The Sum-Capacity of the Linear Deterministic Three-User Cognitive Interference Channel

Diana Maamari, Daniela Tuninetti and Natasha Devroye
Department of Electrical and Computer Engineering
University of Illinois at Chicago, Chicago IL 60607, USA
Email: dmaama2, danielat, devroye @ uic.edu

***Abstract*—Inspired by cognitive networks, we consider the linear deterministic three-user cognitive interference channel with one primary and two secondary/cognitive transmitters which approximates the Gaussian channel at high SNR. Outer bounds on the sum-rate are derived and matching transmission schemes are provided in all interference regimes, thereby completely characterizing the sum-capacity. Significant increase in the sum-capacity is demonstrated when comparing the three-user cognitive channel to the classical (non-cognitive) three-user interference channel and to the two-user cognitive channel. The paper discusses extensions to an arbitrary number of users, the relationship between the cognitive channel and the (fully cooperative) broadcast channel, and observations on the behavior of the cognitive transmitters in the different interference scenarios.**

I. Introduction

The increase in number of wireless services over the past decade along with the shortage of frequency spectrum has spurred the wireless communication community to explore the possibility of several users/devices coexisting in the same frequency band. With the goal of enhancing spectral efficiency and allowing sophisticated secondary users to exploit the same frequency band without causing significant degradation of performance of licensed/primary users, the *cognitive radio technology* has emerged. In cognitive networks, smart devices are capable of sensing and adapting to their environment and therefore better utilize the available spectrum. Cognitive radios are able to interweave with primary signals (i.e., search for available unused spectrum), underlay (i.e., in which they can operate simultaneously with primary users as long as the interference caused is within an acceptable level) or overlay (i.e., in which cognitive radios exploit message knowledge through encoding schemes to mitigate interference) [1].

The cognitive radio channel, first introduced in [2], consists of two source-destination pairs in which one of the transmitters called the *secondary transmitter* has non-causal a priori knowledge of the message of the other transmitter known as the *primary transmitter*. For the state-of-the-art on the two-user cognitive channel we refer the reader to [3], [4]. In particular, the capacity of the deterministic two-user cognitive channel is known [3] and that the capacity of the Gaussian noise channel is known exactly for most channel parameters, and to within one bit otherwise [4].

In this paper we are interested in the extension of the two-user cognitive interference channel to the case of three users. The three-user cognitive interference channel analyzed in this work consists of one primary and two secondary opportunistic cognitive radios. We assume a *cumulative* message cognition structure introduced in [5] whereby user 2 (the first cognitive user) knows the message of user 1 (the primary user), and user 3 (the second cognitive user) knows the messages of both user 1 and user 2. We term this channel the three-user cognitive interference channel with cumulative message sharing (3-user CIFC with CMS).

The cumulative message cognition model is inspired by the concept of overlaying, or layering, cognitive networks. In particular, we consider multiple types of devices sharing the spectrum. The first "layer" consists of the primary users. Each additional cognitive layer transmits simultaneously with the previous layers (overlay), and given the lower layers' codebooks (and are oblivious to higher layer operation). This may enable them to learn the lower layers' messages and use this to aid the lower layers' transmission, or to combat interference at their own receivers. In this setting, the three-user cumulative message sharing model where messages are known non-causally, forms an outer bound for the more realistic causal obtaining of messages possible in a network with layered codebook knowledge.

For this model, we are interested in the impact of this cumulative message knowledge on the sum-rate, and how it improves the two-user cognitive model. We obtain the sum-rate capacity of the linear deterministic channel that approximates the Gaussian noise channel at high SNR [6], and it is seen to significantly increase that of the two-user model, suggesting that, as the number of users increases, this sum-capacity tends to that of the multiple-input multiple-out broadcast channel.

