Lattices and list decoding for relay networks

Natasha Devroye, Assistant Professor, UIC

joint work with Yiwei Song, Ph.D. candidate, UIC
Motivation: Gaussian networks
Motivation: Gaussian networks

Broadcast
Motivation: Gaussian networks
Motivation: Gaussian networks

Interference
Motivation: Gaussian networks
Motivation: Gaussian networks

\[ Y = X_1 + X_2 + Z \]
Motivation: Gaussian networks

\[ Y = X_1 + X_2 + Z \]

Additive Gaussian
Motivation: Gaussian networks

Relaying
Motivation: Gaussian networks

Two-way Relaying
Two-way communication applications - wired

Video conferencing
Two-way communication applications - wired

Video conferencing

Telesurgery
Two-way communication applications - wired

Video conferencing

Telesurgery

Necessary Elements of a Telerobotic Surgery Team

In its current state, the pieces that make up telesurgery are complex, expensive, and relatively fragmented. To success...
Two-way communication applications - wired

Video conferencing

Telesurgery

Data synchronization
Two-way communication applications - wired

Video conferencing

Telesurgery

Data synchronization
Two-way communication applications - wireless
Two-way communication applications - wireless
Two-way communication applications - wireless

Video conferencing
Two-way communication applications - wireless

Battlefield telesurgery

Rural telesurgery

Video conferencing
Most basic channel model

Power constraint $P_1$

1

$X_1$

$Y_1$

2

$Y_2$

$X_2$

Power constraint $P_2$

$Y_1 = aX_1 + bX_2 + N_1 \sim \mathcal{N}(0, \sigma_1^2)$

$Y_2 = cX_1 + dX_2 + N_2 \sim \mathcal{N}(0, \sigma_2^2)$

“Gaussian two-way channel”
Most basic channel model

Power constraint $P_1$

$$Y_1 = aX_1 + bX_2 + N_1 \sim \mathcal{N}(0, \sigma_1^2)$$

$$Y_2 = cX_1 + dX_2 + N_2 \sim \mathcal{N}(0, \sigma_2^2)$$

Power constraint $P_2$

Capacity region:

$$R_1 \leq (1/2) \log(1 + c^2 P_1 / \sigma_2^2)$$

$$R_2 \leq (1/2) \log(1 + b^2 P_2 / \sigma_1^2)$$

``Gaussian two-way channel``
Most basic channel model

``Two-way relay channel''

[Wu, Chou, Kung 2004]
Most basic channel model

[Wy, Chou, Kung 2004]

``Two-way relay channel``
Key idea: structured coding

[Nazer, Gastpar, ArXiv 2009]

Multiple access channel?
Key idea: structured coding

\[ X_1(W_1) + X_2(W_2) + Z \]

Multiple access channel?

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Key idea: structured coding

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Multiple access channel? 

\textit{depends on what the relay needs}
Exploit own-message side-information
Exploit own-message side-information

\[ W_1 \oplus W_2 \]

Diagram:

- Node 1 connected to node R by \( W_1 \)
- Node 2 connected to node R by \( W_2 \)
- Node R connected to nodes 1 and 2 by \( W_1 \oplus W_2 \)
Exploit own-message side-information

\[ \hat{W}_2 = W_1 \oplus W_2 \oplus W_1 \]
Exploit own-message side-information

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Don’t necessarily need individual messages!
If we use random codes....

\[
\begin{align*}
X_1(W_1) &\rightarrow R & X_2(W_2) \\
1 &\rightarrow R & 2
\end{align*}
\]
If we use random codes....
If we use random codes....
If we use random codes....

\[ X_1(W_1) + X_2(W_2) + Z \]
If we use random codes....

\[ X_1(W_1) + X_2(W_2) + Z \]
If we use random codes....

\[ X_1(W_1) + X_2(W_2) + Z \]

Decode both messages

images taken from [Tse, slides]
If we use structured codes....
If we use structured codes....
If we use structured codes....
If we use structured codes....
If we use structured codes...

\[ X_1(W_1) + X_2(W_2) + Z \]

SUM of signals

If we use structured codes....
If we use structured codes....

**Decode the SUM**

\[ X_1(W_1) + X_2(W_2) + Z \]

**SUM of signals**

**R**

1

2

\[ X_1(W_1) \]

\[ X_2(W_2) \]
\[ f(W_1, W_2) \]


VS

\[ f(X_1(W_1) + X_2(W_2)) \]

Gain by decoding the sum?
Gain by decoding the sum?

Don’t decode what you don’t need
Random codes

\[ R_1 \leq \frac{1}{2} \log \left( 1 + \frac{P_1}{N_R} \right) \]

\[ R_2 \leq \frac{1}{2} \log \left( 1 + \frac{P_2}{N_R} \right) \]

\[ R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{P_1 + P_2}{N_R} \right) \]
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Structured codes

\[ R_1 \leq \frac{1}{2} \log \left( \frac{P_1}{P_1 + P_2} + \frac{P_1}{N_R} \right) \]

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No sum-rate!
Does this generalize?
Lattices codes good for multi-user AWGN channels

- achieve capacity of AWGN channel [Erez, Zamir, Trans. IT, 2004]
Lattices codes good for multi-user AWGN channels

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- AWGN inference channel: interference decoding / interference alignment in K>2 interference channels [Bresler, Parekh, Tse, ArXiv 2008] [Sridharan, Jafarian, Jafar, Shamai, arXiv 2008]
Lattices codes for two-way AWGN relay channels

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- AWGN multi-way relay channels
  - [Gunduz, Yener, Goldsmith, Poor, arXiv 2010]
  - [Sezgin, Avestimehr, Khajehnejad, Hassibi, arXiv 2010]
  - achieve to within 2 bit/sez/Hz/user gap of cut-set outer bound in absence of direct links
Lattice codes missing in?

- AWGN relay channel?
Lattice codes missing in?

- AWGN relay channel?

