Improved Capacity Scaling in Wireless Networks With Infrastructure

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***Abstract*—This paper analyzes the impact and benefits of infrastructure support in improving the throughput scaling in networks of n randomly located wireless nodes. The infrastructure uses multiantenna base stations (BSs), in which the number of BSs and the number of antennas at each BS can scale at arbitrary rates relative to n. Under the model, capacity scaling laws are analyzed for both dense and extended networks. Two BS-based routing schemes are first introduced in this study: an infrastructure-supported single-hop (ISH) routing protocol with multiple-access uplink and broadcast downlink and an infrastructure-supported multihop (IMH) routing protocol. Then, their achievable throughput scalings are analyzed. These schemes are compared against two conventional schemes without BSs: the multihop (MH) transmission and hierarchical cooperation (HC) schemes. It is shown that a linear throughput scaling is achieved in dense networks, as in the case without help of BSs. In contrast, the proposed BS-based routing schemes can, under realistic network conditions, improve the throughput scaling significantly in extended networks. The gain comes from the following advantages of these BS-based protocols. First, more nodes can transmit simultaneously in the proposed scheme than in the MH scheme if the number of BSs and the number of antennas are large enough. Second, by improving the long-distance signal-to-noise ratio (SNR), the received signal power can be larger than that of the HC, enabling a better throughput scaling under extended networks. Furthermore, by deriving the corresponding information-theoretic cut-set upper bounds, it is shown under extended networks that a combination of four schemes IMH, ISH, MH, and HC is order-optimal in all operating regimes.**

***Index Terms*—Base station (BS), cut-set upper bound, hierarchical cooperation (HC), infrastructure, multiantenna, multihop (MH), single-hop, throughput scaling.**

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I. Introduction

In [1], Gupta and Kumar introduced and studied the throughput scaling in a large wireless *ad hoc* network. They showed that, for a network of n source-destination (S-D) pairs randomly distributed in a unit area, the total throughput scales as $\Theta(\sqrt{n/\log n})$.[1] This throughput scaling is achieved using a multihop (MH) communication scheme. Recent results have shown that an almost linear throughput in the network, i.e., $\Theta(n^{1-\epsilon})$ for an arbitrarily small $\epsilon > 0$, is achievable by using a hierarchical cooperation (HC) strategy [3]–[6]. Besides the schemes in [3]–[6], there has been a steady push to improve the throughput of wireless networks up to a linear scaling in a variety of network scenarios by using novel techniques such as networks with node mobility [7], interference alignment schemes [8], and infrastructure support [9].

Although it would be good to have such a linear scaling with only wireless connectivity, in practice there will be a price to pay in terms of higher delay and higher cost of channel estimation. For these reasons, it would still be good to have infrastructure aiding wireless nodes. Such hybrid networks consisting of both wireless *ad hoc* nodes and infrastructure nodes, or equivalently base stations (BSs), have been introduced and analyzed in [9]–[13]. BSs are assumed to be interconnected by high capacity wired links.

While it has been shown that BSs can be beneficial in wireless networks, the impact and benefits of infrastructure support are not fully understood yet. This paper features analysis of the capacity scaling laws for a more general hybrid network where there are l antennas at each BS, allowing the exploitation of the spatial dimension at each BS.[2] By allowing the number m of BSs and the number l of antennas to scale at arbitrary rates relative to the number n of wireless nodes, achievable rate scalings

[1]We use the following notation: i) $f(x) = O(g(x))$ means that there exist constants C and c such that $f(x) \le Cg(x)$ for all $x > c$. ii) $f(x) = o(g(x))$ means that $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$. iii) $f(x) = \Omega(g(x))$ if $g(x) = O(f(x))$. iv) $f(x) = \omega(g(x))$ if $g(x) = o(f(x))$. v) $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $g(x) = O(f(x))$ [2].

[2]When the carrier frequency is very high, it is possible to deploy many antennas at each BS since the wavelength becomes very small.

and information-theoretic upper bounds are derived as a function of these scaling parameters. To show our achievability results, two new routing protocols utilizing BSs are proposed. In the first protocol, multiple sources (nodes) transmit simultaneously to each BS using a direct single-hop multiple-access in the uplink and a direct single-hop broadcast from each BS in the downlink. In the second protocol, the high-speed BS links are combined with nearest-neighbor routing via MH among the wireless nodes. The obtained results are also compared to two conventional schemes without using BSs: the MH protocol [1] and HC protocol [3].

The proposed schemes are evaluated and their achievability results are analyzed in two different networks: dense networks [1], [3], [14] of unit area, and extended networks [3], [15]–[18] of unit node density. In dense networks, it is shown that an almost linear throughput scaling can be achieved without BS support, as in [3], which is rather obvious. On the contrary, in extended networks, depending on the network configurations and the path-loss attenuation, the proposed BS-based protocols can improve the throughput scaling significantly, compared to the case without help of BSs. Part of the improvement comes from the following two advantages over the conventional schemes: having more antennas enables more transmit pairs that can be activated simultaneously (compared to those of the MH scheme), i.e., enough degree-of-freedom (DoF) gain is obtained, provided the number m of BSs and the number l of antennas per BS are large enough. In addition, the BSs help to improve the long-distance signal-to-noise ratio (SNR)[3], first termed in [19], which leads to a larger received signal power than that of the HC scheme, i.e., enough power gain is obtained, thus allowing for a better throughput scaling in extended networks.

To show the optimality of our proposed schemes, cut-set upper bounds on the throughput scaling are derived for a network with infrastructure. Note that the previous upper bounds in [3], [15]–[17], [20], and [21] assume pure *ad hoc* networks and those for BS-based networks are not rigorously characterized in both dense and extended networks. In dense networks, it is shown that the obtained upper bound is the same as that of [3] assuming no BSs, which is not obvious. Hence, it is seen that the BSs cannot improve the throughput scaling and the HC scheme is order-optimal for all the operating regimes. In extended networks, the proposed approach is based in part on the characteristics at power-limited regimes shown in [3], [19]. It is shown that our upper bounds match the achievable throughput scalings for all the operating regimes within a factor of n^ϵ with an arbitrarily small exponent $\epsilon > 0$. To achieve the order-optimal scaling, using one of the two BS-based routings, conventional MH transmission, and HC strategy is needed depending on the operating regimes.

The rest of this paper is organized as follows. Section II describes the proposed network model with infrastructure support. The main results are briefly shown in Section III. The two proposed BS-based protocols are characterized in Section IV and their achievable throughput scalings are analyzed in Section V. The corresponding information-theoretic cut-set upper bounds are derived in Section VI. Finally, Section VII summarizes this paper with some concluding remarks.

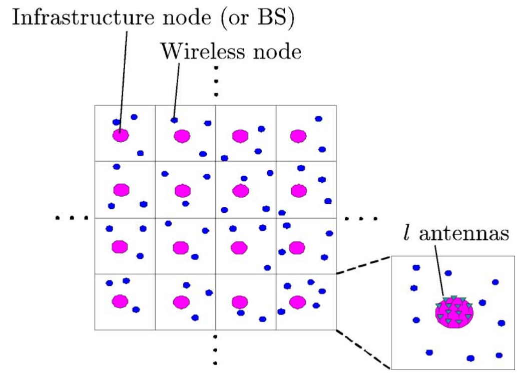

Fig. 1. The wireless ad hoc network with infrastructure support.

Throughout this paper, the superscripts T and $\dagger$ denote the transpose and conjugate transpose, respectively, of a matrix (or a vector). $\mathbf{I}_n$ is the identity matrix of size $n \times n$, $[\cdot]_{ki}$ is the (k,i)th element of a matrix, $\text{tr}(\cdot)$ is the trace, and $\det(\cdot)$ is the determinant. $\mathbb{C}$ is the field of complex numbers and $E[\cdot]$ is the expectation. Unless otherwise stated, all logarithms are assumed to be to the base 2.

II. System and Channel Models

Consider a two-dimensional wireless network that consists of n S-D pairs uniformly and independently distributed on a square except for the area covered by BSs. Then, no nodes are physically located inside the BSs. The network is assumed to have an area of one and n in dense and extended networks, respectively. Suppose that the whole area is divided into m square cells, each of which is covered by one BS with l antennas at its center (see Fig. 1). It is assumed that the total number of antennas in the network scales at most linearly with n, i.e., $ml = O(n)$. For analytical convenience, we would like to state that parameters n, m, and l are then related according to

$$n = m^{1/\beta} = l^{1/\gamma}$$

where $\beta, \gamma \in [0,1)$ satisfying $\beta + \gamma \le 1$. The number of antennas is allowed to grow with the number of nodes and BSs in the network. The placement of these l antennas depends on how the number of antennas scales as follows:

1) l antennas are regularly placed on the BS boundary if $l = O(\sqrt{n/m})$, and
2) $\sqrt{n/m}$ antennas are regularly placed on the BS boundary and the rest are uniformly placed inside the boundary if $l = \omega(\sqrt{n/m})$ and $l = O(n/m)$.[4]

Furthermore, we assume that the BSs are connected to each other with sufficiently large capacity such that the communication between the BSs is not the limiting factor to overall throughput scaling. The required transmission rate of each

[3]In [19], the long-distance SNR is defined as n times the received SNR between two farthest nodes across the largest scale in wireless networks. In our BS-based network, it can be interpreted as the total SNR transferred to any given node (or BS antenna) over a certain smaller scale reduced by infrastructure support.

[4]Such an antenna deployment strategy guarantees both the nearest neighbor transmission from/to each BS antenna and enough space among the antennas, and thus enables our BS-based routing protocol via MH to work well, which will be discussed in Section IV-A.

wired BS-to-BS link will be specified later (in Remark 4). It is also assumed that these BSs are neither sources nor destinations. Suppose that the radius of each BS scales as $\epsilon_0/\sqrt{m}$ for dense networks and as $\epsilon_0\sqrt{n/m}$ for extended networks, where $\epsilon_0 > 0$ is an arbitrarily small constant independent of n, which means that it is independent of m and l as well. This radius scaling would ensure enough separation among the antennas since the per-antenna distance scales at least as the average per-node distance for any parameters $n, m,$ and l.