Past Work. The literature on the fundamental performance of multi-user cognitive interference channels is limited, in parts due to the fact that the two-user counterpart is not yet fully understood [3], [4]. In [5], [7]–[10] different three-user cognitive channels are considered; for sake of space we note that the models differ from the one considered here either in the number of transmitter/receivers, or in the message sharing / cognition structure in all but [5], [7]. In [5], [7] two types of three-user cognitive interference channels with 3 transmitters, 3 receivers, and 3 messages are proposed: that with "cumulative message sharing" (as considered here) and that with "primary message sharing" where the message of

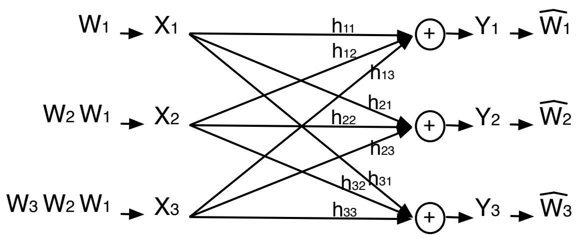

Fig. 1. Three-user Cognitive Interference Channel with CMS.

the one primary user is known at both cognitive transmitters (who do not know each others' messages). An achievable rate region for the discrete channel is provided and evaluated in Gaussian noise.

Contributions. The main contributions of this work for the 3-user CIFC with CMS are:

- we derive a novel and general three-user outer bound region that reduces to the outer bound of [3] for the two-user case,
- we evaluate the sum-rate outer bound for the Linear Deterministic approximation of the Gaussian noise Channel (LDC) at high SNR and provide matching achievable schemes that include elements of bit-cancellation and bit-self cleaning [11],
- we discuss an extension to an arbitrary number of users and relations with the generalized degrees of freedom of the Gaussian noise channel at finite SNR.

Paper Organization. The paper is organized as follows. Section II describes the channel model. Section III presents our novel outer bound region. Section IV considers the LDC and provides sum-rate optimal achievable schemes. In Section V comparisons between the sum-capacity of different cognitive models are presented. Section VI concludes the paper.

II. CHANNELMODEL

The 3-user CIFC with CMS channel (a general example is shown in Fig. 1) consists of

- Channel inputs X_1, X_2, X_3 in alphabets $\mathcal{X}_1$, $\mathcal{X}_2$, and $\mathcal{X}_3$,
- Channel outputs Y_1, Y_2, Y_3 in alphabets $\mathcal{Y}_1$, $\mathcal{Y}_2$, and $\mathcal{Y}_3$,
- A memoryless channel described by $P_{Y_1,Y_2,Y_3|X_1,X_2,X_3}$,
- Encoder i, $i \in [1:3]$, has message W_i, uniformly distributed over $[1 : 2^{NR_i}]$ and independent of everything else, to be decoded at receiver i,
- Encoder 2 knows message W_1 non-causally,
- Encoder 3 knows messages (W_1, W_2) non-causally,
- Encoding functions $f_i : X_i^N := f_i(W_1, \ldots, W_i)$, $i \in [1:3]$,
- Decoding functions $g_i : \widehat{W}_i := g_i(Y_1^N)$, $i \in [1:3]$.

The probability of error is defined as $P_e^{(N)} := \max_{i\in[1:3]} \mathbb{P}[\widehat{W}_i \neq W_i]$. A rate triple (R_1, R_2, R_2) is achievable if there exists a code such the probability of error $P_e^{(N)} \to 0$ as $N \to \infty$. The capacity region is the closure of the set of achievable rates.

III. OUTERBOUNDS

In this section we derive an outer-bound for the general 3-user CIFC with CMS.

