i^* = \mathcal{X}_i \cup \{\varnothing\}$ and channel output alphabet $\mathcal{Y}_i^* = \mathcal{Y}_i \cup \{\varnothing\}$, which are related through a discrete memoryless channel[1]. Lower case letters x_i denote instances of the upper case X_i which lie in the calligraphic alphabets $\mathcal{X}_i^*$. Boldface $\mathbf{x}_i$ represents a vector indexed by time at node i. Finally, it is convenient to denote by $\mathbf{x}_S := \{\mathbf{x}_i | i \in S\}$, a set of vectors indexed by time, and $\bigotimes$ as the cartesian product, i.e., $\bigotimes_{i=1}^3 \mathcal{X}_i = \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3$.

During phase ℓ we use $X_i^{(\ell)}$ to denote the input distribution and $Y_i^{(\ell)}$ to denote the distribution of the received signal of node i, and we use the dummy symbol $\varnothing$ to denote that there is no input or no output at a particular node during a particular phase. $\Delta_{i,n}$ is the phase duration of phase i with block size n and Δ_i is the phase duration of phase i when $n \to \infty$. It is also convenient to define $X_S^{(\ell)} := \{X_i^{(\ell)} | i \in S\}$, a set of input distributions during phase ℓ.

For a block length n, encoders and decoders are functions $X_i^k(W_{\{i\},\mathcal{M}}, Y_i^1, \cdots, Y_i^{k-1})$ producing an encoded message at node i, and $\tilde{W}_{i,j}(Y_j^1, \cdots, Y_j^n, W_{\{j\},\mathcal{M}})$ producing a decoded message or error at node j when it wishes to decode the message $W_{i,j}$ from node i. Let $S(j) := \{i | i < j, i \in S\}$.

III. Protocols for a multi-pair bi-directional relay network

The total transmission time is divided into two time *periods*, each of which may consist of one or more *phases*. During the first *multiple access* period, the terminal nodes transmit to the relay. During the second *broadcast* period the relay transmits to the terminal nodes. We consider three transmission schemes for the multiple-access period: 1) *Full Multiple Access Broadcast* (FMABC) protocol: all terminal nodes transmit for the whole duration, 2) *Partial Multiple Access Broadcast* (PMABC) protocol: 0 uses the whole duration and the other terminal nodes $1, \cdots, \mathsf{m}$ transmit sequentially, and 3) *Full Time Division Broadcast* (FTDBC) protocol: all nodes transmit sequentially, as shown in Fig. 2.

For comparison purposes in our simulations, we also introduce what we call the *simplest sequential protocol* where all terminal nodes sequentially transmit information to the relay, i.e., $0 \to \mathsf{r}$, $1 \to \mathsf{r}, \cdots, \mathsf{m} \to \mathsf{r}$, then the relay sequentially transmits them to the proper destinations, i.e., $\mathsf{r} \to 0$, $\mathsf{r} \to 1, \cdots, \mathsf{r} \to \mathsf{m}$.

The FMABC, PMABC and FTDBC protocols describe the temporal phases or periods of the transmission scheme but

[1] Extensions to Gaussian noise channels will be addressed in Section V.

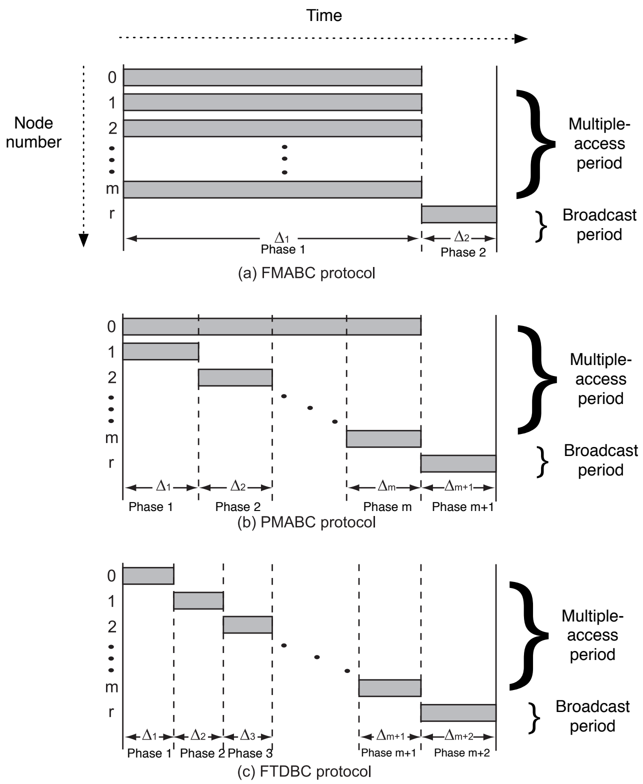

Fig. 2. Three proposed half-duplex protocols - the time phases of the different protocols are seen; the encoders and decoders in the different phases may vary.