- Two-way relay channel in presence of direct links?
Why?
Decode and forward relaying

\[ R_{DF} = \max_{p(x_1, x_R)} \{ \min\{I(X_1; Y_R | X_R), I(X_1, X_R; Y_2)\} \} \]
Decode and forward relaying

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R_{DF} = \max_{p(x_1, x_R)} \left\{ \min \left\{ I(X_1; Y_R | X_R), I(X_1, X_R; Y_2) \right\} \right\}
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- Irregular Markov Encoding with Successive Decoding
  [Cover, ElGamal 1979]
Decide and forward relaying

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- Irregular Markov Encoding with Successive Decoding
  [Cover, El Gamal 1979]

- Regular Encoding with Backward Decoding
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- Nice survey
  [Kramer, Gastpar, Gupta 2005]
Irregular Markov Encoding with Successive Decoding

**Encoding**

<table>
<thead>
<tr>
<th>Transmitter</th>
<th>$x_1(w_1, 1)$</th>
</tr>
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<tbody>
<tr>
<td>Relay</td>
<td>$x_R(1)$</td>
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**Decoding**

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Irregular Markov Encoding with Successive Decoding

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\( s(w) \) is the partition index of message \( w \)
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| Decoding | |
|----------|---------|---------|---------|
| Relay | $w_1$ | $w_2$ | $w_3$ |
| Receiver | $s(w)$ | $s(w)$ | $s(w)$ |

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<tr>
<td>Relay</td>
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<tr>
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</tr>
<tr>
<td>Receiver</td>
</tr>
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$s(w)$ is the partition index of message $w$
# Irregular Markov Encoding with Successive Decoding

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
<tr>
<td><strong>Transmitter</strong></td>
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<td></td>
<td></td>
<td>INTERSECT</td>
<td></td>
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<td><strong>Relay</strong></td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$w_3$</td>
<td></td>
</tr>
<tr>
<td><strong>Receiver</strong></td>
<td>$\mathcal{L}(w_1)$</td>
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<tr>
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<td>$x_1(1, s(w_3))$</td>
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$s(w)$ is the partition index of message $w$
Can we mimic with lattices?

- nested lattices achieve capacity of AWGN channel [Erez, Zamir, Trans. IT, 2004]
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Receiver: \(s(w_1), s(w_2), s(w_3)\)
List decoder?

• ambiguity set of DF scheme of [Cover, El Gamal 1979]

\[ \mathcal{L}(w_i) = \text{set of } w_i \text{ such that } (x_1(w_i, s(w_{i-1})), x_R(s(w_{i-1})), y_2(i)) \text{ are jointly typical} \]
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• concept of "joint typicality" for lattices?
List decoder?

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- concept of "joint typicality" for lattices?

- we propose an alternative "lattice list decoder"
Outline

• **List decoder** for nested lattices:
  
  *decode a list of particular size which contains correct codeword*
Outline

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  decode a list of particular size which contains correct codeword

• **one-way relay channel:** use list decoder to achieve the DF rates of the AWGN relay channel with nested lattices
Outline

• **List decoder** for nested lattices: 
  *decode a list of particular size which contains correct codeword*

• **one-way relay channel**: *use list decoder to achieve the DF rates of the AWGN relay channel with nested lattices*

• **two-way relay channel**: *use list decoder to obtain new achievable rate region and finite gap results for “degraded” cases of two-way relay channel with direct links*
Lattice notation

- $\Lambda = \{\lambda = G \mathbf{i} : \mathbf{i} \in \mathbb{Z}^n\}$, $G$ the generator matrix
Lattice notation

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- lattice quantizer of $\Lambda$:

$$Q(\mathbf{X}) = \arg \min_{\lambda \in \Lambda} ||\mathbf{X} - \lambda||$$
Lattice notation

- \( \Lambda = \{ \lambda = G \mathbf{i} : \mathbf{i} \in \mathbb{Z}^n \} \), \( G \) the generator matrix

- **lattice quantizer** of \( \Lambda \):
  \[
  Q(\mathbf{X}) = \arg \min_{\lambda \in \Lambda} \| \mathbf{X} - \lambda \|
  \]

- \( \mathbf{x} \mod \Lambda := \mathbf{x} - Q(\mathbf{x}) \)
Lattice notation

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- \( x \mod \Lambda := x - Q(x) \)

- fundamental region \( \mathcal{V} := \{ x : Q(x) = 0 \} \) of volume \( V \)
Lattice notation

- $\Lambda = \{\lambda = G \, i : i \in \mathbb{Z}^n\}$, $G$ the generator matrix

- **lattice quantizer of $\Lambda$:**
  $$Q(X) = \arg \min_{\lambda \in \Lambda} ||X - \lambda||$$

- $x \mod \Lambda := x - Q(x)$

- **fundamental region $\mathcal{V} := \{x : Q(x) = 0\}$ of volume $V$**

- **second moment per dimension of a uniform distribution over $\mathcal{V}$:**
  $$\sigma^2(\Lambda) := \frac{1}{V} \cdot \frac{1}{n} \int_{\mathcal{V}} ||x||^2 \, dx$$
Nested lattice codes

• Nested lattice pair: $\Lambda \subseteq \Lambda_c$ ( $\Lambda$ is Rogers-good and Poltyrev-good, $\Lambda_c$ is Poltyrev-good)
Nested lattice codes

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- The code book \( \mathcal{C} = \{ \Lambda_c \cap \mathcal{V}(\Lambda) \} \) is used to achieve the capacity of AWGN channel

[Erz+Zamir, Trans. IT, 2004]
Nested lattice codes

- Nested lattice pair: \( \Lambda \subseteq \Lambda_c \) (\( \Lambda \) is Rogers-good and Poltyrev-good, \( \Lambda_c \) is Poltyrev-good)

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  \[ [\text{Erez+Zamir, Trans. IT, 2004}] \]

- Coding rate: \( R = \frac{1}{n} \log |\mathcal{C}| = \frac{1}{n} \log \frac{V(\Lambda)}{V(\Lambda_c)} \) arbitrary (\# of \( \star \))
Nested lattice chains

- $\Lambda_1 \subseteq \Lambda_2 \subseteq \cdots \subseteq \Lambda_K$ ( $\Lambda_1, \Lambda_2 \ldots \Lambda_{K-1}$ are Rogers-good and Poltyrev-good, $\Lambda_K$ is Poltyrev-good ). The nesting rates between any pairs in the chain can attain any arbitrary values as the dimension $n \to \infty$.

[Nam, Chung, Lee, Trans. IT, 2010]
Nested lattice chains

- $\Lambda_1 \subseteq \Lambda_2 \subseteq \cdots \subseteq \Lambda_K$ ($\Lambda_1, \Lambda_2 \ldots \Lambda_{K-1}$ are Rogers-good and Poltyrev-good, $\Lambda_K$ is Poltyrev-good). The nesting rates between any pairs in the chain can attain any arbitrary values as the dimension $n \to \infty$.