Theorem 1. *The capacity of a 3-user CIFC with CMS is contained in the region defined by*

$$R_1 \le I(Y_1; X_1, X_2, X_3), \quad \text{(1a)}$$
$$R_2 \le I(Y_2; X_2, X_3|X_1), \quad \text{(1b)}$$
$$R_3 \le I(Y_3; X_3|X_1, X_2), \quad \text{(1c)}$$
$$R_2 + R_3 \le I(Y_2; X_2, X_3|X_1) + I(Y_3; X_3|X_1, X_2, Y_2), \quad \text{(1d)}$$
$$R_1 + R_2 + R_3 \le I(Y_1; X_1, X_2, X_3) + I(Y_2; X_2, X_3|X_1, Y_1) + I(Y_3; X_3|X_1, X_2, Y_1, Y_2), \quad \text{(1e)}$$

for some input distribution P_{X_1,X_2,X_3}. The joint conditional distribution $P_{Y_1,Y_2,Y_3|X_1,X_2,X_3}$ can be chosen so as to tighten the different bounds as long as the conditional marginal distributions $P_{Y_i|X_1,X_2,X_3}$ are preserved for $i \in [1:3]$.

Note that by setting $X_3 = Y_3 = \emptyset$ in (1), the region in Th. 1 reduces to the outer bound of [3]. As we shall explain later, Th. 1 can be extended to a general memoryless K-user CIFC with CMS.

Proof: The proof is provided in the Appendix. ■

IV. THE SUM-CAPACITY OF THE LINEAR DETERMINISTIC 3-USER CIFC WITH CMS

A. Channel Model

The Linear Deterministic approximation of the Gaussian noise Channel (LDC) at high SNR was first introduced in [12], and allows one to focus on the signal interactions rather than on the additive noise. The proposed framework has proven to be powerful in understanding communication over interference networks, and the insights gained for the LDC have often been translated into capacity results to within constant gaps for any finite SNR [4], [13], [14]. In light of these success stories we also start our investigation from the LDC.

In the LDC model, the input-output relationship is given by, for $u \in [1:3]$

$$Y_u = \sum_{i\in[1:3]} \mathbf{S}^{m-n_{ui}} X_i, \quad m := \max\{n_{ij}\} \quad (2)$$

where $\mathbf{S}$ is the binary shift matrix of dimension m, all inputs and outputs are binary column vectors of dimension m, and the summation is bit-wise over the binary field.

B. Sum-rate Outer Bound

For sake of space we only evaluate the sum-rate capacity, which we will show to be achievable. Since the channel is deterministic, the sum-rate upper bound in (1e) reduces to the following maximization over the set of joint input distributions P_{X_1,X_2,X_3}:

$$R_\Sigma := R_1 + R_2 + R_3 \le \max\Big\{H(Y_1) + H(Y_2|X_1, Y_1) + H(Y_3|X_1, Y_1, X_2, Y_2)\Big\}.$$

For the LDC

$$R_\Sigma \le \max_{P_{X_1,X_2,X_3}} \Big\{ H(\mathbf{S}^{m-n_{11}}X_1 + \mathbf{S}^{m-n_{12}}X_2 + \mathbf{S}^{m-n_{13}}X_3)$$
$$+ H(\mathbf{S}^{m-n_{22}}X_2 + \mathbf{S}^{m-n_{23}}X_3 | \mathbf{S}^{m-n_{12}}X_2 + \mathbf{S}^{m-n_{13}}X_3)$$
$$+ H(\mathbf{S}^{m-n_{33}}X_3 | \mathbf{S}^{m-n_{13}}X_3, \mathbf{S}^{m-n_{23}}X_3) \Big\}$$
$$= \max\{n_{11}, n_{12}, n_{13}\} + f(n_{22}, n_{23} | n_{12}, n_{13}) +$$
$$[n_{33} - \max\{n_{13}, n_{23}\}]^+ \quad (3)$$

where the function f is defined as

$$f(c, d|a, b) := H(\mathbf{S}^{m-c}X_2 + \mathbf{S}^{m-d}X_3 | \mathbf{S}^{m-a}X_2 + \mathbf{S}^{m-b}X_3),$$

computed for iid Bernoulli$(1/2)$ inputs as in [15, eq.(5)], as:

$$f(c, d|a, b) := \begin{cases} \max\{c+b, a+d\} - \max\{a, b\} & \text{if } c - d \neq a - b, \\ \max\{a, b, c, d\} - \max\{a, b\} & \text{if } c - d = a - b. \end{cases}$$

Depending on the channel gains, different interference scenarios are identified for R_Σ. In the next section we provide transmission schemes that are capable of achieving the sum-rate upper bound we just evaluated.