Let us first describe the uplink channel model. Let $I \subseteq \{1, \ldots, n\}$ denote the set of simultaneously transmitting wireless nodes. Then, the $l \times 1$ received signal vector $\mathbf{y}_s$ of BS $s \in \{1, \ldots, m\}$ and the $l \times 1$ uplink complex channel vector $\mathbf{h}^u_{si}$ between node $i \in \{1, \ldots, n\}$ and BS s are given by

$$\mathbf{y}_s = \sum_{i \in I} \mathbf{h}^u_{si} x_i + \mathbf{n}_s$$

and

$$\mathbf{h}^u_{si} = \left[\frac{e^{j\theta^u_{si,1}}}{r^{u\alpha/2}_{si,1}} \; \frac{e^{j\theta^u_{si,2}}}{r^{u\alpha/2}_{si,2}} \cdots \frac{e^{j\theta^u_{si,l}}}{r^{u\alpha/2}_{si,l}} \right]^T \tag{1}$$

respectively, where x_i is the transmit signal of node i, and $\mathbf{n}_s$ denotes the circularly symmetric complex additive white Gaussian noise (AWGN) vector whose element has zero-mean and variance N_0. Here, $\theta^u_{si,t}$ represents the random phases uniformly distributed over $[0, 2\pi)$ and independent for different i, s, t, and time (transmission symbol), i.e., fast fading. Note that this random phase model is based on a far-field assumption, which is valid if the wavelength is sufficiently small. $r^u_{si,t}$ and $\alpha > 2$ denote the distance between node i and the tth antenna of BS s, and the path-loss exponent, respectively. Similarly, the $1 \times l$ downlink complex channel vector $\mathbf{h}^d_{is}$ between BS s and node i ($s \in \{1, \ldots, m\}$ and $i \in \{1, \ldots, n\}$) and the complex channel h_{ki} between nodes i and k ($i, k \in \{1, \ldots, n\}$) are given by

$$\mathbf{h}^d_{is} = \left[\frac{e^{j\theta^d_{is,1}}}{r^{d\alpha/2}_{is,1}} \; \frac{e^{j\theta^d_{is,2}}}{r^{d\alpha/2}_{is,2}} \cdots \frac{e^{j\theta^d_{is,l}}}{r^{d\alpha/2}_{is,l}} \right]$$

and

$$h_{ki} = \frac{e^{j\theta_{ki}}}{r^{\alpha/2}_{ki}} \tag{2}$$

respectively, where $\theta^d_{is,t}$ and θ_{ki} have uniform distribution over $[0, 2\pi)$, and are independent for different $i, s, t, k,$ and time. $r^d_{is,t}$ and r_{ki} denote the distance between the tth antenna of BS s and node i, and the distance between nodes i and k, respectively.

Suppose that each node and BS should satisfy an average transmit power constraint P and nP/m, respectively, during transmission. This assumption is reasonable since the balance between uplink and downlink would be maintained for the case where the transmit power of one BS in a cell increases proportionally to the total power consumed by all the users covered by the cell.[5] Then, the total transmit powers allowed for the n wireless nodes and the m BSs are the same. In this case, although we allow additional power for BSs, the total transmit power used by all wireless nodes and BSs still remains as $\Theta(n)$.

[5]Note that such a downlink transmit power constraint has been similarly used in the vector Gaussian broadcast channel [22], [23].

Channel state information (CSI) is assumed to be available both at the receivers and the transmitters for downlink transmissions from BSs but only at the receivers for transmissions from wireless nodes. Let $T_n(\alpha, \beta, \gamma)$ denote the total throughput of the network for the parameters α, β, and γ, and then its scaling exponent is defined by [3], [19]

$$e(\alpha, \beta, \gamma) = \lim_{n \to \infty} \frac{\log T_n(\alpha, \beta, \gamma)}{\log n} \tag{3}$$

which captures the dominant term in the exponent of the throughput scaling.[6] It is assumed that each node transmits at a rate $T_n(\alpha, \beta, \gamma)/n$.

III. Main Results

This section presents the formal statement of our results, which are divided into two parts to show the capacity scaling laws: achievable throughput scalings and information-theoretic upper bounds. We simply state these results here and derive them in later sections. The following summarizes our main results. In dense networks, the optimal scaling exponent is given by $e(\alpha, \beta, \gamma) = 1$, while the optimal scaling exponent $e(\alpha, \beta, \gamma)$ in extended networks is given by

$$e(\alpha, \beta, \gamma) = \max\left\{1 + \gamma - \frac{(1-\beta)\alpha}{2},\ \min\left\{\beta + \gamma, \frac{\beta + 1}{2}\right\}, \frac{1}{2}, 2 - \frac{\alpha}{2}\right\} \tag{4}$$

where the details are shown in the following two subsections.

A. Achievable Throughput Scaling

In this subsection, the throughput scaling for both dense and extended networks under our routing protocols is shown. The following theorem first presents a lower bound on the total capacity scaling T_n in dense networks.

Theorem 1: In a dense network

$$T_n = \Omega(n^{1-\epsilon}) \tag{5}$$

is achievable with high probability (whp) for an arbitrarily small $\epsilon > 0$.

Equation (5) is achievable by simply using the HC strategy [3].[7] Although the HC provides an almost linear throughput scaling in dense networks, it may degrade throughput scalings in extended (or power-limited) networks. An achievable throughput under extended networks is specifically given as follows.

Theorem 2: In an extended network

$$T_n = \Omega\left(\max\left\{ml\left(\frac{m}{n}\right)^{\alpha/2-1}, m\min\left\{l, \left(\frac{n}{m}\right)^{1/2-\epsilon}\right\}, n^{1/2-\epsilon}, n^{2-\alpha/2-\epsilon}\right\}\right) \tag{6}$$

is achievable whp for an arbitrarily small $\epsilon > 0$.

[6]To simplify the notation, $T_n(\alpha, \beta, \gamma)$ will be written as T_n if dropping α, β, and γ does not cause any confusion.

[7]Note that the HC always outperforms the proposed BS-based routing protocols in terms of throughput performance under dense networks, even though the details are not shown in this paper.

TABLE I
ACHIEVABLE RATES FOR AN EXTENDED NETWORK WITH INFRASTRUCTURE

Regime	Condition	Scheme	$e(\alpha,\beta,\gamma)$
A	$2<\alpha<3$	HC	$2-\frac{\alpha}{2}$
	$\alpha\geq 3$	MH	$\frac{1}{2}$
B	$2<\alpha<4-2\beta-2\gamma$	HC	$2-\frac{\alpha}{2}$
	$\alpha\geq 4-2\beta-2\gamma$	IMH	$\beta+\gamma$
C	$2<\alpha<3-\beta$	HC	$2-\frac{\alpha}{2}$
	$\alpha\geq 3-\beta$	IMH	$\frac{1+\beta}{2}$
D	$2<\alpha<\frac{2(1-\gamma)}{\beta}$	HC	$2-\frac{\alpha}{2}$
	$\frac{2(1-\gamma)}{\beta}\leq\alpha<1+\frac{2\gamma}{1-\beta}$	ISH	$1+\gamma-\frac{\alpha(1-\beta)}{2}$
	$\alpha\geq 1+\frac{2\gamma}{1-\beta}$	IMH	$\frac{1+\beta}{2}$

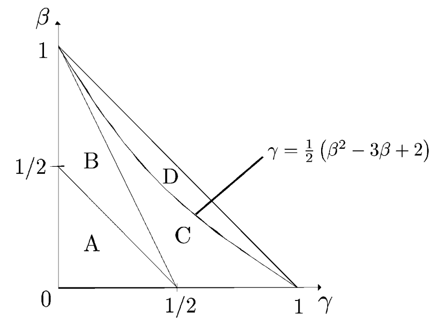

Fig. 2. Operating regimes on the achievable throughput scaling with respect to β and γ.

The first to fourth terms in (6) correspond to the achievable rate scalings of the infrastructure-supported single-hop (ISH), infrastructure-supported multihop (IMH), MH, and HC protocols, respectively, where the two BS-based schemes will be described in detail later (in Section IV). As a result, the best strategy among the four schemes ISH, IMH, MH, and HC depends on the path-loss exponent α, and the parameters m and l under extended networks. Let us give an intuition for the achievability result above. For the first term in (6), ml represents the total number of simultaneously active sources in the ISH protocol while $(m/n)^{\alpha/2-1}$ comes from a performance degradation due to power limitation. The second term in (6) represents the total number of sources that can send their own packets simultaneously using the IMH protocol. From the achievable rates of each scheme, the interesting result below is obtained under each network condition.

Remark 1: The best achievable one among the four schemes and its scaling exponent $e(\alpha,\beta,\gamma)$ in (3) are shown in Table I according to the two-dimensional operating regimes on the achievable throughput scaling with respect to the scaling parameters β and γ (see Fig. 2). This result is analyzed in Appendix A. Operating regimes A–D are shown in Fig. 2. It is important to verify the best protocol in each regime. In Regime A, where β and γ are small, the infrastructure is not helpful. In other regimes, we observe BS-based protocols are dominant in some cases depending on the path-loss exponent α. For example, Regime D has the following characteristics: the HC protocol has the highest throughput when the path-loss attenuation is small, but as the path-loss exponent α increases, the best scheme becomes the ISH protocol. This is because the penalty for long-range multiple-input multiple-output (MIMO) transmissions of the HC increases. Finally, the IMH protocol becomes dominant when α is large since the ISH protocol has a power limitation at the high path-loss attenuation regime.

B. Cut-Set Upper Bound

We now turn our attention to presenting the cut-set upper bound of the total throughput T_n. The upper bound [3] for pure ad hoc networks of unit area is extended to our dense network model.

Theorem 3: The total throughput T_n is upper-bounded by $n\log n$ whp in a dense network with infrastructure.

Note that the same upper bound as that of [3] assuming no BSs is found in dense networks. This upper bound means that n S-D pairs can be active with genie-aided interference removal between simultaneously transmitting nodes, while providing a power gain of $\log n$. In addition, it is examined how the upper bound is close to the achievable throughput scaling.

Remark 2: Based on the above result, it is easy to see that the achievable rate in (5) and the upper bound are of the same order up to a factor $\log n$ in dense networks with the help of BSs, and thus the exponent of the capacity scaling is given by $e(\alpha,\beta,\gamma)=1$. The HC is therefore order-optimal and we may conclude that infrastructure cannot improve the throughput scaling in dense networks.

In extended networks, an upper bound is established based on the characteristics at power-limited regimes shown in [3], [19], and is presented in the following theorem.

Theorem 4: In an extended network, the total throughput T_n is upper-bounded by

$$T_n=O\left(n^{\epsilon}\max\left\{ml\left(\frac{m}{n}\right)^{\alpha/2-1},m\min\left\{l,\sqrt{\frac{n}{m}}\right\},\sqrt{n},n^{2-\alpha/2}\right\}\right) \quad (7)$$

whp for an arbitrarily small $\epsilon>0$.

The relationship between the achievable throughput and the cut-set upper bound is now examined as follows.

Remark 3: The upper bound matches the achievable throughput scaling within n^{ϵ} in extended networks with infrastructure, and thus the scaling exponent in (4) holds. In other words, it is shown that choosing the best of the four schemes ISH, IMH, MH, and HC is order-optimal for all the operating regimes shown in Fig. 2 (see Table I).