not what each node sends, or how its messages are encoded during those phases. The central technical concepts employed in deriving achievable rate regions are:

1) Extended Marton's region for broadcasting: Due to the presence of a base-station with multiple messages (one to each of the terminal nodes), and a relay with multiple decoded messages (traveling to multiple end users and the base-station), we use a *modified* version of a generalization of Marton's broadcasting scheme [13] to >2 messages/users, which takes into account own-message side-information at each node. A full statement of this generalization may be found in [2].

2) Network Coding: Network coding on a *flow-by-flow* (each flow consists of two bi-directional messages $W_{i,0}$ and $W_{0,i}$) basis is used at the relay r, which decodes $\{w_{0,i}\}$ and $\{w_{i,0}\}$, at the end of the multiple access period, and constructs $w_{\mathsf{r}_i} = w_{0,i} \oplus w_{i,0}$, $\forall i \in \mathcal{B}$. Next, the decode-and-forward (DF) relay r constructs $w_\mathsf{r} = (w_{\mathsf{r}_1}, w_{\mathsf{r}_2}, \cdots, w_{\mathsf{r}_\mathsf{m}})$ and broadcasts $\mathbf{x}_\mathsf{r}(w_\mathsf{r})$ during the broadcast period.

3) Random binning: Random binning is further used to exploit over-heard signals from the direct links in the PMABC and FTDBC protocols. We apply random binning to combine, at an end user, the information received along from the direct link, and that received along the relaying link. For example, in the PMABC protocol, node 1 uses m independent jointly typical decoders with sequences $(\mathbf{x}_0^{(2)}(w_{0,1}, w_{\{0\},\mathcal{B}\setminus\{1\}}), \mathbf{y}_1^{(2)}), \cdots, (\mathbf{x}_\mathsf{r}^{(m+1)}(w_{0,1} \oplus w_{1,0}, w_{\mathsf{r}_2}, \cdots, w_{\mathsf{r}_\mathsf{m}}), \mathbf{y}_1^{(m+1)})$, thereby exploiting the received signals $\mathbf{y}_1^{(2)}, \cdots, \mathbf{y}_1^{(m+1)}$ overheard in phases $2, 3, \cdots (m+1)$ to decode $\tilde{w}_{0,1}$.

4) Cooperation: "Over-heard" transmissions received at a terminal node when it is not transmitting may be used to allow them to cooperate in decoding the messages $W_{0,i}$ for $i \in \mathcal{B}$. Cooperation is enabled through a compress and forward strategy in which each terminal node in $\mathcal{B}$ compresses the signals received during the relay broadcast period using an auxiliary message set, which it then transmits during the next multiple access period. If other nodes can decode this auxiliary message, they are able to obtain the compressed received signals which in turn may be used to decode messages from the relay. We note that not all nodes need to cooperate or compress the received signals - our results allow for any subset of the terminal nodes in $\mathcal{B}$ to cooperate.

IV. ACHIEVABLE RATE REGIONS AND OUTER BOUNDS

The achievable rate regions for the *simplest*, and the FMABC, PMABC, FTDBC without any network coding or binning, along with the much more involved PMABC-NRC and FTDBC-NRC schemes with cooperation (the last 'C stands for cooperation) are omitted due to space constraints but may all be found in [2], available online. Instead, we present the core achievable rate regions for the FMABC-N, PMABC-NR, and FTDBC-NR protocols, where 'N' stands for Network coding and 'R' stands for Random binning, which exploit the two-way nature of the data and over-heard side information which is possible when a node is not transmitting. All regions are first presented for discrete memoryless channels and will be evaluated in Gaussian noise in the following section.

A. FMABC-N Protocol

We consider the FMABC protocol in which Network coding is employed at the relay to combine messages on a flow-by-flow basis - i.e. the message from node i to node 0 and vice-versa are combined at the relay. The U_i variables are the auxiliary random variables playing a role similar to those in Marton's region [13] and its >2 user extension in [2].