[Nam, Chung, Lee, Trans. IT, 2010]

- A “good” lattice chain with length 3 is used in our list decoding scheme:

$$\Lambda \subseteq \Lambda_s \subseteq \Lambda_c$$
Lattice list decoder

• IDEA: decode to rather than to
Lattice list decoder

- IDEA: decode to rather than to

- results in a list of codewords
Lattice list decoder

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- require correct codeword to be in list
Lattice list decoder

- IDEA: decode to \[ \Lambda \subseteq \Lambda_s \subseteq \Lambda_c \] rather than to \[ \Lambda_s \subseteq \Lambda_c \]
- results in a list of codewords
- require correct codeword to be in list
- how many are in list?
• message of rate $R$ over the AWGN channel $Y = X + Z$ subject to the average power constraint $P$
Encoding

- message of rate $R$ over the AWGN channel $\mathbf{Y} = \mathbf{X} + \mathbf{Z}$ subject to the average power constraint $P$

- **Encoding:** take $t \in \mathcal{C}_{\Lambda_e, \nu}$ associated with message of rate $R$ and $\mathbf{X} = (t - U) \mod \Lambda$

- $U$ is a dither signal uniformly distributed over $\nu$. 
Decoding
Decoding

\[ \Lambda \subseteq \Lambda_s \subseteq \Lambda_c \]

\[ S_{\Lambda_s,\Lambda_c}(Y') = \{ \Lambda_c \cap (Y' + \mathcal{V}_s) \} \]

\[ \star = Y' \]
Decoding

- Receiver first computes

\[ Y' = (\alpha Y + U) \mod \Lambda \]
\[ = (t - (1 - \alpha)X + \alpha Z) \mod \Lambda \]
\[ = (t + Z') \mod \Lambda \]
Decoding

- Receiver first computes

\[ Y' = (\alpha Y + U) \mod \Lambda \]
\[ = (t - (1 - \alpha)X + \alpha Z) \mod \Lambda \]
\[ = (t + Z') \mod \Lambda \]

- Receiver then decodes the list of codewords \( \hat{t} \):

\[ L(\hat{t}) := S_{Y'_s, \Lambda_c}(Y') \mod \Lambda \]

\[ S_{Y'_s, \Lambda_c}(Y') = \{ \Lambda_c \cap (Y' + Y'_s) \} \]

\[ \star = Y' \]
Lattice list decoder

- Probability of error for list decoding: \( P_e := \Pr\{t \notin L(\hat{t})\} \)
Lattice list decoder

- Probability of error for list decoding: \( P_e := \Pr\{t \notin L(\hat{t})\} \)

\[ \Lambda \subseteq \Lambda_s \subseteq \Lambda_c \]

\[ S_{V_s,\Lambda_c}(Y') = \{\Lambda_c \cap (Y' + V_s)\} \]

SAME list as

\[ Q_{V_s,\Lambda_c}(Y') = \bigcup_{\lambda_c \in \Lambda_c} \{\lambda_c | Y' \in (\lambda_c + V_s)\} \]

\( \star = Y' \)

- easy to count \# in list
Lattice list decoder

- Probability of error for list decoding: \( P_e := \Pr\{t \notin L(\hat{t})\} \)

\[ \Lambda \subseteq \Lambda_s \subseteq \Lambda_c \]

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\[ Q_{\mathcal{V}_s, \Lambda_c}(Y') = \bigcup_{\lambda_c \in \Lambda_c} \{\lambda_c Y' \in (\lambda_c + \mathcal{V}_s)\} \]

\( \star = Y' \)

- Easy to count \# in list
- Easy to bound probability of error
Lattice list decoder

- Theorem 1: Using the encoding and decoding scheme defined above, the receiver decodes a list of codewords of size $2^n(R-C(P/N))$ with probability of error $P_e \to 0$ as $n \to \infty$
Can lattices achieve the DF rates for the AWGN relay channel?
Application I: AWGN DF rates

- in [Cover, El Gamal, Trans. IT, 1979] proven using:
  - superposition coding
  - Slepian-Wolf partitioning
  - coding for cooperative multiple access channel
  - Block-Markov coding
  - list decoding
  - successive decoding
Application I: AWGN DF rates

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*all using RANDOM codes*
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  **all using RANDOM codes**

  **can we use NESTED LATTICE codes instead?**
Irregular Markov Encoding with Successive Decoding

Block $i$

Encoding

$x_1(w_i, s(w_{i-1}))$

$x_R(s(w_{i-1}))$
Irregular Markov Encoding with Successive Decoding

Block $i$

\[ x_1(w_i, s(w_{i-1})) \quad \rightarrow \quad \text{Superposition of two lattice codes} \]

\[ x_R(s(w_{i-1})) \]

Diagram:

- Node 1 connects to Node 2 with a direct edge.
- Node 1 connects to Node 2 with another edge through a circular symbol labeled $R$.
- Node 1 connects to Node 2 with a different edge through a circular symbol labeled $R$.

Wednesday, November 3, 2010
Irregular Markov Encoding with Successive Decoding

Block $i$

Encoding

$x_1(w_i, s(w_{i-1}))$

$x_R(s(w_{i-1}))$

$\sigma^2(\Lambda_1) = \alpha P$

$\Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1}$

Superposition of two lattice codes
Irregular Markov Encoding with Successive Decoding

\[ \sigma^2(\Lambda_1) = \alpha P \]
\[ \sigma^2(\Lambda_2) = \bar{\alpha} P \]

\[ \Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1} \]
\[ \Lambda_2 \subseteq \Lambda_{c2}, \]

Superposition of two lattice codes
Irregular Markov Encoding with Successive Decoding

Block $i$

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$\sigma^2(\Lambda_1) = \alpha P$

$\sigma^2(\Lambda_2) = \bar{\alpha} P$

$\Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1}$

$\Lambda_2 \subseteq \Lambda_{c2}$

Superposition of two lattice codes

Lattice code

$\sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2 \subseteq \sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_{c2}$
Irregular Markov Encoding with Successive Decoding

Block i

Encoding

\[ x_1(w_i, s(w_{i-1})) \]

\[ x_R(s(w_{i-1})) \]

\[ \sigma^2(\Lambda_1) = \alpha P \]

\[ \Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1} \]

\[ \Lambda_2 \subseteq \Lambda_{c2} \]