IV. ROUTING PROTOCOLS

This section explains the two BS-based protocols. Two conventional schemes [1], [3] with no infrastructure support are also introduced for comparison. Each routing protocol operates in different time slots to avoid huge mutual interference. We focus on the description for extended networks since using the HC scheme [3] is enough to provide a near-optimal throughput in dense networks.

A. Protocols With Infrastructure Support

We generalize the conventional BS-based transmission scheme in [9]–[13]: a source node transmits its packet to the

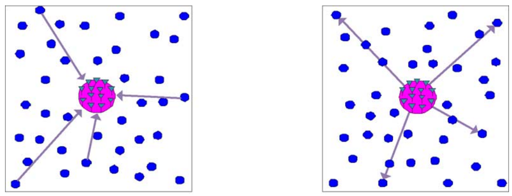

Fig. 3. The ISH protocol.

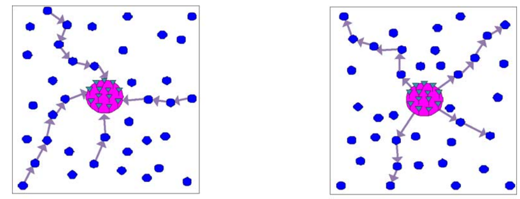

Fig. 4. The IMH protocol.

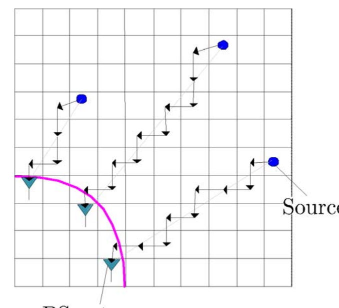

Fig. 5. The access routing in the IMH protocol.

closest BS, the BS having the packet transmits it to the BS that is nearest to the destination of the source via wired BS-to-BS links, and the destination finally receives its data from the nearest BS. Since there exist both access (to BSs) and exit (from BSs) routings, different time slots are used, e.g., even and odd time slots, respectively. We start from the following lemma.

Lemma 1: Suppose $m = n^\beta$ where $\beta \in [0, 1)$. Then, the number of nodes inside each cell is between $((1-\delta_0)n^{1-\beta}, (1+\delta_0)n^{1-\beta})$, i.e., $\Theta(n/m)$, with probability larger than

$$1 - n^\beta e^{-\Delta(\delta_0)n^{1-\beta}} \tag{8}$$

where $\Delta(\delta_0) = (1+\delta_0)\ln(1+\delta_0) - \delta_0$ for $0 < \delta_0 < 1$ independent of n.

The proof of this lemma is given by slightly modifying the proof of Lemma 4.1 in [3]. Note that (8) tends to one as n goes to infinity.

1) Infrastructure-Supported Single-Hop (ISH) Protocol: In contrast with previous works, the spatial dimensions enabled by having multiple antennas at each BS are exploited here, and thus multiple transmission pairs can be supported using a single BS. Under extended networks, the ISH transmission protocol shown in Fig. 3 is now proposed as follows:

- For the access routing, all source nodes in each cell, given by n/m nodes whp from Lemma 1, transmit their independent packets simultaneously via single-hop multiple-access to the BS in the same cell.
- Each BS receives and jointly decodes packets from source nodes in the same cell. Signals received from the other cells are treated as noise.
- The BS that completes decoding its packets transmits them to the BS closest to the corresponding destination by wired BS-to-BS links.
- For the exit routing, each BS transmits all packets received from other BSs, i.e., n/m packets, via single-hop broadcast to the destinations in the cell.

Since the network is power-limited, the proposed ISH scheme is used with the full power, i.e., the transmit powers at each node and BS are P and $\frac{nP}{m}$, respectively.

For the ISH protocol, more DoF gain is provided compared to transmissions via MH if m and l are large enough. The power gain can also be obtained compared to that of the HC scheme in certain cases.

2) Infrastructure-Supported Multihop (IMH) Protocol: The fact that the extended network is power-limited motivates the introduction of an IMH transmission protocol in which multiple source nodes in a cell transmit their packets to BS in the cell via MH, thereby having much higher received power, i.e., more power gain, than that of the direct one-hop transmission in extended networks. That is, better long-distance SNR is provided with the IMH protocol. Similarly, each BS delivers its packets to the corresponding destinations by IMH transmissions. Under extended networks, the IMH transmission protocol in Fig. 4 is proposed as follows:

- Divide each cell into smaller square cells of area $2\log n$ each, where these smaller cells are called routing cells (which include at least one node whp [1], [14]).
- For the access routing, $\min\{l, \sqrt{n/m}\}$ source nodes in each cell transmit their independent packets using MH routing to the corresponding BS in the cell as shown in Fig. 5. It is assumed that each antenna placed only on the BS boundary receives its packet from one of the nodes in the nearest neighbor routing cell. It is easy to see that $\min\{l, \sqrt{n/m}\}$ MH paths can be supported due to our antenna placement within BSs. Exit routing is similar, where each antenna on the BS boundary uses power P that satisfies the power constraint.
- The BS-to-BS transmissions are the same as the ISH case.
- Each routing cell operates based on 9-time division multiple access (TDMA) to avoid causing huge interference to its neighbor cells.

Note that the transmit power $\min\{l, \sqrt{n/m}\}$ at each BS, but not full power, is sufficient to perform the IMH protocol in the downlink.

For the IMH protocol, more DoF gain is possible compared to the MH scheme for large m and l. In addition, more power gain can also be obtained compared to the HC and ISH schemes in certain cases.

B. Protocols Without Infrastructure Support

The upper bound in Theorem 3 is only determined by the number n of wireless nodes in dense networks. The upper bound in Theorem 4 also indicates that either the number m of BSs or the number l of antennas per BS should be greater than a certain level in order to obtain improved throughput scalings in extended networks. This is because otherwise less DoF gain may be provided compared to that of the conventional schemes without help of BSs. Thus, transmissions only using wireless nodes may be sufficient to achieve the capacity scalings in dense networks or in extended networks with small m and l. In this case, the existing MH and HC protocols, which were introduced in [1] and [3], respectively, are also directly applied to our network with infrastructure.

V. Achievable Throughput in Extended Networks

In this section, the achievable throughput scaling in Theorem 2 is rigorously analyzed in extended networks. It is demonstrated that the throughput scaling can be improved under some conditions by applying two BS-based transmissions in extended networks.

The transmission rate of the ISH protocol in extended networks will be shown first.

Lemma 2: Suppose that an extended network uses the ISH protocol. Then, the rate of

$$\Omega\left(l\left(\frac{m}{n}\right)^{\alpha/2-1}\right)$$

is achievable at each cell for both access and exit routings.

Proof: In order to prove the result, we need to quantify the total amount of interference when the ISH scheme is used. We first introduce the following lemma and refer to Appendix B for the detailed proof.

Lemma 3: In an extended network with the ISH protocol, the total interference power P_I^u in the uplink from nodes in other cells to each BS antenna is upper-bounded by $\Theta((m/n)^{\alpha/2-1})$ whp. Each node also has interference power $P_I^d = \Theta((m/n)^{\alpha/2-1})$ whp in the downlink from BSs in other cells.

Note that when $\alpha > 2$ the term $(m/n)^{\alpha/2-1}$ tends to zero as $n \to \infty$. Now, the transmission rate for the access routing is derived as in the following. The signal model from nodes in each cell to the BS with multiple antennas corresponds to the single-input multiple-output (SIMO) multiple-access channel (MAC). Since the maximum Euclidean distance among links of the above SIMO MAC scales as $\Theta(\sqrt{n/m})$, it is upper-bounded by $\delta_1\sqrt{n/m}$, where $\delta_1 > 0$ is a certain constant. Let N_I denote the sum of total interference power P_I^u received from the other cells and noise variance N_0. Then, the worst case noise of this channel has an uncorrelated Gaussian distribution with zero-mean and variance N_I [24]–[26], which lower-bounds the transmission rate. By assuming full CSI at the receiver (BS s) and performing a minimum mean-square error (MMSE) estimation [22], [23], [27] with successive interference cancellation (SIC) at BS s, the sum-rate of the SIMO MAC is given by [22], [23]

$$\begin{aligned} I(\mathbf{x}_s;\mathbf{y}_s,\mathbf{H}_s) &\ge E\left[\log\det\left(\mathbf{I}_l+\frac{P}{N_I}\mathbf{H}_s\mathbf{H}_s^\dagger\right)\right] \\ &\ge E\left[\log\det\left(\mathbf{I}_l+\frac{P}{\delta_1^\alpha(n/m)^{\alpha/2}N_I}\mathbf{G}\mathbf{G}^\dagger\right)\right] \end{aligned} \quad (9)$$

where $\mathbf{x}_s$ denotes the $\frac{n}{m}\times 1$ transmit signal vector, whose elements are nodes in the cell covered by BS s, $\mathbf{y}_s$ is the $l\times 1$ received signal vector at BS s, and $\mathbf{H}_s = [\mathbf{h}_{s1}^u\ \mathbf{h}_{s2}^u\ \cdots\ \mathbf{h}_{s(n/m)}^u]$ [$\mathbf{h}_{si}^u$ for $i=1,\ldots,n/m$ is given in (1)]. $\mathbf{G}$ is the normalized matrix, whose element g_{ti} is given by $e^{j\theta_{si,t}^u}$ and represents the phase between node i and the tth antenna of BS s. Note that rotating the decoding order among n/m nodes in the cell leads to the same rate of each node. Then, the above sum-rate is rewritten as

$$\begin{aligned} I(\mathbf{x}_s;\mathbf{y}_s,\mathbf{H}_s) &\ge E\left[\sum_{i=1}^{l}\log\left(1+\frac{P}{\delta_1^\alpha(n/m)^{\alpha/2-1}N_I}\lambda_i\right)\right] \\ &= \sum_{i=1}^{l}E\left[\log\left(1+\frac{P}{\delta_1^\alpha(n/m)^{\alpha/2-1}N_I}\lambda_i\right)\right] \\ &= lE\left[\log\left(1+\frac{P}{\delta_1^\alpha(n/m)^{\alpha/2-1}N_I}\lambda_1\right)\right] \\ &\ge l\log\left(1+\frac{P}{\delta_1^\alpha(n/m)^{\alpha/2-1}N_I}\bar{\lambda}\right)\Pr\left(\lambda_1>\bar{\lambda}\right) \end{aligned} \quad (10)$$

where $\{\lambda_1,\ldots,\lambda_l\}$ are the unordered eigenvalues of $\frac{m}{n}\mathbf{G}\mathbf{G}^\dagger$ [28] and $\bar{\lambda}$ is any nonnegative constant. Here, the second equality holds since the eigenvalues are unordered (see Section 4 in [28] for more details). Due to the fact that $\log(1+x) = (\log e)x + O(x^2)$ for small $x>0$, (10) is given by

$$I(\mathbf{x}_s;\mathbf{y}_s,\mathbf{H}_s) \ge c_0 l\left(\frac{m}{n}\right)^{\alpha/2-1}\Pr\left(\lambda_1>\bar{\lambda}\right) \quad (11)$$

for some constant $c_0>0$ independent of n, since N_I has a constant scaling from Lemma 3. By the Paley-Zygmund inequality [29], it is possible to lower-bound the sum-rate in the left-hand side (LHS) of (11) by following the same line as Appendix I in [3], thus yielding

$$I(\mathbf{x}_s;\mathbf{y}_s,\mathbf{H}_s) \ge c_1 l\left(\frac{m}{n}\right)^{\alpha/2-1}$$

where $c_1>0$ is some constant independent of n.