Theorem 1: An achievable rate region of the multi-pair half-duplex bi-directional relay network under the FMABC-N protocol with decode and forward relaying is the closure of the set of all points $(R_{0,b}, R_{b,0})$ for all $b \in \mathcal{B}$ satisfying

$$R_{S,\mathcal{M}} < \Delta_1 I(X_S^{(1)}; Y_\mathsf{r}^{(1)} | X_{\bar{S}}^{(1)}, Q) \quad (1)$$

$$R_{\{0\},T} < \sum_{i\in T} \Delta_2 I(U_i^{(2)}; Y_i^{(2)}) - \Delta_2 I(U_i^{(2)}; U_{T(i)}^{(2)}) \quad (2)$$

$$R_{T,\{0\}} < \Delta_2 I(U_T^{(2)}; Y_0^{(2)}, U_{\bar{T}}^{(2)}) \quad (3)$$

for $S \subseteq \mathcal{M}$ and $T \subseteq \mathcal{B}$ over all joint distributions $p(q)\prod_{i=0}^{m} p^{(1)}(x_i|q)p^{(2)}(u_1, \cdots, u_m, x_\mathsf{r})$, where U_j's are the auxiliary random variables, and $|\mathcal{Q}| \leq 2^{m+1} - 1$ over the alphabet $\bigotimes_{i=0}^{m} \mathcal{X}_i \times \bigotimes_{j=1}^{m} \mathcal{U}_j \times \mathcal{X}_\mathsf{r} \times \mathcal{Q}$. □

We note that for the FMABC random-binning to exploit over-heard information is impossible as there is no over-heard side information: during each phase every node is either transmitting or receiving - none are just listening. Under the PMABC and FTDBC protocols however, side-information may be exploited using random binning, as described next.

B. PMABC-NR Protocol

We now consider the PMABC protocol in which Network coding is employed at the relay to combine messages on a flow-by-flow basis, along with Random Binning at the base-station node 0 to allow the end-nodes to exploit information over-heard in the phases during which they are not transmitting. In the following theorem, the U_i variables are the auxiliary random variables similar to those seen in Marton's BC-channel region [13] and its extension [2], while V_{0i} are auxiliary random variables used for binning the message $W_{0,i}$ at the base-station node 0 for node i. We note that binning is only possible at the base-station for the end-users as in the PMABC protocol the base-station is always transmitting during the multiple-access period.

Theorem 2: An achievable rate region of the multi-pair half-duplex bi-directional relay network under the PMABC-NR protocol is the closure of the set of all points $(R_{0,b}, R_{b,0})$ for all $b \in \mathcal{B}$ satisfying

$$R_{\{0\},T} + R_{S,\{0\}} < \sum_{s\in S} \Delta_s I(V_{0T}^{(s)}, X_s^{(s)}; Y_{\mathsf{r}}^{(s)}, V_{0\bar{T}}^{(s)}|Q) + \sum_{s\in \bar{S}} \Delta_s I(V_{0T}^{(s)}; Y_{\mathsf{r}}^{(s)}, V_{0\bar{T}}^{(s)}|X_s^{(s)}, Q) \quad (4)$$

$$R_{\{0\},S} < \sum_{i\in S}\sum_{j=1}^{m} \Big(\Delta_j I(V_{0i}^{(j)}; Y_i^{(j)}|Q) - \Delta_j I(V_{0i}^{(j)}; V_{0S(i)}^{(j)}|Q)\Big) + \Delta_{m+1} I(U_i^{(m+1)}; Y_i^{(m+1)}) - \Delta_{m+1} I(U_i^{(m+1)}; U_{S(i)}^{(m+1)}) \quad (5)$$

$$R_{S,\{0\}} < \Delta_{m+1} I(U_S^{(m+1)}; Y_0^{(m+1)}, U_{\bar{S}}^{(m+1)}) \quad (6)$$

for all $i \in \mathcal{B}$ and $S, T \subseteq \mathcal{B}$ over all joint distributions $p(q) \cdot \left[\prod_{i=1}^m p^{(i)}(v_{01}, \cdots, v_{0\mathsf{m}}, x_0|q) p^{(i)}(x_i|q)\right] \cdot p^{(m+1)}(u_1, \cdots, u_m, x_{\mathsf{r}})$, where $V_{0\mathsf{j}}$ are the Random binning auxiliary random variables at node 0, U_{j}'s are the auxiliary Marton-like random variables used at node r and $V_{0T} := \{V_{0s}|s \in T\}$ with $|\mathcal{Q}| \le 2^{2m} + 2^m$ over the alphabet $\bigotimes_{i=0}^m \mathcal{X}_i \times \bigotimes_{j=1}^m (\mathcal{V}_{0j} \times \mathcal{U}_j) \times \mathcal{X}_{\mathsf{r}} \times \mathcal{Q}$. □

Equation (4) ensures correct decoding at the relay, (5) ensures correct combining of overheard and relayed messages at the end users, while (6) ensures correct decoding at the base-station of the messages relayed (no side-information).

