WAVEFORM SCHEDULING VIA DIRECTED INFORMATION IN COGNITIVE RADAR

Pawan Setlur, Natasha Devroye and Zhiyu Cheng

ECE Department, University of Illinois at Chicago
Email: {setlurp, devroye, zcheng3}@uic.edu

ABSTRACT

The objective of waveform scheduling is to achieve maximal information extraction of the radar scene, which typically changes from one measurement to the next, by exploiting prior statistics and waveform diversity. In this paper waveform scheduling is addressed using information theoretic concepts. A pre-defined waveform library is assumed to be given, or designed *a priori*. To keep the analysis simple, we constrain ourselves to a library comprised of two waveforms scheduled over two consecutive time intervals. We propose selecting the waveforms to maximize the *directed information*, a metric not previously considered in this context, which directly incorporates the feedback present in the radar system. Analog and discrete models are discussed; the former allows for a spectral domain interpretation, whereas, the latter permits analogies to Bayesian error metrics.

Index Terms— Waveform diversity, Cognitive radar, Waveform scheduling, Directed information

1. INTRODUCTION

Active sensing systems such as radar transmit waveforms to illuminate a scene. Transmitting different waveforms at different times may aid in better understanding the radar scene [1]. However, as radar scenes are dynamic, and the goals of different systems vary, a consensus on how to best design and schedule waveforms, and how to incorporate or fuse the received signals for a general system does not yet exist. In this paper, we address the *closed loop* waveform scheduling problem from an information theoretic perspective. In particular, we seek to schedule waveforms to maximize information gain in a way which incorporates feedback, or previous radar returns, thereby closing the loop in active radar sensing.

It is argued here that in general, the information theoretic quantity *directed information (DI)* [2] – which naturally incorporates feedback – related to, but not always equal to the more commonly used *mutual information* (MI), should be maximized. Typical radar returns include clutter, and it is readily acknowledged that target-clutter interactions exist [3]-[5]. When such interactions are ignored or assumed absent, then, as we show, maximizing the directed information is equivalent to maximizing the mutual information.

Past work. Bell's seminal work first formalized the relation between information theory and radar waveform design [6], where he argued that to design waveforms for maximal information extraction of an extended target over one scheduling epoch, mutual information should be used as metric. Waveform design in the presence of signal dependent interference was treated in [7]. Mutual information and (Rényi) entropy have been used in other applications as surrogate metrics, for example sensor / resource scheduling [8]-[11].

Contributions. In this work we propose, for the first time, the use of directed information for waveform scheduling over multiple time epochs, which incorporates the feedback from past radar echoes. We note that while we are interested in maximizing information gain, and not waveform design, our formulation nevertheless allows for it, when the maximization is performed on the waveforms themselves.

2. MODEL

Consider a single complex target consisting of many point scatterers and whose spatial extent spans multiple range cells. We assume that we are given a waveform library consisting of two waveforms (more may be handled analogously but for simplicity we start with two), $\{s_1(t), s_2(t)\}$, where t indexes continuous time. If waveform $s_i(t)$ is transmitted at the first scheduling instant, the radar return in baseband is

$$y_1^{(i)}(t) = \alpha_1(t) * s_i(t) + \beta_1(t) * s_i(t) + v_1(t), \qquad (1)$$

for $t \in [\tau_{\min}, \tau_{\max}]$, are the set of time delays (or equivalently range) under consideration, $*$ denotes the convolution operator and the noise $v_1(t)$ is a zero mean complex stationary Gaussian random processes independent of $\alpha_1(t)$ and $\beta_1(t)$.

The impulse responses, $\alpha_1(t)$ and $\beta_1(t)$ are complex, finite duration and finite energy Gaussian random processes, modeling the reflectivities of the target, and the clutter plus the target interactions with the clutter (or its environment), respectively. Hence $\alpha_1(t)$ and $\beta_1(t)$ may in general be correlated (e.g. multipath). For ease of analysis, we will assume that the processes, $\alpha_1(t), \beta_1(t)$ are locally covariance stationary within their respective temporal supports.