C. Achievable Scheme

Our main result is the achievability of the proposed sum-rate upper bound.

Theorem 2. *The upper bound for the LDC 3-user CIFC with CMS given by* (3) *is achievable.*

Proof: The proof distinguishes two cases. **Case1**: If

$$n_{33} \le \max\{n_{13}, n_{23}\} \quad (4)$$

the sum-rate becomes

$$R_\Sigma \le \max\{n_{11}, n_{12}, n_{13}) + f(n_{22}, n_{23} | n_{12}, n_{13}). \quad (5)$$

The condition in (4) corresponds to the case $H(Y_3|X_1, Y_1, X_2, Y_2) = 0$, i.e., conditioned on (X_1, X_2) the signal received at the "most cognitive" receiver is a degraded version of the signal received at the other two receivers. For this condition, the useful signal at receiver 3 is weak with respect to the interfering signals. One might thus suspect that $R_3 = 0$ is optimal; we show that this is the case.

Since the third user has a weak channel, we set its rate to zero and we therefore convert the three-user channel into a deterministic two-user cognitive channel where user 1 is the primary user and the cognitive user has a vector input given by $[X_2, X_3]$ to its output Y_2. The capacity of a general deterministic two-user cognitive channel is [3]

$$R_1 \le H(Y_1),$$
$$R_2 \le H(Y_2|X_1),$$
$$R_1 + R_2 \le H(Y_1) + H(Y_2|X_1, Y_1)$$

for some input distribution $P_{X_1,[X_2,X_3]}$. The sum-rate achievable with this scheme is

$$R_1 + R_2 = \max_{P_{X_1,[X_2,X_3]}} \Big\{ H(Y_1) + H(Y_2|X_1, Y_1) \Big\}$$
$$= \max\{n_{11}, n_{12}, n_{13}) + f(n_{22}, n_{23} | n_{12}, n_{13}).$$

Case2: If

$$n_{33} > \max\{n_{13}, n_{23}\} \quad (6)$$

the sum-rate is upper bounded by

$$R_\Sigma \le \max\{n_{11}, n_{12}, n_{13}) + f(n_{22}, n_{23} | n_{12}, n_{13})$$
$$+ n_{33} - \max\{n_{13}, n_{23}\}. \quad (7)$$

Is this case, the condition in (6) suggests that the intended signal at receiver 3 is sufficiently strong to be able to support a non-zero R_3. The form of the sum-rate also suggests that a plausible strategy is to use the optimal strategy for Case1 and "sneak in" extra bits for user 3 in such a way that they appear below the noise level at the other receivers. We next show that this is optimal.

Split the signal of user 3 in two parts

$$X_3 := X_{3a} + X_{3b}$$

where X_{3a} is intended to mimic the scheme for Case1 (i.e., as if user 2 had input $[X_2, X_{3a}]$) and X_{3b} is

$$X_{3b} := S^{\max\{n_{13}, n_{23}\}} V_3$$

for some iid Bernoulli$(1/2)$ input V_3. Note that the shift caused by $S^{\max\{n_{13}, n_{23}\}}$ keeps V_3 below noise level at Y_1 and Y_2. We implement the optimal strategy for Case1 with $[X_1, X_2, X_{3a}]$, the remaining bits then achieve the sum-rate in (7). ■

D. Examples

In order to present some concrete examples of the achievability scheme, we consider here the symmetric LDC defined as $n_{ii} = n_S > 0$ for $i \in [1:3]$ and $n_{ij} = n_S\alpha$ for some $\alpha \ge 0$ for $j \neq i$. We also define,

$$d_i := R_i / n_S, \; i \in [1:K],$$
$$d_\Sigma(\alpha; K) := \sum_{i \in [1:K]} d_i.$$