For the exit routing, the signal model from the BS with multiple antennas in one cell to nodes in the cell corresponds to the multiple-input single-output (MISO) broadcast channel (BC). From Lemma 3, it is seen that the total interference power received from the other BSs is bounded. Hence, it is possible to derive the transmission rate for the exit routing by exploiting an uplink-downlink duality [22], [23], [30], [31]. In this case, the transmitters in the downlink are designed by an MMSE transmit precoding with dirty paper coding [32]–[34] at each BS, and the rate of the MISO BC is then equal to that of the dual SIMO MAC with a sum power constraint. More precisely, with full CSI at the transmitter (BS) and the total transmit power $\frac{nP}{m}$ in the downlink, the sum-rate of the MISO BC is lower-bounded by [22]

$$\begin{aligned} &\max_{\mathbf{Q}_x\ge 0} E\left[\log\det\left(\mathbf{I}_l+\frac{1}{N_I'}\mathbf{H}_s'^{\dagger}\mathbf{Q}_x\mathbf{H}_s'\right)\right] \\ &\ge E\left[\log\det\left(\mathbf{I}_l+\frac{P}{N_I'}\mathbf{H}_s'^{\dagger}\mathbf{H}_s'\right)\right] \end{aligned} \quad (12)$$

where $\mathbf{H}'_s = [\mathbf{h}_{1s}^{dT}\ \mathbf{h}_{2s}^{dT}\ \cdots\ \mathbf{h}_{(n/m)s}^{dT}]^T$, N'_I denotes the sum of total interference power P_I^d from BSs in the other cells and noise variance N_0, and $\mathbf{Q}_x$ is the $\frac{n}{m} \times \frac{n}{m}$ positive semidefinite input covariance matrix which is diagonal and satisfies $\mathrm{tr}(\mathbf{Q}_x) \le \frac{nP}{m}$. Here, the inequality holds since the rate is reduced by simply applying the same average power of each user. Due to the fact that (12) is equivalent to the right-hand side (RHS) of (9) (with a change of variables), $\Omega(l(m/n)^{\alpha/2-1})$ is achievable in the downlink of each cell by following the same approach as that for the access routing, which completes the proof of Lemma 2. ■

Note that l corresponds to the DoF at each cell provided by the ISH protocol while $(m/n)^{\alpha/2-1}$ represents the throughput degradation due to power loss.

The transmission rate for the access and exit routings of IMH protocol will now be analyzed in extended networks. The number of source nodes that can be active simultaneously is examined under the IMH protocol, while maintaining a constant throughput $\Theta(1)$ per S-D pair.

Lemma 4: When an extended network uses the IMH protocol, the rate of

$$\Omega\left(\min\left\{l, \left(\frac{n}{m}\right)^{1/2-\epsilon}\right\}\right) \tag{13}$$

is achievable at each cell for both access and exit routings, where $\epsilon > 0$ is an arbitrarily small constant.

Proof: This result is obtained by modifying the analysis in [1], [14], and [35] on scaling laws under our BS-based network. We mainly focus on the aspects that are different from the conventional schemes. From the 9-TDMA operation, the signal-to-interference-and-noise ratio (SINR) seen by any receiver is given by $\Omega(1)$ with a transmit power P. It can be interpreted that when the worst case noise [24]–[26] is assumed as in the ISH protocol, the achievable throughput per S-D pair is lower-bounded by $\log(1 + \mathrm{SINR})$, thus providing a constant scaling. First consider the case $l = o(\sqrt{n/m})$ where the number l of antennas scales slower than the number n/m of nodes in a cell. Then, it is possible to activate up to l source nodes at each cell because there exist l routes for the last hop to each BS antenna in the uplink. On the other hand, when $l = \Omega(\sqrt{n/m})$, the maximum number of simultaneously transmitting sources per BS is equal to the number of routing cells on the BS boundary, which scales with $(n/m)^{1/2-\epsilon}$ for an arbitrarily small $\epsilon > 0$. In the downlink of each cell, the same number of S-D pairs as that in the uplink is active simultaneously. Therefore, the transmission rate per each BS is finally given by (13), which completes the proof of Lemma 4. ■

By using Lemmas 2 and 4, we are ready to show the achievable throughput scaling in extended networks. The achievable throughputs of the ISH and IMH protocols are given by

$$T_n = \Omega\left(ml\left(\frac{m}{n}\right)^{\alpha/2-1}\right) \tag{14}$$

and

$$T_n = \Omega\left(m\min\left\{l, \left(\frac{n}{m}\right)^{1/2-\epsilon}\right\}\right) \tag{15}$$

respectively, since there are m cells in the network. Throughput scalings of two conventional protocols that do not utilize the BSs are also considered. From the results of [1], [3]

$$T_n = \Omega\left(n^{1/2-\epsilon}\right) \tag{16}$$

and

$$T_n = \Omega(n^{2-\alpha/2-\epsilon}) \tag{17}$$

are yielded for the MH and HC schemes, respectively. Hence, the throughput scaling in extended networks is simply lower-bounded by the maximum of (14)–(17), which completes the proof of Theorem 2.

In addition, we would like to examine the required rate of each BS-to-BS transmission.

Remark 4: To see how much data traffic flows on each BS-to-BS link, we first show the following lemma.

Lemma 5: Let X_{ki} denote the number of destinations in the kth cell whose source nodes are in the ith cell, where $i, k \in \{1, \dots, m\}$. Then, for all $i, k \in \{1, \dots, m\}$, the following equation holds whp:

$$X_{ki} = \begin{cases} O\left(\frac{n}{m^2}\right) & \text{if } n = \omega(m^2) \\ O(\log n) & \text{if } n = O(m^2) \end{cases}. \tag{18}$$

The proof of this lemma is presented in Appendix C. Let C_{BS} denote the rate of each BS-to-BS link. Then, since each S-D pair transmits at a rate T_n/n and the number of packets carried simultaneously through each link is bound by (18) from Lemma 5, the required rate C_{BS} is given by

$$C_{\mathrm{BS}} = \begin{cases} \Omega\left(\frac{T_n}{m^2}\right) & \text{if } n = \omega(m^2) \\ \Omega\left(\frac{T_n \log n}{n}\right) & \text{if } n = O(m^2) \end{cases}.$$

VI. Cut-Set Upper Bound

To see how closely the proposed schemes approach the fundamental limits in a network with infrastructure, new BS-based cut-set outer bounds on the throughput scaling are analyzed based on the information-theoretic approach [36].

A. Dense Networks

Before showing the main proof of Theorem 3, we start from the following lemma.

Lemma 6: In our two-dimensional dense network where n nodes are uniformly distributed and there are m BSs with l regularly spaced antennas, the minimum distance between any two nodes or between a node and an antenna on the BS boundary is larger than $1/n^{1+\epsilon_1}$ whp for an arbitrarily small $\epsilon_1 > 0$.

The proof of this lemma is presented in Appendix D. Now we present the cut-set upper bound of the total throughput T_n in dense networks. The proof steps are similar to those of [3], [37]. The throughput per S-D pair is simply upper-bounded by the capacity of the SIMO channel between a source node and

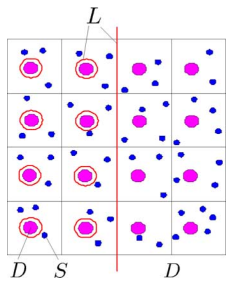

Fig. 6. The cut L in the two-dimensional random network.

the rest of nodes including BSs. Hence, the total throughput for n S-D pairs is bounded by

$$T_n \le \sum_{i=1}^{n} \log\left(1+\frac{P}{N_0}\left(\sum_{\substack{k=1\\k\neq i}}^{n}|h_{ki}|^2+\sum_{s=1}^{m}\|\mathbf{h}_{si}^u\|^2\right)\right)$$
$$\le n\log\left(1+\frac{P}{N_0}n^{(1+\epsilon_1)\alpha}(n-1+ml)\right)$$
$$= c_2 n\log n$$

where $\|\cdot\|$ denotes L_2-norm of a vector and $c_2>0$ is some constant independent of n. The second inequality holds due to Lemma 6. This completes the proof of Theorem 3.

B. Extended Networks

Consider the cut L in Fig. 6 dividing the network area into two halves in an extended random network. Let S_L and D_L denote the sets of sources and destinations, respectively, for the cut in the network. More precisely, under L, (wireless) source nodes S_L are on the left half of the network, while all nodes on the right half and all BS antennas are destinations D_L.[8] In this case, we get the $n\times(n+ml)$ MIMO channel between the two sets of nodes and BSs separated by the cut.

In extended networks, it is necessary to narrow down the class of S-D pairs according to their Euclidean distance to obtain a tight upper bound. In this subsection, the upper bound based on the power transfer arguments in [3] and [19] is shown, where an upper bound is proportional to the total received signal power from source nodes. The present problem is not equivalent to the conventional extended setup since a network with infrastructure support is taken into account. A new upper bound based on hybrid approaches that consider either the sum of the capacities of the multiple-input single-output (MISO) channel between transmitters and each receiver or the amount of power transferred across the network according to operating regimes, is thus derived. We start from the following lemma.

Lemma 7: Assume a two-dimensional extended network. When the network area with the exclusion of BS area is divided into n squares of unit area, there are less than $\log n$ nodes in each square whp.

[8]The other cut $\tilde{L}$ can also be considered in the network. In this case, sources $S_{\tilde{L}}$ represent antennas at each BS as well as ad hoc nodes on the left half. The (wireless) destination nodes $D_{\tilde{L}}$ are on the right half. Since the cut L provides a tight upper bound compared to the achievable rate, the analysis for the cut $\tilde{L}$ is not shown in this paper.

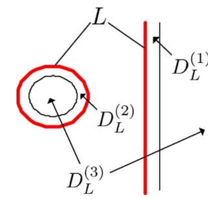

Fig. 7. The partition of destinations in the two-dimensional random network. To simplify the figure, one BS is shown in the left half network.