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If waveform $s_j(t)$ is transmitted at the second time instant, then the radar return can be written similar to (1) as,

$$y_2^{(j)}(t) = \alpha_2(t) * s_j(t) + \beta_2(t) * \ s_j(t) + v_2(t) \qquad (2)$$

for $t \in [T + \tau_{\min}, T + \tau_{\max}]$, where T is the period between the two scheduling epochs, (typically in the order of μs). Here $\alpha_2(t)$ and $\beta_2(t)$ are complex finite energy Gaussian random processes similar to $\alpha_1(t)$ and $\beta_1(t)$ but defined in the second time scheduling instant. The noise in (2) is $v_2(t)$, again assumed to be Gaussian, and it may be correlated with $v_1(t)$. Statistical assumptions on $\alpha_2(t)$ and $\beta_2(t)$ are identical to those imposed on $\alpha_1(t)$ and $\beta_1(t)$, and the four impulse responses may in general be correlated. We do distinguish the impulses responses at the first and second scheduling epochs as we allow for moving targets, therefore giving rise to a different radar cross section (RCS) fading process of the target and its interaction with the clutter. As we analyze only two scheduling instants, the Doppler cannot be estimated satisfactorily. Nevertheless, the phase progression arising from the Doppler can be easily absorbed into say $\alpha_m(t), m = 1, 2$. For ease of exposition, we assume that the same set of time delays $[\tau_{\min}, \tau_{\max}]$, are valid for each scheduling instance. In practice they may be different and this may be accounted for in our model. The discrete model is derived next.

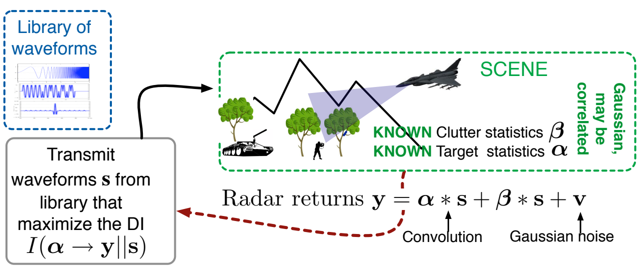

Fig. 1. Closed loop radar waveform scheduling to maximize the directed information (DI).

2.1. Discrete model

By sampling the continuous-time model we obtain a discrete model. Without loss of generality, assume that N denotes the data length of both the returns, and there are K discrete scatterers representing the random processes $\alpha_m(t)$ and $\beta_m(t), m = 1, 2$ in each scheduling instant[1], then we can write both (1), (2) in an equivalent discrete form,

$$\begin{aligned} \mathbf{y}_1^{(i)} &= \bar{\mathbf{S}}_i[\boldsymbol{\alpha}_1^H, \boldsymbol{\beta}_1^H]^H + \mathbf{v}_1, \quad i = 1, \text{ or } 2 \\ \mathbf{y}_2^{(j)} &= \bar{\mathbf{S}}_j[\boldsymbol{\alpha}_2^H, \boldsymbol{\beta}_2^H]^H + \mathbf{v}_2, \quad j = 1, \text{ or } 2 \end{aligned} \qquad (3)$$

where, $\boldsymbol{\alpha}_m = [\alpha_{m1}, \alpha_{m2}, \ldots, \alpha_{mK}]^H \in \mathbb{C}^{K\times 1}$ and $\boldsymbol{\beta}_m = [\beta_{m1}, \beta_{m2}, \ldots, \beta_{mK}]^H \in \mathbb{C}^{K\times 1}, m = 1, 2$ now represent the reflectivities of the scatterers, and the reflectivities of the clutter and target interactions, respectively. We note that the discrete model allows us to consider range cells which are target and target+clutter only. In other words, range cells which consist of noise only contributions are ignored. The matrices, $\bar{\mathbf{S}}_i$ and $\bar{\mathbf{S}}_j$ are defined as,