C. FTDBC-NR Protocol

The U_i and V_{0i} variables have the same interpretation as in Theorem 2.

Theorem 3: An achievable rate region of the multi-pair half-duplex bi-directional relay network under the FTDBC-NR protocol is the closure of the set of all points $(R_{0,b}, R_{b,0})$ for all $b \in \mathcal{B}$ satisfying

$$R_{\{0\},S} < \Delta_1 I(V_{0S}^{(1)}; Y_{\mathsf{r}}^{(1)}, V_{0\bar{S}}^{(1)}) \quad (7)$$

$$R_{i,0} < \Delta_{i+1} I(X_i^{(i+1)}; Y_{\mathsf{r}}^{(i+1)}) \quad (8)$$

$$R_{\{0\},S} < \sum_{i\in S} \Delta_1 I(V_{0i}^{(1)}; Y_i^{(1)}) - \Delta_1 I(V_{0i}^{(1)}; V_{0S(i)}^{(1)}) + \Delta_{m+2} I(U_i^{(m+2)}; Y_i^{(m+2)}) - \Delta_{m+2} I(U_i^{(m+2)}; U_{S(i)}^{(m+2)}) \quad (9)$$

$$R_{S,\{0\}} < \sum_{i\in S} \Delta_{i+1} I(X_i^{(i+1)}; Y_0^{(i+1)}) + \Delta_{m+2} I(U_S^{(m+2)}; Y_0^{(m+2)}, U_{\bar{S}}^{(m+2)}) \quad (10)$$

for all $i \in \mathcal{B}$ and $S \subseteq \mathcal{B}$ over all joint distributions $p^{(1)}(v_{01}, \cdots, v_{0\mathsf{m}}, x_0) \cdot \left(\prod_{j=1}^m p^{(j+1)}(x_j)\right) \cdot p^{(m+2)}(u_1, \cdots, u_{\mathsf{m}}, x_{\mathsf{r}})$, where V_{0j}, U_j's are the auxiliary random variables and $V_{0T} := \{V_{0s}|s \in T\}$ over the alphabet $\bigotimes_{i=0}^m \mathcal{X}_i \times \bigotimes_{j=1}^m (\mathcal{V}_{0j} \times \mathcal{U}_j) \times \mathcal{X}_{\mathsf{r}}$. □

Remark 4: (7) and (8) correspond to the transmissions from $\mathcal{M}$ to the relay r, while (9) – (10) correspond to the relay broadcast phase.

D. Compress-and-Forward based terminal node cooperation

The achievable rate regions for the PMABC-NRC and FTDBC-NRC protocols, where the 'C' stands for Cooperation are omitted due to space limitations but are available in [2] online. Note that due to the half-duplex constraint FMABC-NR and FMABC-NRC regions are not possible.

E. Outer bounds

The FMABC, PMABC and FTDBC outer bounds are obtained by applying the cut-set bound lemma tailored to half-duplex multi-phase protocols first derived in [1] to the different protocols, where the "cuts" will look different depending on what nodes are permitted to transmit during each phase. The bounds are omitted for brevity and may be found in [2].

V. Numerical Analysis

We assume an additive white Gaussian noise (AWGN) channel model, assume Gaussian input distributions for the achievability schemes, which may or may not be optimal, and evaluate the mutual information terms. The corresponding mathematical channel model is, for each channel use k :

$$\mathbf{Y}[k] = \mathbf{H}\mathbf{X}[k] + \mathbf{Z}[k]$$

where $\mathbf{Y}[k]$, $\mathbf{X}[k]$ and $\mathbf{Z}[k]$ are independent, of unit power, additive, white Gaussian, complex and circularly symmetric, and $\mathbf{H} \in \mathbb{C}^{(m+2)\times(m+2)}$ relate the vector channel inputs and output, which are placed in the order $0, 1, 2, \cdots m, \mathsf{r}$. In phase ℓ, if node i is in transmission mode $X_i[k]$ follows the input distribution $X_i^{(\ell)} \sim \mathcal{CN}(0, P_i)$. Otherwise, $X_i[k] = \varnothing$, which means that the input symbol does not exist in the above mathematical channel model. We assume full CSI.