This result can be obtained by applying our BS-based network and slightly modifying the proof of Lemma 1 in [18]. For the cut L, the total throughput T_n for sources on the left half is bounded by the capacity of the MIMO channel between S_L and D_L, and thus

$$T_n \le \max_{\mathbf{Q}_L\ge 0} E[\log\det(\mathbf{I}_{n+ml}+\mathbf{H}_L\mathbf{Q}_L\mathbf{H}_L^\dagger)]$$
$$= \max_{\mathbf{Q}_L\ge 0} E\left[\log\det\left(\mathbf{I}_{\Theta(n)}+\mathbf{H}_L\mathbf{Q}_L\mathbf{H}_L^\dagger\right)\right]$$

where the equality holds since $n=\Omega(ml)$.[9] $\mathbf{H}_L$ consists of $\mathbf{h}_{si}^u$ in (1) for $i\in S_L$, $s\in B$, and h_{ki} in (2) for $i\in S_L$, $k\in D_r$. Here, B and D_r represent the set of BSs in the network and the set of (wireless) nodes on the right half, respectively. $\mathbf{Q}_L$ is the positive semidefinite input covariance matrix whose kth diagonal element satisfies $[\mathbf{Q}_L]_{kk}\le P$ for $k\in S_L$. The set D_L $(=B\cup D_r)$ is partitioned into three groups according to their location, as shown in Fig. 7. By generalized Hadamard's inequality [38] as in [3], [16]

$$T_n \le \max_{\mathbf{Q}_L\ge 0} E\left[\log\det\left(\mathbf{I}_{\sqrt{n}}+\mathbf{H}_L^{(1)}\mathbf{Q}_L\mathbf{H}_L^{(1)\dagger}\right)\right]$$
$$+\max_{\mathbf{Q}_L\ge 0} E\left[\log\det\left(\mathbf{I}_{O(\sqrt{ml})}+\mathbf{H}_L^{(2)}\mathbf{Q}_L\mathbf{H}_L^{(2)\dagger}\right)\right]$$
$$+\max_{\mathbf{Q}_L\ge 0} E\left[\log\det\left(\mathbf{I}_{\Theta(n)}+\mathbf{H}_L^{(3)}\mathbf{Q}_L\mathbf{H}_L^{(3)\dagger}\right)\right] \quad (19)$$

where $\mathbf{H}_L^{(t)}$ is the matrix with entries $[\mathbf{H}_L^{(t)}]_{ki}$ for $i\in S_L$, $k\in D_L^{(t)}$, and $t=1,\dots,3$. Here, $D_L^{(1)}$ and $D_L^{(2)}$ denote the sets of destinations located on the rectangular slab with width 1 immediately to the right of the centerline (cut) and on the ring with width 1 immediately inside each BS boundary (cut) on the left half, respectively. $D_L^{(3)}$ is given by $D_L\setminus(D_L^{(1)}\cup D_L^{(2)})$. Note that the sets ($D_L^{(1)}$ and $D_L^{(2)}$) of destinations located very close to the cut are considered separately since otherwise their contribution to the total received power will be excessive, resulting in a loose bound.

Each term in (19) will be analyzed below in detail. Before that, to get the total power transfer of the set $D_L^{(3)}$, the same technique as that in [3, Section V] is used, which is the relaxation of the individual power constraints to a total weighted power constraint, where the weight assigned to each source corresponds to the total received power on the other side of the cut. Specifically, each column i of the matrix $\mathbf{H}_L^{(3)}$ is normalized by the square root of the total received power on the other side of the cut from

[9]Here and in the sequel, the noise variance N_0 is assumed to be 1 to simplify the notation.

source $i \in S_L$. The total weighted power $P_{L,i}^{(3)}$ by source i is then given by

$$P_{L,i}^{(3)} = Pd_{L,i}^{(3)} \tag{20}$$

where

$$d_{L,i}^{(3)} = \sum_{k \in \bar{D}_r \setminus D_L^{(1)}} r_{ki}^{-\alpha} + \sum_{s \in B_l, t \in [1,l]} r_{si,t}^{u-\alpha}. \tag{21}$$

Here, $\bar{D}_r$ is the set of destination nodes including BS antennas on the right half and B_l represents the set of BSs on the left half. Then, the third term in (19) is rewritten as

$$\max_{\tilde{\mathbf{Q}}_L \ge 0} E\left[\log\det\left(\mathbf{I}_n + \mathbf{F}_L^{(3)} \tilde{\mathbf{Q}}_L \mathbf{F}_L^{(3)\dagger}\right)\right] \tag{22}$$

where $\mathbf{F}_L^{(3)}$ is the matrix with entries $[\mathbf{F}_L^{(3)}]_{ki} = \frac{1}{\sqrt{d_{L,i}^{(3)}}}[\mathbf{H}_L^{(3)}]_{ki}$, which are obtained from (21), for $i \in S_L$, $k \in D_L^{(3)}$. Then, $\tilde{\mathbf{Q}}_L$ is the matrix satisfying

$$[\tilde{\mathbf{Q}}_L]_{ki} = \sqrt{d_{L,k}^{(3)} d_{L,i}^{(3)}}[\mathbf{Q}_L]_{ki}$$

which means $\mathrm{tr}(\tilde{\mathbf{Q}}_L) \le \sum_{i \in S_L} P_{L,i}^{(3)}$ (equal to the sum of the total received power from each source).

We next examine the behavior of the largest singular value for the normalized channel matrix $\mathbf{F}_L^{(3)}$. From the fact that $\mathbf{F}_L^{(3)}$ is well-conditioned whp, this shows how much it essentially affects an upper bound of (22), which will be analyzed later in Lemma 9.

Lemma 8: Let $\mathbf{F}_L^{(3)}$ denote the normalized channel matrix whose element is given by $[\mathbf{F}_L^{(3)}]_{ki} = \frac{1}{\sqrt{d_{L,i}^{(3)}}}[\mathbf{H}_L^{(3)}]_{ki}$. Then,

$$E\left[\left\|\mathbf{F}_L^{(3)}\right\|_2^2\right] \le c_3(\log n)^3 \tag{23}$$

where $\|\cdot\|_2$ denotes the largest singular value of a matrix and $c_3 > 0$ is some constant independent of n.

The proof of this lemma is presented in Appendix E. Using Lemma 8 yields the following result.

Lemma 9: The term shown in (22) is upper-bounded by

$$n^\epsilon \sum_{i \in S_L} P_{L,i}^{(3)} \tag{24}$$

whp where $\epsilon > 0$ is an arbitrarily small constant and $P_{L,i}^{(3)}$ is given by (20).

Proof: Equation (22) is bounded by

$$\max_{\tilde{\mathbf{Q}}_L \ge 0} E\left[\log\det\left(\mathbf{I}_n + \mathbf{F}_L^{(3)} \tilde{\mathbf{Q}}_L \mathbf{F}_L^{(3)\dagger}\right) 1_{\mathcal{E}_{\mathbf{F}_L^{(3)}}}\right] + \max_{\tilde{\mathbf{Q}}_L \ge 0} E\left[\log\det\left(\mathbf{I}_n + \mathbf{F}_L^{(3)} \tilde{\mathbf{Q}}_L \mathbf{F}_L^{(3)\dagger}\right) 1_{\mathcal{E}^c_{\mathbf{F}_L^{(3)}}}\right] \tag{25}$$

where the event $\mathcal{E}_{\mathbf{F}_L^{(3)}}$ is given by

$$\mathcal{E}_{\mathbf{F}_L^{(3)}} = \left\{\left\|\mathbf{F}_L^{(3)}\right\|_2^2 > n^\epsilon\right\}$$

for an arbitrarily small constant $\epsilon > 0$. Then, by using the result of Lemma 8 and applying the proof technique similar to that in [3, Section V], it is possible to prove that the first term in (25) decays polynomially to zero as n tends to infinity, and for the second term in (25), it follows that: (see the equation at the bottom of the page), which completes the proof. ■

Note that (24) represents the power transfer from the set S_L of sources to the set $D_L^{(3)}$ of the corresponding destinations for a given cut L. For notational convenience, let $d_{L,i}^{(4)}$ and $d_{L,i}^{(5)}$ denote the first and second terms in (21), respectively. Then, $Pd_{L,i}^{(4)}$ and $Pd_{L,i}^{(5)}$ correspond to the total received power from source i to the destination sets $\bar{D}_r \setminus D_L^{(1)}$ and $D_L \setminus (D_L^{(2)} \cup \bar{D}_r)$, respectively. The computation of the total received power of the set $D_L^{(3)}$ will now be computed as follows:

$$\sum_{i \in S_L} P_{L,i}^{(3)} = \sum_{i \in S_L} Pd_{L,i}^{(4)} + \sum_{i \in S_L} Pd_{L,i}^{(5)} \tag{26}$$

which is eventually used to compute (24).

First, to get an upper bound on $\sum_{i \in S_L} Pd_{L,i}^{(4)}$ in (26), the network area is divided into n squares of unit area. By Lemma 7, since there are less than $\log n$ nodes inside each square whp, the power transfer can be upper-bounded by that under a regular network with at most $\log n$ nodes at each square (see [3] for the detailed description). Such a modification yields the following upper bound [3] for $\sum_{i \in S_L} Pd_{L,i}^{(4)}$

$$\sum_{i \in S_L} Pd_{L,i}^{(4)} \le \begin{cases} c_4 n^{2-\alpha/2}(\log n)^2 & \text{if } 2 < \alpha < 3 \\ c_4\sqrt{n}(\log n)^3 & \text{if } \alpha = 3 \\ c_4\sqrt{n}(\log n)^2 & \text{if } \alpha > 3 \end{cases} \tag{27}$$

whp for some constant $c_4 > 0$ independent of n. Next, the second term $\sum_{i \in S_L} Pd_{L,i}^{(5)}$ in (26) can be derived as in the following lemma.

Lemma 10: The term $\sum_{i \in S_L} Pd_{L,i}^{(5)}$ is given by

$$\sum_{i \in S_L} Pd_{L,i}^{(5)} = \begin{cases} 0 & \text{if } l = o(\sqrt{n/m}) \\ O\left(nl\left(\frac{m}{n}\right)^{\alpha/2}\log n\right) & \text{if } l = \Omega(\sqrt{n/m}) \text{ and } 2 < \alpha < 3 \\ O\left(ml\sqrt{\frac{m}{n}}(\log n)^2\right) & \text{if } l = \Omega(\sqrt{n/m}) \text{ and } \alpha = 3 \\ O\left(\frac{n}{\sqrt{l}}\left(\frac{ml}{n}\right)^{\alpha/2}\log n\right) & \text{if } l = \Omega(\sqrt{n/m}) \text{ and } \alpha > 3 \end{cases} \tag{28}$$

whp.