With these definitions, the sum-capacity can be expressed as

$$d_\Sigma(\alpha; 3) = \max\{2 + [1 - \alpha]^+, 2\alpha\}. \quad (8)$$

Figs. 2 and 3 show examples of the achievable strategy for Case1, corresponding to $\alpha > 1$, and Case2, corresponding to $\alpha < 1$, respectively. We show the different linear combinations of the shifted transmit signals X_1, X_2, X_3 received at the three receivers. In both figures, notice the important role of cognition: in Fig. 3 the third transmitter (cognitive of all 3 messages) sends either a combination of the messages of users 1 and 2, thereby simultaneously "cleaning" the interference at the respective receivers, while at the same time pre-canceling the interference seen at his own receiver to obtain the green bits. In Fig. 2 the two cognitive transmitters aid in transmitting the first two messages (a form of broadcast strategy).

Additional interesting observations regarding the role of cognition for the 3-user CIFC with CMS may be made. The second cognitive transmitter, cognizant of the its message and the message of primary transmitter, is only required to use its cognitive abilities in the strong interference scenarios. The

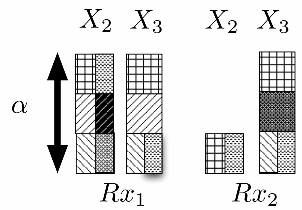

Fig. 2. 3-user CIFC with CMS in strong interference ($\alpha = 3$). Receiver 3 is not shown as $R_3 = 0$. The achievable rates are $d_1 = d_2 = 3 = \alpha, d_3 = 0$ thereby achieving (7).

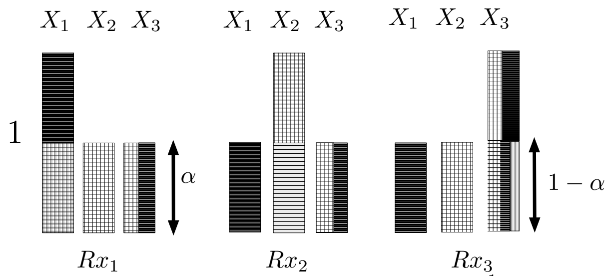

Fig. 3. 3-user CIFC with CMS in weak interference ($\alpha = \frac{1}{2}$). The achievable rates are $d_1 = 1, d_2 = 1, d_3 = 1 - \alpha$ thereby achieving (5)

third cognitive transmitter, cognizant of all messages, applies bit cancellation and bit self cleaning in weak interference regimes to cancel interference at their receivers, allowing the other two transmitters to attain their maximal rates.

V. Comparison of sum-capacity between CIFC and other known channels

We compare the sum-rate of different channel models with different number of users and different levels of cognition. Our base line for comparison is the K-user interference channel without any cognition; for this model we have [16]

$$d_\Sigma^{(\text{no cognition})}(\alpha; K) = \frac{K}{2} d_\Sigma^{(\text{no cognition})}(\alpha; 2) \qquad (9)$$

and where $d_\Sigma^{(\text{no cognition})}(\alpha; 2)$ is the so-called W-curve of Etkin, Tse and Wang [13] except for a discontinuity at $\alpha = 1$ where $d_\Sigma^{(\text{no cognition})}(\alpha; K) = 1$ for all K. Note that, except at $\alpha = 1$, the sum-rate $d_\Sigma^{(\text{no cognition})}(\alpha; K)$ normalized by the number of users does not depend on K.

At the other end of the spectrum we have the case where all users are cognitive of all messages; in this case the channel is equivalent to a MIMO BC with K transmit antennas and K single-antenna receivers; since the system has enough degrees of freedom to zero-force the interference we have

$$d_\Sigma^{(\text{BC})}(\alpha; K) = K \max\{1, \alpha\}. \qquad (10)$$

Also in this case the sum-rate normalized by the number of users does not depend on K.