Superposition of two lattice codes

Lattice code

\[ \sqrt{\frac{P_R}{\bar{\alpha}P}} \Lambda_2 \subseteq \sqrt{\frac{P_R}{\bar{\alpha}P}} \Lambda_{c2} \]

\[ \sigma^2(\sqrt{\frac{P_R}{\bar{\alpha}P}} \Lambda_2) = P_R \]
Irregular Markov Encoding with Successive Decoding

**Encoding**

Block $i$

$$x_1(w_i, s(w_{i-1}))$$

$$x_R(s(w_{i-1}))$$

**Decoding**

Block $i+1$

$$\sigma^2(\Lambda_1) = \alpha P$$

$$\Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1}$$

$$\Lambda_2 \subseteq \Lambda_{c2},$$

Superposition of two lattice codes

Lattice code

$$\sqrt{\frac{P_R}{\bar{\alpha}P}} \Lambda_2 \subseteq \sqrt{\frac{P_R}{\bar{\alpha}P}} \Lambda_{c2}$$

$$\sigma^2(\sqrt{\frac{P_R}{\bar{\alpha}P}} \Lambda_2) = P_R$$
Irregular Markov Encoding with Successive Decoding

**Block i**

**Encoding**

\[ x_1(w_i, s(w_{i-1})) \]

\[ x_R(s(w_{i-1})) \]

\[ \sigma^2(\Lambda_1) = \alpha P \]

\[ \sigma^2(\Lambda_2) = \bar{\alpha} P \]

\[ \Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1} \]

\[ \Lambda_2 \subseteq \Lambda_{c2} \]

Superposition of two lattice codes

**Decoding**

\[ \sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2 \subseteq \sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_{c2} \]

\[ \sigma^2(\sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2) = P_R \]

\[ \Lambda \]

\[ w_i \]
Irregular Markov Encoding with Successive Decoding

Encoding

Block $i$

$x_1(w_i, s(w_{i-1}))$

$x_R(s(w_{i-1}))$

Decoding

Block $i+1$

$w_i$

Superposition of two lattice codes

$L_1 \subseteq L_{s1} \subseteq L_{c1}$

$L_2 \subseteq L_{c2}$

$L$-lattice code

$L_2 \subseteq L_{c2}$

$\sigma^2(L_1) = \alpha P$

$\sigma^2(L_2) = \bar{\alpha} P$

$\sqrt{\frac{P_R}{\bar{\alpha} P}} L_2 \subseteq \sqrt{\frac{P_R}{\bar{\alpha} P}} L_{c2}$

$\sigma^2(\sqrt{\frac{P_R}{\bar{\alpha} P}} L_2) = P_R$

Regular nested lattice decoder
Irregular Markov Encoding with Successive Decoding

Block $i$

Encoding

$x_1(w_i, s(w_{i-1}))$

$x_R(s(w_{i-1}))$

Decoding

$w_i$

$s(w_{i-1})$

Superposition of two lattice codes

$\sigma^2(\Lambda_1) = \alpha P$

$\sigma^2(\Lambda_2) = \bar{\alpha} P$

$\Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1}$

$\Lambda_2 \subseteq \Lambda_{c2}$

Lattice code

$\sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2 \subseteq \sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_{c2}$

Regular nested lattice decoder

$\sigma^2(\sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2) = P_R$
Irregular Markov Encoding with Successive Decoding

Block $i$

Encoding

$x_1(w_i, s(w_{i-1}))$

$x_R(s(w_{i-1}))$

Decoding

$w_i$

$s(w_{i-1})$, $\mathcal{L}(w_i)$

\[ \sigma^2(\Lambda_1) = \alpha P \]

\[ \sigma^2(\Lambda_2) = \bar{\alpha} P \]

$\Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1}$

$\Lambda_2 \subseteq \Lambda_{c2}$

Superposition of two lattice codes

Lattice code

\[ \sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2 \subseteq \sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_{c2} \]

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Regular nested lattice decoder
Irregular Markov Encoding with Successive Decoding

**Encoding**

- Block $i$
  - $x_1(w_i, s(w_{i-1}))$
  - $x_R(s(w_{i-1}))$

**Decoding**

- Block $i+1$
  - $w_i$
  - $s(w_{i-1}) \mathcal{L}(w_i)$

Superposition of two lattice codes

$L_1 \subseteq L_{s1} \subseteq L_{c1}$

$L_2 \subseteq L_{c2}$

Lattice code

$\sigma^2(L_1) = \alpha P$

$\sigma^2(L_2) = \bar{\alpha} P$

Regular nested lattice decoder

Sequential list and nested lattice decoders
Irregular Markov Encoding with Successive Decoding

- **Encoding**
  - Block $i$
    - $x_1(w_i, s(w_{i-1}))$
    - $x_R(s(w_{i-1}))$

- **Decoding**
  - Block $i+1$
    - $w_i$
    - $s(w_{i-1}) \ L(w_i)$

- Superposition of two lattice codes
- Regular nested lattice decoder
- Sequential list and nested lattice decoders

Mathematical Expressions:
- $\sigma^2(\Lambda_1) = \alpha P$
- $\Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1}$
- $\Lambda_2 \subseteq \Lambda_{c2}$
- $\sigma^2(\Lambda_2) = \bar{\alpha} P$
- $\sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2 \subseteq \sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_{c2}$
- $\sigma^2(\sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2) = P_R$

Sequential list and nested lattice decoders

- $\mathcal{L}(w_i) = \{w_i : \hat{\mathbf{t}}(w_i) \in L(\hat{\mathbf{t}}(w_i))\}$
Irregular Markov Encoding with Successive Decoding

**Encoding**

Block $i$

1. $x_1(w_i, s(w_{i-1}))$

2. $x_R(s(w_{i-1}))$

**Decoding**

Block $i+1$

- $w_i$
- $w_{i+1}$
- $s(w_{i-1})$
- $\mathcal{L}(w_i)$
- $s(w_i)$

\[ \sigma^2(\Lambda_1) = \alpha P \]
\[ \sigma^2(\Lambda_2) = \bar{\alpha} P \]

$\Lambda_1 \subseteq \Lambda_{s1} \subseteq \Lambda_{c1}$

$\Lambda_2 \subseteq \Lambda_{c2}$

Superposition of two lattice codes

Regular nested lattice decoder

Sequential list and nested lattice decoders

$(\sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2) \subseteq (\sqrt{\frac{P_R}{\alpha P}} \Lambda_{c2})$