The proof of this lemma is presented in Appendix F.

$$\max_{\tilde{\mathbf{Q}}_L \ge 0} E\left[\log\det\left(\mathbf{I}_n + \mathbf{F}_L^{(3)} \tilde{\mathbf{Q}}_L \mathbf{F}_L^{(3)\dagger}\right) 1_{\mathcal{E}^c_{\mathbf{F}_L^{(3)}}}\right] \le n^\epsilon \sum_{i \in S_L} P_{L,i}^{(3)}$$

It is now possible to derive the cut-set upper bound in Theorem 4 by using Lemmas 9 and 10. For notational convenience, let $T_n^{(i)}$ denote the ith term in the RHS of (19) for $i \in \{1,2,3\}$. By generalized Hadamard's inequality [38] as in [3] and [16], the first term $T_n^{(1)}$ in (19) can be easily bounded by

$$T_n^{(1)} \le \sum_{k\in D_L^{(1)}} \log\left(1+\frac{P}{N_0}\sum_{i\in S_L}|h_{ki}|^2\right) \le \bar{c}_1\sqrt{n}(\log n)^2 \quad (29)$$

where $\bar{c}_1 > 0$ is some constant independent of n. Note that this upper bound does not depend on β and γ. The second inequality holds since the minimum distance between any source and destination is larger than $1/n^{1/2+\epsilon_1}$ whp for an arbitrarily small $\epsilon_1 > 0$, which is obtained by the derivation similar to that of Lemma 6, and there exist no more than $\sqrt{n}\log n$ nodes in $D_L^{(1)}$ whp by Lemma 7. The upper bound for $T_n^{(2)}$ is now derived. Since some nodes in $D_L^{(2)}$ are located very close to the cut and the information transfer to $D_L^{(2)}$ is limited in DoF, the second term $T_n^{(2)}$ of (19) is upper-bounded by the sum of the capacities of the MISO channels. More precisely, by generalized Hadamard's inequality

$$T_n^{(2)} \le \begin{cases} \bar{c}_2 ml\log n & \text{if } l=o(\sqrt{n/m}) \\ \bar{c}_2\sqrt{nm}\log n & \text{if } l=\Omega(\sqrt{n/m}) \end{cases} \le \bar{c}_2 m\min\left\{l,\sqrt{\frac{n}{m}}\right\}\log n \quad (30)$$

where $\bar{c}_2 > 0$ is some constant independent of n. Next, the third term $T_n^{(3)}$ of (19) will be shown by using (24), (27), and Lemma 10. If $l=o(\sqrt{n/m})$, which corresponds to operating regimes A and B shown in Fig. 2, then $T_n^{(3)}$ is given by

$$T_n^{(3)} = \begin{cases} O(n^{2-\alpha/2+\epsilon}) & \text{if } 2<\alpha<3 \\ O(n^{1/2+\epsilon}) & \text{if } \alpha\ge 3. \end{cases}$$

Hence, under this network condition

$$T_n \le c_5 n^{\epsilon}\max\left\{ml,\sqrt{n},n^{2-\alpha/2}\right\}$$

for some constant $c_5 > 0$ independent of n, which is upper-bounded by the RHS of (7). Now we focus on the case for $l=\Omega(\sqrt{n/m})$ (regimes C and D in Fig. 2). In this case, $T_n^{(3)}$ is upper-bounded by

$$T_n^{(3)} \le \begin{cases} \bar{c}_3 n^{\epsilon}\left(n^{2-\alpha/2}(\log n)^2 + nl\left(\frac{m}{n}\right)^{\alpha/2}\log n\right) & \text{if } 2<\alpha<3 \\ \bar{c}_3 n^{\epsilon}\left(\sqrt{n}(\log n)^3 + ml\sqrt{\frac{m}{n}}(\log n)^2\right) & \text{if } \alpha=3 \\ \bar{c}_3 n^{\epsilon}\left(\sqrt{n}(\log n)^2 + \frac{n}{\sqrt{l}}\left(\frac{ml}{n}\right)^{\alpha/2}\log n\right) & \text{if } \alpha>3 \end{cases}$$
$$\le \begin{cases} \bar{c}_3 n^{\epsilon_2}\max\left\{n^{2-\alpha/2}, nl\left(\frac{m}{n}\right)^{\alpha/2}\right\} & \text{if } 2<\alpha<3 \\ \bar{c}_3 n^{\epsilon_2}\max\left\{\sqrt{n}, \frac{n}{\sqrt{l}}\left(\frac{ml}{n}\right)^{\alpha/2}\right\} & \text{if } \alpha\ge 3 \end{cases} \quad (31)$$

for some constant $\bar{c}_3 > 0$ and an arbitrarily small constant $\epsilon_2 > \epsilon > 0$. From (29), (30), and (31), we thus get the following result:

$$T_n \le \begin{cases} \bar{c}_4 n^{\epsilon}\max\left\{\sqrt{nm}, n^{2-\alpha/2}, nl\left(\frac{m}{n}\right)^{\alpha/2}\right\} & \text{if } 2<\alpha<3 \\ \bar{c}_4 n^{\epsilon}\max\left\{\sqrt{nm}, \frac{n}{\sqrt{l}}\left(\frac{ml}{n}\right)^{\alpha/2}\right\} & \text{if } \alpha\ge 3 \end{cases}$$
$$\le \bar{c}_4 n^{\epsilon}\max\left\{\sqrt{nm}, n^{2-\alpha/2}, ml\left(\frac{m}{n}\right)^{\alpha/2-1}\right\}$$

where the first and second inequalities hold since $\sqrt{nm}=\Omega(\sqrt{n})$ and $\sqrt{nm}=\Omega(\frac{n}{\sqrt{l}}(\frac{ml}{n})^{\alpha/2})$, respectively, which results in (7). This completes the proof of Theorem 4.

Now we would like to examine in detail the amount of information transfer by each separated destination set.

Remark 5: The information transfer by the BS antennas on the left half, i.e., the destination set $D_L\setminus\bar{D}_r$, becomes dominant under operating regimes B–D (especially at the high path-loss attenuation regimes) in Fig. 2. More specifically, compared to the pure network case with no BSs, as m and l increases (i.e., regimes B–D), enough DoF gain is obtained by exploiting multiple antennas at each BS, while the power gain is provided since all the BSs are connected by the wired BS-to-BS links. In addition, note that the first to fourth terms in (7) represent the amount of information transferred to the destination sets $D_L\setminus(D_L^{(2)}\cup\bar{D}_r)$, $D_L^{(2)}$, $D_L^{(1)}$, and $\bar{D}_r\setminus D_L^{(1)}$, and can be achieved by the ISH, IMH, MH, HC schemes, respectively.

VII. Conclusion

The paper has analyzed the benefits of infrastructure support for generalized hybrid networks. Provided the number m of BSs and the number l of antennas at each BS scale at arbitrary rates relative to the number n of wireless nodes, the capacity scaling laws were derived as a function of these scaling parameters. Specifically, two routing protocols using BSs were proposed, and their achievable rate scalings were derived and compared with those of the two conventional schemes MH and HC in both dense and extended networks. Furthermore, to show the optimality of the achievability results, new information-theoretic upper bounds were derived. In both dense and extended networks, it was shown that our achievable schemes are order-optimal for all the operating regimes.

Appendix

A. Achievable Throughput With Respect to Operating Regimes

Let e_{ISH}, e_{IMH}, $_{\text{MH}}$, and e_{HC} denote the scaling exponents for the achievable throughput of the ISH, IMH, MH, and HC protocols, respectively. The scaling exponents among the above schemes are compared according to operating regimes A–D shown in Fig. 2 (ϵ is omitted for notational convenience). From the result of Theorem 2, note that e_{ISH}, e_{MH}, and e_{HC} are given by $1+\gamma-\frac{(1-\beta)\alpha}{2}$, $\frac{1}{2}$, and $2-\frac{\alpha}{2}$, respectively, regardless of operating regimes.

1) Regime A ($0\le\beta+\gamma<\frac{1}{2}$): $e_{\text{IMH}}=\beta+\gamma$ is obtained. Since $e_{\text{MH}} > e_{\text{IMH}} > e_{\text{ISH}}$, pure ad hoc transmis-

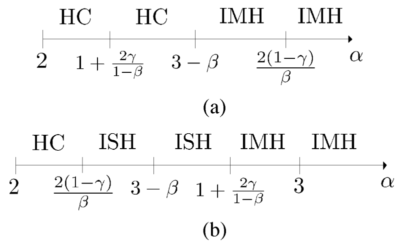

Fig. 8. The best achievable schemes with respect to α. (a) The Regime C. (b) The Regime D.

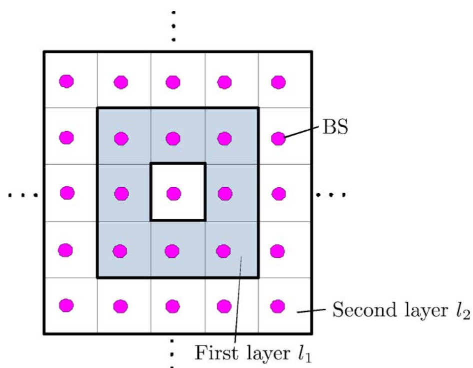

Fig. 9. Grouping of interfering cells. The first layer l_1 of the network represents the outer 8 shaded cells.

sions with no BSs outperform the ISH and IMH protocols. Hence, the results in Regime A of Table I are obtained.

2) Regime B ($\beta+\gamma \geq \frac{1}{2}$ and $\beta + 2\gamma < 1$): $e_{\rm IMH}$ is the same as that under Regime A. Since $e_{\rm IMH} > e_{\rm ISH}$ and $e_{\rm IMH} \geq e_{\rm MH}$, the IMH always outperforms the ISH and the MH. Hence, it is found that the HC scheme has the largest scaling exponent under $2 < \alpha < 4-2\beta-2\gamma$, but if $\alpha \geq 4-2\beta-2\gamma$ the IMH protocol becomes the best.

3) Regime C ($\beta+2\gamma \geq 1$ and $\gamma < \frac{1}{2}(\beta^2-3\beta+2)$): Remark that $e_{\rm IMH} = \frac{1+\beta}{2}$ and $e_{\rm IMH} \geq e_{\rm MH}$. Then, the following inequalities with respect to the path-loss exponent α are found: $e_{\rm ISH} > e_{\rm IMH}$ for $2 < \alpha < 1+\frac{2\gamma}{1-\beta}$ and $e_{\rm ISH} \leq e_{\rm IMH}$ for $\alpha \geq 1+\frac{2\gamma}{1-\beta}$; $e_{\rm HC} > e_{\rm IMH}$ for $2 < \alpha < 3-\beta$ and $e_{\rm HC} \leq e_{\rm IMH}$ for $\alpha \geq 3-\beta$; and $e_{\rm HC} > e_{\rm ISH}$ for $2 < \alpha < \frac{2(1-\gamma)}{\beta}$ and $e_{\rm HC} \leq e_{\rm ISH}$ for $\alpha \geq \frac{2(1-\gamma)}{\beta}$. The best scheme thus depends on the comparison among $1+\frac{2\gamma}{1-\beta}$, $3-\beta$, and $\frac{2(1-\gamma)}{\beta}$. Note that $3-\beta < \frac{2(1-\gamma)}{\beta}$ and $3-\beta > 1+\frac{2\gamma}{1-\beta}$ always hold under Regime C. Finally, the best achievable schemes with respect to α are obtained and are shown in Fig. 8(a).