We use the following channel gain matrix for $m = 2$ case:

$$\mathbf{H} = \begin{bmatrix} 0 & 0.3 & 0.05 & 1 \\ 0.3 & 0 & 1.5 & 1 \\ 0.05 & 1.5 & 0 & 0.2 \\ 1 & 1 & 0.2 & 0 \end{bmatrix} \quad (11)$$

First, in Fig. 3, we examine the effect of using Marton binning and Network coding and compare their performance to the simplest protocol by plotting three achievable rate regions; 1) the simplest protocol (Simple), 2) convex hull of the FMABC, PMABC and FTDBC protocols (MB) and 3) convex hull of the FMABC-N, PMABC-NR and FTDBC-NR protocols (MB-NR). We set $P_0 = P_1 = P_2 = P_{\mathsf{r}} = 0$ dB. For more realistic comparison, we add lower limits of individual data rates, i.e., $R_{0,1} \ge 0.01, R_{0,2} \ge 0.01, R_{1,0} \ge 0.01, R_{2,0} \ge 0.01$ to guarantee minimum information flow in each data link. Without this limitation, the sum-data rate will be maximized when both

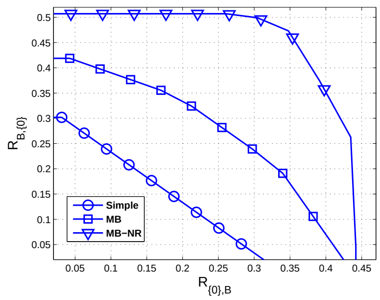

Fig. 3. Effect of Marton binning, per-flow Network coding and Random binning. $P_0 = P_1 = P_2 = P_r = 0$ dB.

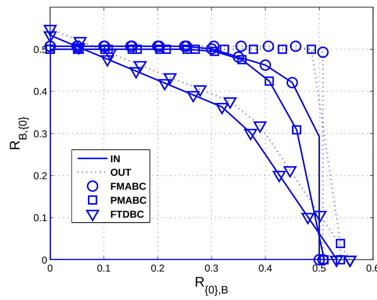

Fig. 4. Overall region comparison - no cooperation. $P_0 = P_1 = P_2 = P_r = 0$ dB.

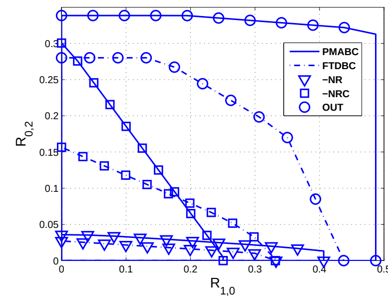

Fig. 5. Effect of cooperation. $P_0 = P_1 = P_2 = P_r = 0$ dB, and $R_{0,1} = 0.19$, $R_{2,0} = 0.01$.

the transmission rates $R_{0,2}$ and $R_{2,0}$ equal zero at least in the Simplest case because the link between the relay and the node 2 is very poor. We see that the proposed protocols using conventional MAC and extended Martons broadcasting largely enhance the performance over straightforward extensions of one-way protocols. Furthermore, we can significantly improve the achievable rate region by Network coding and Random binning schemes (in MB-NR). We emphasize that the inclusion Simple $\subseteq$ MB $\subseteq$ MB-NR is not affected by the minimum rate constraints.

The achievable regions of the FMABC-N, PMABC-NR and FTDBC-NR protocols are plotted in Fig. 4 and compared to our modified cut-set based outer bounds. The 4-dimensional rate regions in $(R_{0,1}, R_{0,2}, R_{1,0}, R_{2,0})$ are projected onto $(R_{0,1} + R_{0,2}, R_{1,0} + R_{2,0})$ 2-dimensional space. While space constraints do not allow the presentation of these plots under different channel conditions and SNRs, as seen in our extended work [2], one main conclusions drawn from plotting various regions of the type seen in Fig. 4 is that different protocols are optimal under different channel conditions - no inclusion relationships appear to exist. In the low SNR regime, the FMABC-N protocol outperforms the other protocols since the amount of both side information and multiple access interference is relatively small. However, in the high SNR regime the FTDBC-NR protocol becomes the best since it exploits side information more effectively. Under asymmetric SNR conditions (if we allow larger input power for the base station (node 0) and relay (node r)) the PMABC-NR protocol outperforms the other two protocols; see [2].