$\sigma^2(\sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_2) = P_R$

$\mathcal{L}(w_i) = \{w_i : \hat{t}(w_i) \in L(\hat{t}(w_i))\}$

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Irregular Markov Encoding with Successive Decoding

Block i

**Encoding**

\[ x_1(w_i, s(w_{i-1})) \]

\[ x_R(s(w_{i-1})) \]

\[ \Lambda_1 \subset \Lambda_s \subset \Lambda_c \]

Superposition of two lattice codes

\[ \sigma^2(\Lambda_1) = \alpha P \]

\[ \sigma^2(\Lambda_2) = \bar{\alpha} P \]

**Decoding**

Block i+1

**Lattice code**

\[ \sqrt{\frac{P_R}{\alpha P}} \Lambda_2 \subset \sqrt{\frac{P_R}{\bar{\alpha} P}} \Lambda_{c2} \]

Regular nested lattice decoder

\[ \sigma^2(\sqrt{\frac{P_R}{\alpha P}} \Lambda_2) = P_R \]

Sequential list and nested lattice decoders

\[ \mathcal{L}(w_i) = \{ w_i : \hat{t}(w_i) \in L(\hat{t}(w_i)) \} \]
Decoding

- At relay: $Y_R = X_1 + X_2 + Z_R$
Decoding

- At relay: \( Y_R = X_1 + X_2 + Z_R \) \( R < \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_R} \right) \)
Decoding

- At relay: \( Y_R = X_1 + X_2 + Z_R \quad R < \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_R} \right) \)

- At destination:

\[
Y_2 = X_1 + X_2 + X_R + Z_2 = \left( 1 + \sqrt{\frac{P_R}{\alpha P}} \right) X_2 + X_1 + Z_2
\]
Decoding

- At relay: $Y_R = X_1 + X_2 + Z_R \quad R < \frac{1}{2} \log \left(1 + \frac{\alpha P}{N_R} \right)$

- At destination:

  $Y_2 = X_1 + X_2 + X_R + Z_2$

  $= \left(1 + \sqrt{\frac{P_R}{\bar{\alpha}P}} \right) X_2 + X_1 + Z_2$

Successive decoding:
Decoding

- At relay: $Y_R = X_1 + X_2 + Z_R \quad R < \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_R} \right)$

- At destination:

  \[
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  \]

Successive decoding:

Same partition index
Decoding

- At relay: \( Y_R = X_1 + X_2 + Z_R \quad R < \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_R} \right) \)

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\[
Y_2 = X_1 + X_2 + X_R + Z_2 \\
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\]

Successive decoding:

Same partition index \( \rightarrow \) List decoded
Decoding

- At relay: \( Y_R = X_1 + X_2 + Z_R \)  \( R < \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_R} \right) \)

- At destination:

\[
Y_2 = X_1 + X_2 + X_R + Z_2 = \left( 1 + \sqrt{\frac{P_R}{\alpha P}} \right) X_2 + X_1 + Z_2
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Successive decoding:

Same partition index  \( \rightarrow \)  List decoded
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Successive decoding:

- Same partition index
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Decoding

• At relay: $Y_R = X_1 + X_2 + Z_R$  \( R < \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_R} \right) \)

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Y_2 = X_1 + X_2 + X_R + Z_2 = \left( 1 + \sqrt{\frac{P_R}{\alpha P}} \right) X_2 + X_1 + Z_2
\]

Successive decoding:  
Same partition index  \( \rightarrow \)  List decoded  \( \rightarrow \)  Desired message
Decoding

- At relay: \( Y_R = X_1 + X_2 + Z_R \)  \( R < \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_R} \right) \)

- At destination:

\[
Y_2 = X_1 + X_2 + X_R + Z_2 = \left( 1 + \sqrt{\frac{P_R}{\alpha P}} \right) X_2 + \left( X_1 + Z_2 \right)
\]

Successive decoding:

- Same partition index
- List decoded

\[
R < \frac{1}{2} \log \left( 1 + \frac{P + P_R + 2\sqrt{\alpha P P_R}}{N + N_R} \right)
\]
Application I: DF rates for relay channel

• Capacity of degraded AWGN relay channel shown to be

\[ R \leq \max_{0 \leq \alpha \leq 1} \min \left\{ \frac{1}{2} \log \left( 1 + \frac{P + P_R + 2\sqrt{\alpha PP_R}}{N + N_R} \right), \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_R} \right) \right\} \]
Application I: DF rates for relay channel

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Thm 2: This can be achieved using NESTED LATTICE CODES!
Application 2: two-way relay channel
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Importance in larger two-way networks
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Need to combine information from various streams using lattice codes
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Simplest example
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Application 2: two-way relay channel with direct links
Application 2: two-way relay channel with direct links

- achieve to within $1/2$ bit/s/Hz gap in absence of direct link
  [Nam, Chung, Lee, Trans. IT, to appear]
Application 2: two-way relay channel with direct links

\[ Y_r = X_1 + X_2 + N_R \]

- achieve to within 1/2 bit/s/Hz gap in absence of direct link
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- uses lattice codes to "decode the sum" at the relay, rather than individual messages
Application 2: two-way relay channel with direct links

\[ Y_r = X_1 + X_2 + N_R \]

- Achieve to within 1/2 bit/s/Hz gap in absence of direct link
  [Nam, Chung, Lee, Trans. IT, to appear]

- Achieve to within 2 bits/s/Hz gap for special cases with direct link
  [Avestimehr, Sezgin, Tse, European Trans. Comm, 2009]

- Uses lattice codes to "decode the sum" at the relay, rather than individual messages

- Uses random codes and quantizers at relay
Application 2: two-way relay channel with direct links
Application 2: two-way relay channel with direct links

\[ Y_R = X_1 + X_2 + Z_R, \quad Z_R \sim \mathcal{N}(0, N_R) \]

Power \( P_R \)

No message, just relay

\[ Y_1 = X_1 + X_2 + X_R + Z_1 \quad Y_2 = X_1 + X_2 + X_R + Z_2 \]

\[ Z_1 \sim \mathcal{N}(0, N_1) \quad Z_2 \sim \mathcal{N}(0, N_2) \]

Degraded if \( Z_1 = Z_R + Z'_1, \quad Z_2 = Z_R + Z'_2 \)
Application 2: two-way relay channel with direct links

\[ Y_R = X_1 + X_2 + Z_R, \quad Z_R \sim \mathcal{N}(0, N_R) \]