4) Regime D ($\beta+\gamma < 1$ and $\gamma \geq \frac{1}{2}(\beta^2-3\beta+2)$): The same scaling exponents for our four protocols are the same as those under Regime C. The result is obtained by comparing $1+\frac{2\gamma}{1-\beta}$, $3-\beta$, and $\frac{2(1-\gamma)}{\beta}$ under Regime D. The following two inequalities $3-\beta \geq \frac{2(1-\gamma)}{\beta}$ and $3-\beta \leq 1+\frac{2\gamma}{1-\beta}$ are satisfied, and the best achievable schemes with respect to α are obtained and shown in Fig. 8(b).

This coincides with the result shown in Table I.

B. Proof of Lemma 3

First consider the uplink case. There are $8k$ interfering cells, each of which includes $\Theta(n/m)$ nodes whp, in the kth layer l_k of the network as illustrated in Fig. 9. Let d_k denote the Euclidean distance between a given BS antenna and any node in l_k, which is a random variable. Since d_k scales as $\Theta(k\sqrt{n/m})$, there exists $c_7 > c_6 > 0$ with constants c_6 and c_7 independent of n, such that $d_k = c_8 k\sqrt{n/m}$, where all c_8 lies in the interval $[c_6, c_7]$. Hence, the total interference power P_I^u at each BS antenna from simultaneously transmitting nodes is upper-bounded by

$$\begin{aligned} P_I^u &\leq \sum_{k=1}^{\infty} \frac{P}{(m/n)^{\alpha/2-1}}(8k)\frac{n}{m}\left(\frac{m}{(c_6 k)^2 n}\right)^{\alpha/2} \\ &= \frac{8P}{c_6^{\alpha}}\sum_{k=1}^{\infty}\frac{1}{k^{\alpha-1}} \\ &\leq c_9 \end{aligned}$$

where $c_9 > 0$ is some constant independent of n. Now let us consider the interference in the downlink. The interfering signal received by node i, which is in the cell covered by BS s, from the simultaneously operating BSs $s' \in \{1,\ldots,m\}\backslash\{s\}$ is given by

$$\sum_{s'\in\{1,\ldots,m\}\backslash\{s\}} \mathbf{h}_{is'}^d \left(\sum_{j=1}^{n/m} \mathbf{u}_j^{s'} x_j^{s'}\right)$$

where $\mathbf{u}_j^{s'}$ denotes the jth transmit precoding vector at BS s' normalized so that its L_2-norm is unity and $x_j^{s'}$ is the jth transmit packet at BS s'. Since $\mathbf{u}_j^{s'}$ is represented by a function of the downlink channel coefficients between BS s' and nodes communicating with BS s', the terms $[\mathbf{h}_{is'}^d]_k \cdot [\sum_j \mathbf{u}_j^{s'} x_j^{s'}]_k$ are independent for all $k \in \{1,\ldots,n/m\}$ and $s' \in \{1,\ldots,m\} \setminus \{s\}$. Using the fact above and the layering technique similar to the uplink case, an upper bound of the average total interference power P_I^d at each node in the downlink is obtained as the following:

$$P_I^d \leq \sum_{k=1}^{\infty} \frac{(n/m)P}{(m/n)^{\alpha/2-1}}(8k)\left(\frac{m}{(c_6 k)^2 n}\right)^{\alpha/2} \leq c_9'$$

where $c_9' > 0$ is some constant independent of n.

C. Proof of Lemma 5

Let X_i denote the number of sources in the ith cell and $\mathcal{E}_x$ denote the event that X_i is between $((1-\delta_0)n/m, (1+\delta_0)n/m)$ for all $i \in \{1,\ldots,m\}$, where $0 < \delta_0 < 1$ is some constant independent of n. Then, we have

$$\begin{aligned} &\Pr\{X_{ki} < a \text{ for all } i,k \in \{1,\ldots,m\}\} \\ &\quad \geq \Pr\{\mathcal{E}_x\}\Pr\{X_{ki} < a \text{ for all } i,k|\mathcal{E}_x\} \\ &\quad \geq \Pr\{\mathcal{E}_x\}\left(1 - m^2 \Pr\left\{\sum_{j=1}^{(1+\delta_0)n/m} B_j \geq a\right\}\right) \end{aligned} \tag{32}$$

where $\sum_j B_j$ is the sum of $(1+\delta_0)n/m$ independent and identically distributed (i.i.d.) Bernoulli random variables with probability

$$\Pr\{B_j = 1\} = \frac{1}{m}.$$

Here, the second inequality holds since the union bound is applied over all $i,k \in \{1,\ldots,m\}$. We first consider the case where $n/m = \omega(m)$, i.e., $0 \le \beta < 1/2$. By setting $a = (1+\delta_0)^2 n/m^2$, we then get

$$\begin{aligned}\Pr\left\{\sum_{j=1}^{(1+\delta_0)n/m} B_j \ge (1+\delta_0)^2\frac{n}{m^2}\right\} &= \Pr\left\{e^{s\sum_{j=1}^{(1+\delta_0)n/m} B_j} \ge e^{s(1+\delta_0)^2 n/m^2}\right\}\\ &\le e^{-(1+\delta_0)n/m^2(s(1+\delta_0)-e^s+1)} \end{aligned}\tag{33}$$

which is derived from the steps similar to the proof of Lemma 4.1 in [3], where the first inequality comes from an application of Chebyshev's inequality. Hence, using (8), (32), and (33) yields

$$\begin{aligned}&\Pr\left\{X_{ki} < (1+\delta_0)^2\frac{n}{m^2} \text{ for all } i,k \in \{1,\ldots,m\}\right\}\\ &\ge \left(1-n^\beta e^{-\Delta(\delta_0)n^{1-\beta}}\right)\left(1-m^2 e^{-(1+\delta_0)\Delta(\delta_0)n/m^2}\right)\\ &= \left(1-n^\beta e^{-\Delta(\delta_0)n^{1-\beta}}\right)\left(1-e^{2\beta\ln n-(1+\delta_0)\Delta(\delta_0)n^{1-2\beta}}\right)\end{aligned}$$

where $\Delta(\delta_0) = (1+\delta_0)\ln(1+\delta_0) - \delta_0$, by choosing $s = \ln(1+\delta_0)$, which converges to one as n goes to infinity. When $n/m = O(m)$, i.e., $1/2 \le \beta < 1$, setting $a = \ln n$ and $s = (2+\delta_0)\beta$ and following the approach similar to the first case, we obtain

$$\begin{aligned}&\Pr\{X_{ki} < \ln n \text{ for all } i,k \in \{1,\ldots,m\}\}\\ &\ge \left(1-n^\beta e^{-\Delta(\delta_0)n^{1-\beta}}\right)\\ &\quad\cdot\left(1-m^2 e^{-(1+\delta_0)(1-e^{(2+\delta_0)\beta})n/m^2-(2+\delta_0)\beta\ln n}\right)\\ &= \left(1-n^\beta e^{-\Delta(\delta_0)n^{1-\beta}}\right)\\ &\quad\cdot\left(1-e^{-\delta_0\beta\ln n-(1+\delta_0)\left(1-e^{(2+\delta_0)\beta}\right)n^{1-2\beta}}\right)\end{aligned}$$

which converges to one as n goes to infinity. This completes the proof.

D. *Proof of Lemma 6*

This result can be obtained by slightly modifying the asymptotic analysis in [3] and [14]. The minimum node-to-node distance is easily derived by following the same approach as that in [3] and is proved to scale at least as $1/n^{1+\epsilon_1}$ with probability $1-\Theta(1/n^{2\epsilon_1})$. We now focus on how the distance between a node and an antenna on the BS boundary scales. Consider a circle of radius $1/n^{1+\epsilon_1}$ around one specific antenna on the BS boundary. Note that there is no antenna inside the circle since the per-antenna distance is greater than $1/n^{1+\epsilon_1}$. Let $\mathcal{E}_d$ denote the event that n nodes are located outside the circle given by the antenna. Then, we have

$$P\{\mathcal{E}_d^{\mathcal{C}}\} \le 1 - \left(1-\frac{c_{10}\pi}{n^{2+2\epsilon_1}}\right)^n$$

where $0 < c_{10} < 1$ is some constant independent of n. Hence, by the union bound, the probability that the event $\mathcal{E}_d$ is satisfied for all the BS antennas is lower-bounded by

$$\begin{aligned}1 - mlP\{\mathcal{E}_d^{\mathcal{C}}\} &\ge 1 - ml\left(1-\left(1-\frac{c_{10}\pi}{n^{2+2\epsilon_1}}\right)^n\right)\\ &\ge 1 - n\left(1-\left(1-\frac{c_{10}\pi}{n^{2+2\epsilon_1}}\right)^n\right)\end{aligned}$$

where the second inequality holds since $ml = O(n)$, which tends to one as n goes to infinity. This completes the proof.