To show the cooperation coding gain, we plot the achievable rate region of the different protocols with and without cooperation. In Fig. 5, we fixed the data rates $(R_{0,1}, R_{2,0})$ to the rate pair ((0.19,0.01) and plot rate regions in the $(R_{1,0}, R_{0,2})$ domain. We do this to highlight the cooperation gain, which comes from re-allocating node 1's transmission resources (i.e. relative power) to the two information flows; $1 \rightarrow$ r $(R_{1,0})$ and $1 \rightarrow 2$ $(R_{0,2})$. As expected -NRC protocols achieve much better performance than -NR protocols. Notably, the cooperation protocols improve $R_{0,2}$ without any degradation of $R_{1,0}$ in the FTDBC protocol. In contrast, the maximum $R_{1,0}$ of the PMABC-NRC protocol is less than that of its PMABC-NR only protocol. We explain this by the fact that in our achievable rate region, we used a simplified and sub-optimal (successive decoding like) receiver in the PMABC-NRC protocol instead of using a fully general joint-decoder (as is done in the simpler PMABC-NR protocol), which limits the $R_{1,0}$. If we were to enhance the PMABC-NRC scheme by using the general joint decoder, the maximum $R_{1,0}$ would be reached and the overall performance would improve - a technically challenging task left for future work. We furthermore expect the gains of cooperation to increase if many more terminal nodes are able to exploit node 1's cooperative broadcasting; however these situations with current regions are too complex to be evaluated numerically.

REFERENCES

[1] S. J. Kim, P. Mitran, and V. Tarokh, "Performance bounds for bi-directional coded cooperation protocols," *IEEE Trans. Inf. Theory*, vol. 54, no. 11, pp. 5235–5241, Nov. 2008.

[2] S. Kim, B. Smida, and N. Devroye, "Achievable rate regions and outer bounds for a multi-pair bi-directional relay network," January, 2010, posted on ArXiv, short versions also at http://www.ece.uic.edu/∼devroye/conferences.html.

[3] S. J. Kim, N. Devroye, and V. Tarokh, "Bi-directional half-duplex protocols with multiple relays," Submitted to *IEEE Trans. Inform. Theory*, 2007, http://arxiv.org/abs/0810.1268.

[4] T. J. Oechtering and H. Boche, "Bidirectional Regenerative Half-duplex Relaying using Relay Selection," *IEEE Transactions on Wireless Communications*, vol. 7, no. 5, pp. 1879–1888, May 2008.

[5] S. Kim, N. Devroye, and V. Tarokh, "A class of bi-directional multi-relay protocols," in *Proc. IEEE Int. Symp. Inform. Theory*, Seoul, Jun. 2009, pp. 349–353.

[6] J. Pooniah and L.-L. Xie, "An achievable rate region for the two-way two-relay channel," in *Proc. IEEE Int. Symp. Inform. Theory*, Jul. 2008, pp. 489–493.

[7] R. Vaze and R. Heath, "Optimal amplify and forward strategy for two-way relay channel with multiple relays," in *Proc. IEEE Inf. Theory Workshop*, Volos, Jun. 2009.

[8] H. Ghozlan, Y. Mohasseb, H. E. Gamal, and G. Kramer, "The MIMO wireless switch: Relaying can increase the multiplexing gain," 2009. [Online]. Available: http://arxiv.org/abs/0901.2588

[9] M. Chen and A. Yener, "Interference management for multiuser two-way relaying," in *Proc. Conf. on Inf. Sci. and Sys.*, Princeton, Mar. 2008, pp. 246–251.

[10] A. Sezgin, A. Avestimehr, M. Khajehnejad, and B. Hassibi, "Divide-and-conquer: Approaching the capacity of the two-pair bidirectional Gaussian relay network," *Arxiv preprint arXiv:1001.4271*, 2010.

[11] D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, "The multi-way relay channel," in *Proc. IEEE Int. Symp. Inform. Theory*, Seoul, Jul. 2009, pp. 339–343.

[12] T. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. IT-25, pp. 572–584, Sep. 1979.

[13] K. Marton, "A coding theorem for the discrete memoryless broadcast channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 306–311, May 1979.