Power \( P_R \)
No message, just relay

\[ Y_1 = X_1 + X_2 + X_R + Z_1 \quad Y_2 = X_1 + X_2 + X_R + Z_2 \]
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- we derive a new achievable rate region using nested lattices, with direct link
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- we derive a new achievable rate region using \textbf{nested lattices}, with \textbf{direct link}

- this region attains \textbf{constant gaps} for certain degraded channel
Application 2: two-way relay channel with direct links

\[ Y_R = X_1 + X_2 + Z_R, \quad Z_R \sim \mathcal{N}(0, N_R) \]

- Power \( P_R \)
- No message, just relay

\[ Y_1 = X_1 + X_2 + X_R + Z_1 \quad Y_2 = X_1 + X_2 + X_R + Z_2 \]

- Message \( W_1 \)
- Power \( P_1 \)

- Message \( W_2 \)
- Power \( P_2 \)

\[ Z_1 \sim \mathcal{N}(0, N_1) \quad Z_2 \sim \mathcal{N}(0, N_2) \]

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- this region attains constant gaps for certain degraded channel
Application 2: two-way relay channel with direct links

\[ \hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1 \]

- Decoding sum
  [Nam, Chung, Lee Trans. IT 2010]

Diagram:
- \( w_1 \leftrightarrow t_1 \)
- \( w_2 \leftrightarrow t_2 \)
Application 2: two-way relay channel with direct links

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  [Nam, Chung, Lee, Trans. IT 2010]

- Random binning
  [Xie, CWIT 2007]
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  [Song, D 2010]
Application 2: two-way relay channel with direct links

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New achievable rate region
Decoding the sum

\[ w_1 \leftrightarrow t_1 \quad \text{and} \quad w_2 \leftrightarrow t_2 \]

Image taken from [Nam, Chung, Lee Trans. IT 2010]
Decoding the sum

\[ w_1 \leftrightarrow t_1 \in \mathcal{C}_1 \quad \text{and} \quad w_2 \leftrightarrow t_2 \]

Image taken from [Nam, Chung, Lee Trans. IT 2010]
Decoding the sum

\[ w_1 \leftrightarrow t_1 \in \mathcal{C}_1 \]

\[ w_2 \leftrightarrow t_2 \in \mathcal{C}_2 \]

Image taken from [Nam, Chung, Lee Trans. IT 2010]
Decoding the sum

\[ \hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1 \]

\[ w_1 \leftrightarrow t_1 \in \mathcal{C}_1 \quad w_2 \leftrightarrow t_2 \in \mathcal{C}_2 \]

*Image taken from [Nam, Chung, Lee Trans. IT 2010]*
Decoding the sum

\[ \hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1 \]

\[ w_1 \leftrightarrow t_1 \in C_1 \quad w_2 \leftrightarrow t_2 \in C_2 \]

Broadcast \( x_R(\hat{T}) \)

Image taken from [Nam, Chung, Lee Trans. IT 2010]
Decoding the sum

\[ \hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1 \]

\[ w_1 \leftrightarrow t_1 \in \mathcal{C}_1 \]
\[ w_2 \leftrightarrow t_2 \in \mathcal{C}_2 \]

Broadcast \( x_R(\hat{T}) \)

\[ \hat{w}_2 = f(\hat{T}, w_1) \]
\[ \hat{w}_1 = f(\hat{T}, w_2) \]

Image taken from [Nam, Chung, Lee Trans. IT 2010]
Broadcasting with side information

\[ w_1 \in \{1, 2, \ldots, 2^{nR_1}\} \quad \hat{w}_2, w_1 \quad \hat{w}_1, w_2 \]

\[ \mathcal{W}_1, \mathcal{W}_2 \quad w_2 \in \{1, 2, \ldots, 2^{nR_2}\} \]
Broadcasting with side information

\[ w_1 \in \{1, 2, \ldots, 2^{nR_1}\} \]

\[ w_2 \in \{1, 2, \ldots, 2^{nR_2}\} \]

Send common message

\[ w_1 \oplus w_2 \]
Broadcasting with side information

\[ w_1 \in \{1, 2, \ldots, 2^{nR_1}\} \quad w_2 \in \{1, 2, \ldots, 2^{nR_2}\} \]

Send common message

\[ w_1 \oplus w_2 \]

\[ R_1 \leq \min\{I(X_R; Y_1), I(X_R; Y_2)\} \]
\[ R_2 \leq \min\{I(X_R; Y_1), I(X_R; Y_2)\} \]
Broadcasting with side information

\[ w_1 \in \{1, 2, \cdots, 2^{nR_1}\} \]

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Send common message

\[ w_1 \oplus w_2 \]

Binning / Slepian-Wolf

\[ 2^{nR} \text{ bins, } R > \max\{I(X_R; Y_1), I(X_R; Y_2)\} \]

\[ R_1 \leq \min\{I(X_R; Y_1), I(X_R; Y_2)\} \]

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\[ R_1 \leq I(X_R; Y_1) \]

\[ R_2 \leq I(X_R; Y_2) \]
Application 2: two-way relay channel with direct links

\[ \hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1 \]

- Decoding sum
  [Nam, Chung, Lee Trans. IT 2010]

- Random binning
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- List decoding
  [Song, D 2010]

New achievable rate region
Outline of achievability scheme

- assume WLOG $P_1 \geq P_2$
Outline of achievability scheme

- assume WLOG $P_1 \geq P_2$

$w_1 \leftrightarrow t_1$ $(\Lambda_1, \Lambda_{c1})$
Outline of achievability scheme

• assume WLOG $P_1 \geq P_2$

\[ w_1 \leftrightarrow t_1 \quad (\Lambda_1, \Lambda_{c_1}) \quad 1 \quad 2 \quad (\Lambda_2, \Lambda_{c_2}) \quad w_2 \leftrightarrow t_2 \]

(block Markov coding)
Outline of achievability scheme

- assume WLOG $P_1 \geq P_2$

\[ w_1 \leftrightarrow t_1 \quad (\Lambda_1, \Lambda_{c1}) \quad 1 \quad 2 \quad (\Lambda_2, \Lambda_{c2}) \quad w_2 \leftrightarrow t_2 \]