$$\bar{\mathbf{S}}_i := \text{Diag}\{\mathbf{S}_i, \mathbf{S}_i\}, \ \bar{\mathbf{S}}_j := \text{Diag}\{\mathbf{S}_j, \mathbf{S}_j\},$$

where, $\text{Diag}\{\cdot, \cdot\}$ converts the matrix arguments into a block diagonal matrix. The matrices, $\mathbf{S}_i$ and $\mathbf{S}_j$ consists of the waveform samples, $s_i(\cdot)$ and $s_j(\cdot)$, respectively. Convolution matrices are special cases of $\mathbf{S}_i$ and $\mathbf{S}_j$, but in general their structure depends on the sparsity of both $\boldsymbol{\alpha}_m$ and $\boldsymbol{\beta}_m$.

The question we seek to answer is: do we transmit waveform $s_1(t)$ or $s_2(t)$ in the first scheduling instant, and likewise transmit $s_1(t)$ or $s_2(t)$ in the second scheduling instant?

3. SCHEDULING VIA DIRECTED INFORMATION

Directed information was derived to analyze the performance of communication systems with feedback [2]. Indeed, the directed information (maximized over a suitable input distribution) yields the capacity of channels where transmitters have causal access to the receivers' past symbols (feedback) and hence may adapt current inputs.

Our goal, as in much of the waveform design and mutual-information based sensor scheduling work, is to schedule waveforms so as to maximize the amount of information gained over the two (at the moment) scheduling instances. The DI captures the information causally obtained at the received about the scene (or $\boldsymbol{\alpha}$) and is the natural choice for maximizing the information gain over multiple time steps while incorporating the past radar returns. Mutual information does not preserve the causality of the information flow [2]. However, as we will see, in some relevant scenarios the DI is equal to the (two epoch) MI.

For now, we will discuss only the relevant properties of DI as it pertains to our radar problem. In particular, we wish to select the waveforms $\mathbf{s}_i$ and $\mathbf{s}_j$ at times 1,2 respectively that will maximize the causally conditioned directed information *between the target responses* $\boldsymbol{\alpha}$ *and the received signal* $\mathbf{y}$, $DI(i, j)$, defined for $i, j \in \{1, 2\}$ as

$$\begin{aligned} DI(i, j) &:= I(\boldsymbol{\alpha} \rightarrow \mathbf{y}||\mathbf{s}_i, \mathbf{s}_j) \\ &= I(\boldsymbol{\alpha}_1; \mathbf{y}_1^{(i)}|\mathbf{s}_i) + I(\boldsymbol{\alpha}; \mathbf{y}_2^{(j)}|\mathbf{s}_i, \mathbf{s}_j, \mathbf{y}_1^{(i)}), \end{aligned} \qquad (4)$$

where, $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_1^H, \boldsymbol{\alpha}_2^H]^H$, $\mathbf{y} = [\mathbf{y}_1^{(i)H}, \mathbf{y}_2^{(j)H}]^H$, $\mathbf{s}_m \in \Re^{(N+1-K)\times 1}, m = 1, 2$ represents the vectors comprising the waveform's samples, and $I(X; Y|Z)$ is the mutual information between (X, Y) conditioned on Z defined in the usual manner, see [12]. The mutual information, in contrast, taken

[1] Both N and K may assume different values in the two scheduling instants, but for notational simplicity here we assume they are constant over time.

over the two time steps is given by

$$
\begin{aligned}
MI(i,j) &= I(\boldsymbol{\alpha};\mathbf{y}|\mathbf{s}_i,\mathbf{s}_j) && (5)\\
&= I(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2;\mathbf{y}_1^{(i)}|\mathbf{s}_i,\mathbf{s}_j) + I(\boldsymbol{\alpha};\mathbf{y}_2^{(j)}|\mathbf{s}_i,\mathbf{s}_j,\mathbf{y}_1^{(i)}),\\
&= DI(i,j) + I(\boldsymbol{\alpha}_2;\mathbf{y}_1^{(i)}|\mathbf{s}_i,\mathbf{s}_j,\boldsymbol{\alpha}_1). && (6)
\end{aligned}
$$

Like the MI, DI is non-negative, but unlike MI, DI is not symmetric. From the above it is clear that $DI(i,j) \le MI(i,j)$, and that the mutual information contains a non-causal term $I(\boldsymbol{\alpha}_2;\mathbf{y}_1^{(i)}|\mathbf{s}_i,\mathbf{s}_j,\boldsymbol{\alpha}_1)$ which is non-negative.