For the case of the 2x2 CIFC with CMS, we have [4]

$$d_\Sigma(\alpha; 2) = \max\{2 - \alpha, \alpha\} = 2\max\{1, \alpha\} - \alpha. \qquad (11)$$

We also notice that $d_\Sigma(\alpha; 3)$ characterized in this work can be rewritten as

$$d_\Sigma(\alpha; 3) = 3\max\{1, \alpha\} - \alpha. \qquad (12)$$

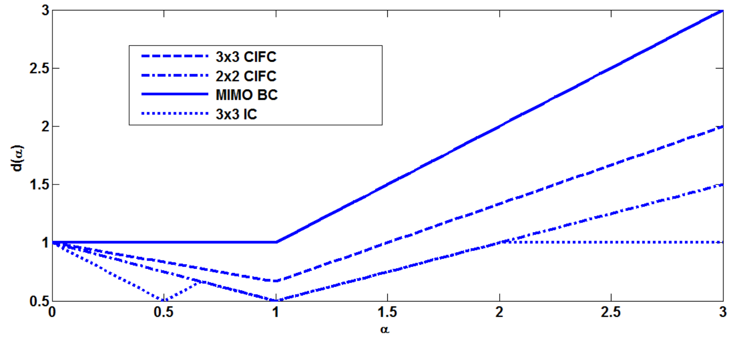

Fig. 4. $d_\Sigma(\alpha; K)/K$ for different channel models.

Therefore, for K-user CIFC with CMS the sum-rate normalized by the number of users is a function of K. The sum-capacity result for the symmetric LDC can in fact be evaluated for the case of K users and is given by (see Appendix)

$$d_\Sigma(\alpha; K) = K \max\{1, \alpha\} - \alpha. \qquad (13)$$

This has the interesting interpretation that the cumulative cognition in the symmetric case looses α/K with respect $d_\Sigma^{(\text{BC})}(\alpha; K)/K$. In other words, as the number of cognitive users increases the network approaches the sum-rate performance of a fully coordinated BC.

Fig. 4 shows the sum-rate normalized by the number of users for different channel models. We note the increase in performance in all interference regimes when compared to that of 2-user CIFC with CMS, the classical K-user interference channel but a loss with respect to the K-user BC.

VI. Conclusion

In this paper we studied the 3-user cognitive interference channel with cumulative message sharing. We derived an outer-bound for the capacity region of a general memoryless channel. We then showed that the sum-rate evaluated for the linear deterministic channel model is achievable for three users, thereby obtaining the sum-capacity. Comparisons between the 3-user channel and other channel models with different cognition models highlight the benefits of cognition, and under what conditions they are most prominent. Our upper bound, evaluated for K users rather than 3, suggests that as the number of users increases the the sum-rate approaches the sum-rate performance of a fully coordinated broadcast channel. Whether this is achievable is the subject of ongoing work, as is the translation of the insights gained from the deterministic channel to the Gaussian noise channel.

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APPENDIX

Proof of Th. 1. The bounds in (1a) to (1c) are a simple application of the cut-set bound. The bound in (1d) is obtained as follows:

$$\begin{aligned}
&N(R_2+R_3-2\epsilon_N) \overset{\text{(a)}}{\leq} I(Y_2^N;W_2)+I(Y_3^N;W_3)\\
&\overset{\text{(b)}}{\leq} I(Y_2^N,W_1;W_2)+I(Y_3^N,Y_2^N,W_1,W_2;W_3)\\
&\overset{\text{(c)}}{=} I(Y_2^N;W_2|W_1)+I(Y_3^N,Y_2^N;W_3|W_1,W_2)\\
&\overset{\text{(d)}}{=} I(Y_2^N;W_2|W_1)+I(Y_2^N;W_3|W_1,W_2)\\
&\quad +I(Y_3^N;W_3|W_1,W_2,Y_2^N)\\
&\overset{\text{(e)}}{=} I(Y_2^N;W_2,W_3|W_1)+I(Y_3^N;W_3|W_1,W_2,Y_2^N)\\
&\overset{\text{(f)}}{=} I(Y_2^N;W_2,W_3|W_1,X_1^N)\\
&\quad +I(Y_3^N;W_3|W_1,W_2,Y_2^N,X_1^N,X_2^N)\\
&\overset{\text{(g)}}{\leq} \sum_{t=1}^{N} H(Y_{2,t}|X_{1,t})-H(Y_{2,t}|X_{1,t},X_{2,t},X_{3,t})\\
&\quad +H(Y_{3,t}|X_{1,t},X_{2,t})-H(Y_{3,t}|X_{1,t},X_{2,t},X_{3,t})\\
&\overset{\text{(h)}}{=} \sum_{t=1}^{N} I(Y_{2,t};X_{2,t},X_{3,t}|X_{1,t})+I(Y_{3,t};X_{3,t}|X_{1,t},X_{2,t})
\end{aligned}$$

where (a) follows from Fano's inequality, (b) the non-negativity of mutual information, (c) from the independence of the messages, (d) and (e) from chain rule (note how we gave side information so that we could recombine different entropy terms), (f) because the inputs are deterministic functions of the messages, (g) follows since conditioning does not reduce entropy, and (h) definition of mutual information. With similar steps (give enough messages to reconstruct the inputs, and also give outputs so that we can recombine terms by using the chain rule of mutual information) we obtain the bound in (1e). The main steps are:

$$\begin{aligned}
&N(R_1+R_2+R_3-3\epsilon_N)\\
&\leq I(Y_1^N;W_1)+I(Y_2^N;W_2)+I(Y_3^N;W_3)\\
&\leq I(Y_1^N;W_1)+I(Y_2^N,Y_1^N,W_1;W_2)\\
&+I(Y_3^N,Y_1^N,W_1,Y_2^N,W_2;W_3)\\
&\leq I(Y_1^N;W_1,W_2,W_3)\\
&+I(Y_2^N;W_2,W_3|Y_1^N,W_1)\\
&+I(Y_3^N;W_3|Y_1^N,W_1,Y_2^N,W_2)\\
&\leq \sum_{t=1}^{N} I(Y_{1,t};X_{1,t},X_{2,t},X_{3,t})\\
&+I(Y_{2,t};X_{2,t},X_{3,t}|X_{1,t},Y_{1,t})\\
&+I(Y_{3,t};X_{3,t}|X_{1,t},X_{2,t},Y_{1,t},Y_{2,t})
\end{aligned}$$

Proof of eq.(13)**.** For the K-user symmetric LDC the sum-rate is upper bounded by

$$\begin{aligned}
&\sum_{k=1}^{K} R_k \leq \sum_{k=1}^{K} H\Big(Y_k|X_1,\ldots,X_{k-1},Y_1,\ldots,Y_{k-1}\Big)\\
&=\sum_{k=1}^{K-1} H\Big(\mathbf{S}^{m-n_D}X_k+\mathbf{S}^{m-n_I}(\sum_{i=k+1}^{K}X_i)+\mathbf{S}^{m-n_I}(\sum_{i=1}^{K-1}X_i)\\
&\quad |X_1,\ldots,X_{k-1},\mathbf{S}^{m-n_I}(\sum_{i=k}^{K}X_i)\Big)\\
&+H\Big(\mathbf{S}^{m-n_D}X_K|\mathbf{S}^{m-n_I}X_K\Big)\\
&\leq \sum_{k=1}^{K-1} H\Big((\mathbf{S}^{m-n_D}+\mathbf{S}^{m-n_I})X_k\Big)+H\Big(\mathbf{S}^{m-n_D}X_K|\mathbf{S}^{m-n_I}X_K\Big)\\
&\leq n_S(K-1)\max\{1,\alpha\}+n_S[1-\alpha]^+\\
&\leq n_S\Big(K\max\{1,\alpha\}-\alpha\Big).
\end{aligned}$$

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