E. *Proof of Lemma 8*

The size of matrix $\mathbf{F}_L^{(3)}$ is $\Theta(n)\times\Theta(n)$ since $ml = O(n)$. Thus, the analysis essentially follows the argument in [3] with a slight modification (see Appendix III in [3] for more precise description). Consider the network transformation resulting in a regular network with at most $\log n$ nodes at each square vertex except for the area covered by BSs. Then, the same node displacement as shown in [3] is performed, which will decrease the Euclidean distance between source and destination nodes. For convenience, the source node positions are indexed in the resulting regular network. It is thus assumed that the source nodes under the cut are located at positions $(-i_x+1, i_y)$ where $i_x, i_y = 1, \cdots \sqrt{n}$. In the following, $\sum_{k\in D_L^{(3)}} |[\mathbf{F}_L^{(3)}]_{ki}|^2$ and an upper bound for $\sum_{i\in S_L} |[\mathbf{F}_L^{(3)}]_{ki}|^2$ are derived:

$$\sum_{k\in D_L^{(3)}} \left|\left[\mathbf{F}_L^{(3)}\right]_{ki}\right|^2 = 1$$

and

$$\begin{aligned}\sum_{i\in S_L}\left|\left[\mathbf{F}_L^{(3)}\right]_{ki}\right|^2 &= \sum_{i\in S_L}\left|\frac{1}{\sqrt{d_{L,i}^{(3)}}}\left[\mathbf{H}_L^{(3)}\right]_{ki}\right|^2\\
&= \begin{cases}\sum_{i\in S_L}\frac{r_{ki}^{-\alpha}}{d_{L,i}^{(3)}} & \text{if } k \in \bar{D}_r \setminus D_L^{(1)}\\ \sum_{i\in S_L}\frac{r_{si,t}^{u-\alpha}}{d_{L,i}^{(3)}} & \text{if } k \in \{t : t\in[1,l]\\ & \quad \text{for } s \in B_l\}\end{cases}\\
&\le \begin{cases}\sum_{i\in S_L}\frac{r_{ki}^{-\alpha}}{\sum_{k\in\bar{D}_r\setminus D_L^{(1)}} r_{ki}^{-\alpha}} & \text{if } k \in \bar{D}_r \setminus D_L^{(1)}\\ \sum_{i\in S_L}\frac{r_{si,t}^{u-\alpha}}{\sum_{k\in\bar{D}_r\setminus D_L^{(1)}} r_{ki}^{-\alpha}} & \text{if } k \in \{t : t\in[1,l]\\ & \quad \text{for } s \in B_l\}\end{cases}\\
&\le \begin{cases}c_{11}\log n\sum_{i\in S_L} x_i^{\alpha-2} r_{ki}^{-\alpha} & \text{if } k \in \bar{D}_r \setminus D_L^{(1)}\\ c_{11}\log n\sum_{i\in S_L} x_i^{\alpha-2} r_{si,t}^{u-\alpha} & \text{if } k \in \{t : t\in[1,l]\\ & \quad \text{for } s \in B_l\}\end{cases}\\
&\le \begin{cases}c_{11}\log n\sum_{i\in S_L} r_{ki}^{-2} & \text{if } k \in \bar{D}_r \setminus D_L^{(1)}\\ c_{11}\log n\sum_{i\in S_L} r_{si,t}^{u-2} & \text{if } k \in \{t : t\in[1,l]\\ & \quad \text{for } s \in B_l\}\end{cases}\\
&\le c_{11}(\log n)^2 \sum_{i_x,i_y=1}^{\sqrt{n}} \frac{1}{i_x^2+i_y^2}\\
&\le c_{12}(\log n)^3\end{aligned}$$

where $\bar{D}_r$ is the set of nodes including BS antennas on the right half, B_l is the set of BSs in the left half network, c_{11} and c_{12} are some positive constants independent of n, and x_i denotes

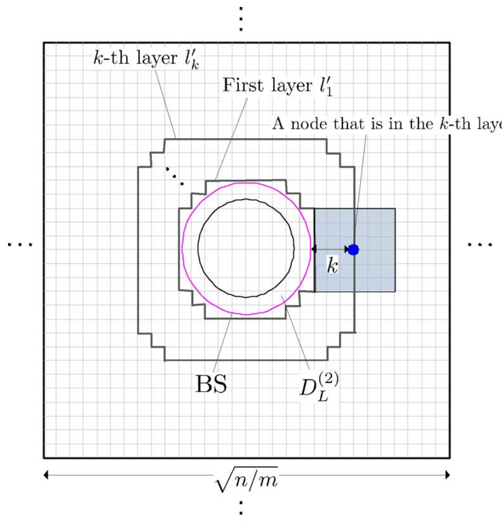

Fig. 10. The displacement of the nodes to square vertices. The antennas are regularly placed at spacing $\sqrt{\frac{n}{ml}}$ outside the shaded square.

the x-coordinate of node $i \in S_L$ for our random network ($x_i = 1, \ldots, \sqrt{n}$). Here, the second and fifth inequalities hold since

$$\sum_{k \in \bar{D}_r \setminus D_L^{(1)}} r_{ki}^{-\alpha} \geq \frac{x_i^{2-\alpha}}{c_{11} \log n}$$

and

$$\sum_{i_x, i_y = 1}^{\sqrt{n}} \frac{1}{i_x^2 + i_y^2} = O(\log n)$$

respectively (see Appendix III in [3] for the detailed derivation). The fourth inequality comes from the result of Lemma 7. Hence, it is proved that both scaling results are the same as the random network case shown in [3].

Now it is possible to prove the inequality in (23) by following the same line as that in Appendix III of [3], which results in

$$E\left[\mathrm{tr}\left(\left(\mathbf{F}_L^{(3)\dagger}\mathbf{F}_L^{(3)}\right)^q\right)\right] \leq C_q n (c_{13} \log n)^{3q}$$

where $C_q = \frac{(2q)!}{q!(q+1)!}$ is the Catalan number for any q and $c_{13} > 0$ is some constant independent of n. Then, from the property $\|\mathbf{F}_L^{(3)}\|_2^2 = \lim_{q\to\infty} \mathrm{tr}((\mathbf{F}_L^{(3)\dagger}\mathbf{F}_L^{(3)})^q)^{1/q}$ (see [39]), the expectation of the term $\|\mathbf{F}_L^{(3)}\|_2^2$ is finally given by (23), which completes the proof.

F. Proof of Lemma 10

When $l = o(\sqrt{n/m})$, there is no destination in $D_L^{(5)}$, and thus $\sum_{i\in S_L} P d_{L,i}^{(5)}$ becomes zero. Hence, the case for $l = \Omega(\sqrt{n/m})$ is the focus from now on. By the same argument as shown in the derivation of $\sum_{i\in S_L} P d_{L,i}^{(4)}$, the network area is divided into n squares of unit area. Then, by Lemma 7, the power transfer under our random network can be upper-bounded by that under a regular network with at most $\log n$ nodes at each square except for the area covered by BSs. As illustrated in Fig. 10, the nodes in each square are moved together onto one vertex of the corresponding square. The node displacement is performed in a sense of decreasing the Euclidean distance between node $i \in S_L$ and the antennas of the corresponding BS, thereby providing an upper bound for $d_{L,i}^{(5)}$. Layers of each cell are then introduced, as shown in Fig. 10, where there exist $8(\epsilon_0\sqrt{n/m} + k)$ vertices, each of which includes $\log n$ nodes, in the kth layer l'_k of each cell. The regular network described above can also be transformed into the other, which contains antennas regularly placed at spacing $\epsilon_0\sqrt{\frac{n\pi}{ml}}$ outside the shaded square for an arbitrarily small $\epsilon_0 > 0$. Note that the shaded square of size $2k \times 2k$ is drawn based on a source node in l'_k at its center (see Fig. 10). The modification yields an increase of the term $d_{L,i}^{(5)}$ by source i. When $d_{L,i(k)}^{(5)}$ is defined as $d_{L,i}^{(5)}$ by node i that lies in l'_k, the following upper bound for $d_{L,i(k)}^{(5)}$ is obtained:

$$\begin{aligned}
d_{L,i(k)}^{(5)} &\leq \sum_{k_x,k_y=\zeta}^{\infty} \frac{1}{\left((\epsilon_0\sqrt{\frac{n\pi}{ml}}k_x)^2 + (\epsilon_0\sqrt{\frac{n\pi}{ml}}k_y)^2\right)^{\alpha/2}} \\
&= \left(\frac{ml}{n}\right)^{\alpha/2} \sum_{k_x,k_y=\zeta}^{\infty} \frac{\eta^\alpha}{(k_x^2 + k_y^2)^{\alpha/2}} \\
&\leq \left(\frac{ml}{n}\right)^{\alpha/2} \sum_{k'=\zeta}^{\infty} \frac{8\eta^\alpha k'}{k'^\alpha} \\
&\leq c_{14} \left(\frac{ml}{n}\right)^{\alpha/2} \left(\frac{1}{\zeta^{\alpha-1}} + \int_\zeta^\infty \frac{1}{x^{\alpha-1}} dx\right) \\
&= c_{14} \left(\frac{1}{\zeta} + \frac{1}{\alpha-2}\right) \left(\frac{ml}{n}\right)^{\alpha/2} \frac{1}{\zeta^{\alpha-2}}
\end{aligned}$$

where $\zeta = 1 + \lfloor k\eta \rfloor$, $\eta = \frac{1}{\epsilon_0}\sqrt{\frac{ml}{n\pi}}$, and c_{14} is some constant independent of n. Here, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. Hence, $d_{L,i(k)}^{(5)}$ is given by

$$d_{L,i(k)}^{(5)} = \begin{cases} O\left(\left(\frac{ml}{n}\right)^{\alpha/2}\right) & \text{if } k = O\left(\sqrt{\frac{n}{ml}}\right) \\ O\left(k^{2-\alpha}\left(\frac{ml}{n}\right)\right) & \text{if } k = \Omega\left(\sqrt{\frac{n}{ml}}\right) \end{cases}$$

finally resulting in

$$\begin{aligned}
&\sum_{i\in S_L} P d_{L,i}^{(5)} \\
&\leq P\frac{m}{2}\log n \sum_{k=1}^{\sqrt{n/m}} 8\left(\epsilon_0\sqrt{\frac{n}{m}} + k\right) d_{L,i(k)}^{(5)} \\
&\leq c_{15} P \sqrt{nm} \log n \\
&\quad \cdot \left[\sum_{k=1}^{\sqrt{n/ml}-1} \left(\frac{ml}{n}\right)^{\alpha/2} + \sum_{k=\sqrt{n/ml}}^{\sqrt{n/m}} k^{2-\alpha}\left(\frac{ml}{n}\right)\right] \\
&\leq c_{15} P \sqrt{nm} \log n \\
&\quad \cdot \left[\left(\frac{ml}{n}\right)^{(\alpha-1)/2}\right. \\
&\quad \left. + \left(\frac{ml}{n}\right)\left(\left(\frac{ml}{n}\right)^{\alpha/2-1} + \int_{\sqrt{n/ml}}^{\sqrt{n/m}} \frac{1}{k^{\alpha-2}} dk\right)\right] \\
&\leq \begin{cases} \frac{3c_{15}P}{3-\alpha} nl \left(\frac{m}{n}\right)^{\alpha/2} \log n & \text{if } 2 < \alpha < 3 \\ \frac{3c_{15}P}{2} ml \sqrt{\frac{m}{n}} (\log n)^2 & \text{if } \alpha = 3 \\ \frac{3c_{15}P}{\alpha-3} \frac{n}{\sqrt{l}} \left(\frac{ml}{n}\right)^{\alpha/2} \log n & \text{if } \alpha > 3 \end{cases} \qquad (34)
\end{aligned}$$

where c_{15} is some constant independent of n. Here, the first inequality holds since there exist $8(\epsilon_0\sqrt{n/m}+k)$ vertices in l'_k and at most $\log n$ nodes at each vertex. Equation (34) yields the result in (28), which completes the proof.

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