The waveform selection or scheduling criteria is then:

$$
(\mathbf{s}_i^*,\mathbf{s}_j^*) = \arg\max_{i,j} \;\; DI(i,j) \qquad (7)
$$

After the first scheduling instant, one has some information about $\boldsymbol{\alpha}_1$ and access to the returns $\mathbf{y}_1^{(i)}$. This is used to select a waveform in the next time slot which will best illuminate $\boldsymbol{\alpha}_2$ in order to maximize the net information transfer from the target to the radar over the two time-steps. However, for some cases, maximizing the DI is equivalent to maximizing the MI. From (5) and (6), $MI(i,j) \neq DI(i,j)$ when $I(\boldsymbol{\alpha}_2;\mathbf{y}_1^{(i)}|\mathbf{s}_i,\mathbf{s}_j,\boldsymbol{\alpha}_1) \neq 0$ or when

$$
H(\boldsymbol{\alpha}_2|\boldsymbol{\alpha}_1,\mathbf{y}_1^{(i)},\mathbf{s}_i,\mathbf{s}_j) \neq H(\boldsymbol{\alpha}_2|\boldsymbol{\alpha}_1,\mathbf{s}_i,\mathbf{s}_j) \qquad (8)
$$

This holds when knowing $\mathbf{y}_1^{(i)}$ (in addition to $\boldsymbol{\alpha}_1$) provides partial additional information about $\boldsymbol{\alpha}_2$. Let us denote $\mathrm{Cov}\{\mathbf{x},\mathbf{y}\}$ as the covariance between $\mathbf{x}$ and $\mathbf{y}$. Then it may be shown that (8) holds when

$$
\mathrm{Cov}\{\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1\} \neq \mathbf{0} \text{ and } \mathrm{Cov}\{\boldsymbol{\alpha}_2,\boldsymbol{\beta}_1\} \neq \mathbf{0}, \qquad (9)
$$

or when the target responses and clutter responses are correlated (within one slot, and over two slots).

4. SPECIAL CASES

In this section we will consider independence of clutter responses from the target responses. In the discrete case, it is shown that DI maximization is related to minimizing the Bayesian mean squared error. In the analog model, we provide a spectral domain interpretation of the DI maximization.

4.1. Independent target and clutter responses: discrete

We now consider the special case of when the target is statistically independent of clutter and its interactions with the clutter are not considered, i.e. when $\mathrm{Cov}\{\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1\} = \mathrm{Cov}\{\boldsymbol{\alpha}_2,\boldsymbol{\beta}_1\} = \mathbf{0}$ and hence $MI(i,j) = DI(i,j)$. For brevity, we can now absorb the $\boldsymbol{\beta}_m$'s into the noise as they are uncorrelated with the $\boldsymbol{\alpha}_m$'s. Then, the waveform scheduling criteria becomes,

$$
\begin{aligned}
(\mathbf{s}_i^*,\mathbf{s}_j^*) &= \arg\max_{i,j} DI(i,j) && (10)\\
&= \arg\min_{i,j} \ln\det\{\mathrm{BMSE}(\boldsymbol{\alpha}|\mathbf{y})\} && (11)\\
&= \arg\min_{i,j} \ln\det\{(\mathbf{C}_{\boldsymbol{\alpha}}^{-1} + \mathbf{H}^{(ij)H}\mathbf{C}_{\mathbf{v}}^{-1}\mathbf{H}^{(ij)})^{-1}\}
\end{aligned}
$$

where, BMSE is the minimum Bayesian mean square error [14], and

$$
\begin{aligned}
&\mathbf{H}^{(ij)} := \begin{bmatrix} \mathbf{S}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_j \end{bmatrix}\\
&\mathbf{C}_{\boldsymbol{\alpha}} = \mathrm{Cov}\{\boldsymbol{\alpha},\boldsymbol{\alpha}\},\ \mathbf{C}_{\mathbf{v}} = \mathrm{Cov}\{\mathbf{v},\mathbf{v}\}\\
&\mathbf{v} = [\mathbf{v}_1^H,\mathbf{v}_2^H]^H
\end{aligned}
$$