- decode $\mathcal{L}(w_2)$
- decode $\mathcal{L}(w_1)$

*(block Markov coding)*
Outline of achievability scheme

- assume WLOG $P_1 \geq P_2$

- decode $\hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1$

\[ w_1 \leftrightarrow t_1 \quad (\Lambda_1, \Lambda_{c1}) \quad 1 \quad \rightarrow \quad 2 \quad (\Lambda_2, \Lambda_{c2}) \quad w_2 \leftrightarrow t_2 \]

- decode $\mathcal{L}(w_2)$
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*(block Markov coding)*
Outline of achievability scheme

- assume WLOG $P_1 \geq P_2$
  - decode $\hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1$
  - send bin index of $\hat{T}$ (random code)

$w_1 \leftrightarrow t_1 \ (\Lambda_1, \Lambda_{c1})$

$w_2 \leftrightarrow t_2 \ (\Lambda_2, \Lambda_{c2})$

- decode $L(w_2)$
  - decode $L(w_1)$

*(block Markov coding)*
Outline of achievability scheme

- assume WLOG $P_1 \geq P_2$

- decode $\hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1$

- send bin index of $\hat{T}$ (random code)

\[ w_1 \leftrightarrow t_1 \quad (\Lambda_1, \Lambda_{c1}) \]

\[ 1 \quad \rightarrow \quad 2 \quad (\Lambda_2, \Lambda_{c2}) \quad w_2 \leftrightarrow t_2 \]

- decode $L(w_2)$

- decode $L(w_1)$

(block Markov coding)
Outline of achievability scheme

- assume WLOG \( P_1 \geq P_2 \)
- decode \( \hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1 \)
- send bin index of \( \hat{T} \) (random code)

\[
\begin{align*}
    w_1 &\leftrightarrow t_1 \quad (\Lambda_1, \Lambda_{c1}) \\
    w_2 &\leftrightarrow t_2 \quad (\Lambda_2, \Lambda_{c2})
\end{align*}
\]

- decode \( \mathcal{L}(w_2) \)
- decode \( \hat{w}_2 \) satisfying:
  1. \( (x_R(\text{bin index of } \hat{T}), Y_1) \) jointly typical
  2. belongs to list decoded from direct link

*(block Markov coding)*
Outline of achievability scheme

- assume WLOG $P_1 \geq P_2$

- decode $\hat{T} = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1$

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  1. $(x_R(\text{bin index of } \hat{T}), Y_2)$ jointly typical
  2. belongs to list decoded from direct link

*(block Markov coding)*
Irregular Markov Encoding with Successive Decoding

(actually transmit dithered codewords)

<table>
<thead>
<tr>
<th>Block 1</th>
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</tr>
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<tbody>
<tr>
<td>Transmitter 1</td>
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</tr>
<tr>
<td>Transmitter 2</td>
<td></td>
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Wednesday, November 3, 2010
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$\hat{T}_i = (t_{1i} + t_{2i} - Q_2(t_{2i} + U_{2i})) \mod \Lambda_1$
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$s(\hat{T}_i)$ is the bin index of $\hat{T}_i$

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<thead>
<tr>
<th>Encoding</th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
<th>Block 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter 1</td>
<td>$t_1(w_{11})$</td>
<td>$t_1(w_{12})$</td>
<td>$t_1(w_{13})$</td>
<td>$t_1(1)$</td>
</tr>
<tr>
<td>Transmitter 2</td>
<td>$t_2(w_{21})$</td>
<td>$t_2(w_{22})$</td>
<td>$t_2(w_{23})$</td>
<td>$t_2(1)$</td>
</tr>
<tr>
<td>Relay</td>
<td>$x_R(1)$</td>
<td>$x_R(s(\hat{T}_1))$</td>
<td>$x_R(s(\hat{T}_2))$</td>
<td>$x_R(s(\hat{T}_3))$</td>
</tr>
</tbody>
</table>

$\hat{T}_i = (t_{1i} + t_{2i} - Q_2(t_{2i} + U_{2i})) \mod \Lambda_1$

Decoding

<table>
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<tr>
<th>Decoding</th>
<th>$\hat{T}_1$</th>
<th>$\hat{T}_2$</th>
<th>$\hat{T}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relay</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Receiver 1</td>
<td>$L(w_{21})$</td>
<td>$s(\hat{T}_1)$</td>
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Intersect
Outline of achievability scheme

- Relay node: \( Y_R = X_1 + X_2 + Z_R \), decodes \( \hat{T} \):

\[
R_1 < \frac{1}{2} \log \left( \frac{P_1}{P_1 + P_2} + \frac{P_1}{N_R} \right)
\]
\[
R_2 < \frac{1}{2} \log \left( \frac{P_2}{P_1 + P_2} + \frac{P_2}{N_R} \right)
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- Node 2: \( Y_2 = X_1 + X_R + Z_2 \), decodes \( \hat{w}_1 \):

\[
R_1 < I(X_R; Y_2 | X_2) + C(P_1 / N_2)
\]
\[
= \frac{1}{2} \log \left( 1 + \frac{P_R}{P_1 + N_2} \right) + \frac{1}{2} \log \left( 1 + \frac{P_1}{N_2} \right)
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- Analogous for node 1
Theorem 3: For the two-way relay channel with direct links, we may achieve:

\[ R_1 \leq \min \left( \left[ \frac{1}{2} \log \left( \frac{P_1}{P_1 + P_2} + \frac{P_1}{N_R} \right) \right]^+, \frac{1}{2} \log \left( 1 + \frac{P_1 + P_R}{N_2} \right) \right) \]

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- eliminates “MAC”-like constraints at relay
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- eliminates “MAC”-like constraints at relay
- combines direct and relayed information using lattice list decoder
Finite-gap results

\[ Y_R = X_1 + X_2 + Z_R, \quad Z_R \sim \mathcal{N}(0, N_R) \]

\[ Y_1 = X_1 + X_2 + X_R + Z_1 \quad Y_2 = X_1 + X_2 + X_R + Z_2 \]

\[ Z_1 \sim \mathcal{N}(0, N_1) \quad Z_2 \sim \mathcal{N}(0, N_2) \]

- Two-way physically degraded: \( Z_1 = Z_R + Z'_1 \) AND \( Z_2 = Z_R + Z'_2 \):

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\frac{1}{2} \text{bit gap.}
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- **Two-way physically degraded**: \( Z_1 = Z_R + Z'_1 \) AND \( Z_2 = Z_R + Z'_2 \):
  \[ \frac{1}{2} \text{bit gap.} \]