We may alternatively view (analog) waveform scheduling (or design) in the spectral domain, which is derived next, while still enforcing independence of target and clutter.

4.2. Independent target and clutter responses: analog

Assume that the radar operates with a bandwidth denoted by W. Let us divide the bandwidth into P consecutive bands each of infinitesimal width denoted by δf. Denote the center frequency of the p-th band as $f_p, p = 1,\ldots,P$. Then, consider the following transformation on (1) and (2),

$$
\begin{aligned}
y_1^{(i)}(t) &= \sum_{p=1}^{P} y_{1p}^{(i)}(t),\ y_{1p}^{(i)}(t) := \alpha_{1p}(t) * s_{ip}(t) + v_{1p}(t)\\
y_2^{(j)}(t) &= \sum_{p=1}^{P} y_{2p}^{(j)}(t),\ y_{2p}^{(j)}(t) := \alpha_{2p}(t) * s_{jp}(t) + v_{2p}(t)
\end{aligned}
$$

where $v_{mp}(t)$ and $\alpha_{mp}(t)$ $(m = 1,2)$ have spectral content in the p-th band only and zero elsewhere. Using identical notation, $s_{ip}(t)$ and $s_{jp}(t)$ are constrained to be in the p-th band and have spectral content defined by $S_{ip}(f) = S_i(f_p)r_p(f)$, and $S_{jp}(f) = S_j(f_p)r_p(f)$, respectively. Here, $S_m(f)$ is the fourier transform of $s_m(t)$, the indicator function is denoted as $\mathbb{1}[\cdot]$, and $r_p(f) := \mathbb{1}[f_p - \delta f/2 \le f \le f_p + \delta f/2]$. Let us define the power spectral density (PSD) of $v_{mp}(t), m = 1,2$ as $V_p^m(f) = V_m(f_p)r_p(f)$ and the energy spectral variance (ESV) of $\alpha_{mp}(t)$ as $\Gamma_p^m(f) = \Gamma_m(f_p)r_p(f)$, where, $V_m(f)$, and $\Gamma_m(f)$ denote the PSD and ESV of $v_m(t)$ and $\alpha_m(t)$, respectively. Similarly, we can define the cross PSD and cross ESV to be $V_{12}^p(f) = V_{12}(f_p)r_p(f)$ and $\Gamma_{12}^p(f) = \Gamma_{12}(f_p)r_p(f)$ of the noise and target impulse responses at the two scheduling instants, respectively, where $V_{12}(f)$ and $\Gamma_{12}(f)$ are the cross PSD and cross ESV of the original random processes. We can now readily show that,

$$I(\alpha_{1p}(t), \alpha_{2p}(t); y_{1p}^{(i)}(t), y_{2p}^{(j)}(t)) = \tilde{T}\delta f \ln\left[1 + \frac{\chi_1(f_p, i, j)}{\chi_2(f_p)\tilde{T}^2}\right]$$

$$\begin{aligned}
&\chi_1(f_p, i, j) = |S_i(f_p)|^2 |S_j(f_p)|^2 \Gamma_1(f_p)\Gamma_2(f_p)\\
&+ \tilde{T}|S_i(f_p)|^2 \Gamma_1(f_p) V_{22}(f_p) + \tilde{T}|S_j(f_p)|^2 \Gamma_2(f_p) V_{11}(f_p)\\
&- 2\tilde{T}\mathrm{Re}\{S_i(f_p) S_j^*(f_p) \Gamma_{12}(f_p) V_{12}(f_p)\} \qquad (12)\\
&- |S_i(f_p)|^2 |S_j(f_p)|^2 |\Gamma_{12}(f_p)|^2\\
&\chi_2(f_p) = V_{11}(f_p) V_{22}(f_p) - |V_{12}(f_p)|^2
\end{aligned}$$