- **Two-way stochastically degraded**: \( N_1 \geq N_R \) AND \( N_2 \geq N_R \):
  \[ \frac{1}{2} \log 3 \text{ bit gap.} \]
What we are missing with direct links

• 4 bit gap for two-way stochastically degraded  [Avestimehr, Sezgin, Tse 2009]

• 0.5 log(3) bit gap for two-way stochastically degraded  [Song, D 2010]

• 0.5 bit gap for two-way physically degraded  [Song, D 2010]
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No known lattice schemes achieve constant gap to cut-set outer bound!
Numerical evaluations

**High SNR**

- $P_1 = P_2 = P_R = 10$, $N_1 = N_2 = N_R = 1$

**High direct-link SNR**

- $P_1 = P_2 = 10$, $P_R = 1$, $N_1 = N_2 = N_R = 1$

**Low direct-link SNR**

- $P_1 = P_2 = 5$, $P_R = 10$, $N_1 = N_2 = 10$, $N_R = 5$

- Comparison with other random coding based Decode-and-Forward schemes which utilize the direct link:
Numerical evaluations

High SNR

High direct-link SNR

Low direct-link SNR

\[ R_1 \leq R_0 = \max_{0 \leq \alpha_1 \leq 1} \min \left\{ \log \left(1 + \alpha_1 P_1 N_2 + P_1 N_2 R \right), 2 \right\} \]

\[ R_2 \leq R_0 = \max_{0 \leq \alpha_2 \leq 1} \min \left\{ \log \left(1 + \alpha_2 P_2 N_1 + P_2 N_2 R \right), 2 \right\} \]

- Comparison with other random coding based Decode-and-Forward schemes which utilize the direct link:

  [Rankov, Wittneben, ISIT 2006]: MAC uplink, superposition downlink
Numerical evaluations

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  [Xie CWIT 2007]: MAC uplink, Slepian-Wolf binning downlink
Numerical evaluations

\[ R_1 \leq R_O = \max_{0 \leq \alpha \leq 1} \min \left( \log \left(1 + \alpha P_1 N_2 + P_1 N_R \right), 1 \right) \] (12)

\[ R_2 \leq R_O = \max_{0 \leq \alpha \leq 1} \min \left( \log \left(1 + \alpha P_2 N_1 + P_2 N_R \right), 1 \right) \] (13)

- Comparison with other random coding based Decode-and-Forward schemes which utilize the direct link:

  [Rankov, Wittneben, ISIT 2006]: MAC uplink, superposition downlink

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  Cut-set
So far

• Thm 1: lattice list decoder

• Thm 2: lattices achieve the DF rates for the relay channel

• Thm 3: new achievable rate region for two-way relay channel with direct links
So far

• Thm 1: lattice list decoder

• Thm 2: lattices achieve the DF rates for the relay channel

• Thm 3: new achievable rate region for two-way relay channel with direct links

• Lattice list decoder: other uses?

• Lattices show promise in two-way scenarios: other extensions?
One-way relay network extension

- extension of one-way DF rates using regular encoding and sliding window decoding of [Xie, Kumar 2002] achievable using lattices in AWGN??
One-way relay network extension

- extension of one-way DF rates using regular encoding and sliding window decoding of [Xie, Kumar 2002] achievable using lattices in AWGN??
- working on it.....
One-way relay network extension

- extension of one-way DF rates using regular encoding and sliding window decoding of [Xie, Kumar 2002] achievable using lattices in AWGN??

- working on it.....

- lattice list decoding is essential in combining information along various paths
Two-way extensions

\[ P_1, N_1 \quad P_R, N_R \quad P_2, N_2 \]

1 \quad R \quad 2
Two-way extensions

\[ P_1, N_1 \quad P_R, N_R \quad P_2, N_2 \]

1 \quad R \quad 2

capacity known to 1/2 bit
Two-way extensions

capacity known to 1/2 bit
Two-way extensions

Capacity known to 1/2 bit

Working on capacity to 1/2 bit
Two-way extensions

capacity known to 1/2 bit

working on capacity to 1/2 bit

More generally?
Reflections on two-way Gaussian networks
Reflections on two-way Gaussian networks

capacity known
Reflections on two-way **Gaussian** networks

- Capacity known
- Capacity known = 2 one-way channels
Reflections on two-way Gaussian networks

1 \rightarrow 2 \quad \text{capacity known}

1 \rightarrow 2 \quad \text{capacity known} = 2 \text{ one-way channels}

1 \rightarrow R \rightarrow 2 \quad \text{capacity known}
Reflections on two-way Gaussian networks

1. Capacity known

2. Capacity known = 2 one-way channels

3. Capacity known

4. Capacity known to within 1/2 bit uses LATTICES
Reflections on two-way **Gaussian** networks

1. \(1 \rightarrow 2\) capacity known
2. \(1 \leftrightarrow 2\) capacity known = 2 one-way channels
3. \(1 \rightarrow R \rightarrow 2\) capacity known
4. \(1 \leftrightarrow R \leftrightarrow 2\) capacity known to within 1/2 bit uses LATTICES
5. \(1 \rightarrow R_1 \rightarrow R_2 \rightarrow R_3 \rightarrow 2\) capacity known
Reflections on two-way **Gaussian** networks

1. 
   - Capacity known
   - 1 \( \rightarrow \) 2

2. 
   - Capacity known = 2 one-way channels
   - 1 \( \leftrightarrow \) 2

3. 
   - Capacity known
   - 1 \( \rightarrow \) R \( \rightarrow \) 2

4. 
   - Capacity known to within 1/2 bit
     - Uses LATTICES
   - 1 \( \leftrightarrow \) R \( \leftrightarrow \) 2

5. 
   - Capacity known
   - 1 \( \rightarrow \) R\(_1\) \( \rightarrow \) R\(_2\) \( \rightarrow \) R\(_3\) \( \rightarrow \) 2

6. 
   - Capacity known
   - 1 \( \leftrightarrow \) R\(_1\) \( \leftrightarrow \) R\(_2\) \( \leftrightarrow \) R\(_3\) \( \leftrightarrow \) 2
Reflections on two-way **Gaussian** networks

1. capacity known

2. capacity known = 2 one-way channels

3. capacity known

4. capacity known to within 1/2 bit

5. can we get to 1/2 bit with lattices?
Questions?

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University of Illinois at Chicago
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