where $\tilde{T}$ is the total time duration of $y_1^{(i)}(t)$ and $y_2^{(j)}(t)$. Considering any two non-overlapping bands, and due to independence, the total MI is the sum of their respective MIs. Hence in the limiting case we have,

$$\begin{aligned}
&I(\alpha_1(t), \alpha_2(t); y_1^{(i)}(t), y_2^{(j)}(t)/s_i(t), s_j(t))\\
&= \sum_p \lim_{\delta f \to 0} I(\alpha_{1p}(t), \alpha_{2p}(t); y_{1p}^{(i)}(t), y_{2p}^{(j)}(t))\\
&= \tilde{T} \int\limits_W \ln\left[1 + \frac{\chi_1(f, i, j)}{\chi_2(f)\tilde{T}^2}\right] \mathrm{d}f \qquad (13)
\end{aligned}$$

The waveform scheduling criteria now becomes,

$$\arg\max_{i,j} DI(i,j) = \arg\max_{i,j} \int\limits_W \ln\left[1 + \frac{\chi_1(f, i, j)}{\chi_2(f)\tilde{T}^2}\right] \mathrm{d}f$$

The optimizations of [6] and [7] (for waveform design over a single epoch, and hence optimized over the waveforms themselves rather than over the selection of waveforms from a given library) may be obtained as special cases of the framework presented here. In particular, for one scheduling epoch only, the first term in the DI is the one-step MI, optimized in [6] and [7], where we note that (1) and (2) allow for signal - clutter interactions.

4.3. An example when waveform diversity is useless.

We consider a special case where transmitting diverse waveforms on the scheduling epochs is unnecessary. Assume that $V_1(f) = V_2(f) = \sigma^2, f \in \mathcal{W}$ and $V_{12}(f) = 0$ (white, Gaussian noise, independent and identical over the two slots). In the same spirit assume $\Gamma_1(f) = \Gamma_2(f) = \sigma_a^2, f \in \mathcal{W}$ and $\Gamma_{12}(f) = 0$. These assumptions imply that the ESV's are flat in the bandwidth, and the cross ESV is zero. Now substituting these assumptions in (13), and using (7), we have

$$\begin{aligned}
&\arg\max_{i,j} DI(i,j) =\\
&= \arg\max_{i,j} \int_{\mathcal{W}} \ln\left[\left(1 + \frac{|S_i(f)|^2\sigma_a^2}{\tilde{T}\sigma^2}\right)\left(1 + \frac{|S_j(f)|^2\sigma_a^2}{\tilde{T}\sigma^2}\right)\right] \mathrm{d}f\\
&= \arg\max_{i} \int_{\mathcal{W}} \ln\left(1 + \frac{|S_i(f)|^2\sigma_a^2}{\tilde{T}\sigma^2}\right) \mathrm{d}f, \quad i = 1, 2 \quad (14)
\end{aligned}$$

From (14), we see that the maximization over the two epochs decouples to a single maximization for one epoch. In other words, pick one waveform which maximizes (14) and schedule it for both transmission epochs. In practice, radar scenes with the aforementioned assumptions are more an exception than the rule.

5. CONCLUSIONS

A cognitive radar framework was proposed to adaptively schedule waveforms by extracting information from the past radar returns. The model assumed was general encompassing clutter and interference which are correlated with the target. Maximizing the directed information, which incorporates feedback, rather than mutual information, was proposed. The optimization problem was considered in several special cases. For simplicity, the analysis assumed a waveform library comprising two distinct waveforms and two scheduling instants. Nevertheless, the conclusions and analysis apply to a larger waveform library and multiple epochs.

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