Information theoretic limits of two-way (relay) networks

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One-way (monologue) vs. Two-way (dialogue)

One-way (monologue)

Two-way (dialogue)
Two-way communication applications - wired

Video conferencing

Telesurgery

Data synchronization
The future application of telesurgery for patients in extreme environments is currently providing the bulk of the funding for the development of telesurgery (Fig. 7). It is often time and cost prohibitive to evacuate soldiers, mariners, submariners, and astronauts from their extreme environments to undergo urgent or emergent surgery. The mortality rate for injured American armed service members during Operations Enduring Freedom (OEF) and Iraqi Freedom (OIF) (10%) was decreased by 67% compared to the mortality rate for soldiers injured in World War II (30%), and by 58% compared with the mortality rate during Operation Desert Storm (24%). The decrease in mortality rate has occurred despite an increase in the severity and complexity of wounds suffered. While the significant increase in survival is in part because of improved medical care, it is primarily the result of the decrease in time required to receive definitive medical care. The majority of modern war deaths occur within the first hour after injury; the “golden hour.” Limited medical assets and unacceptably high human risk suggest we will not be able to address this unmet medical need by placing multiple surgical teams across the front lines of battle. A force quality and type of care available to patients in rural settings as well as patients in extreme and remote environments such as the battlefield, at and under the sea, and in extraterrestrial locations.

As telesurgery gains acceptance within the surgical community, we envision networks of telesurgeons operating on patients located in both remote mobile and fixed telesurgery suites. Widespread application necessitates cooperation of multiple telecommunication providers; network issues increase with the addition of every network provider and interfaces. Unfortunately, the QoS provided by the entire network is only as good as the QoS on the worst leg of the network. Providing telesurgery to underserved rural patients is currently difficult as the final leg or “last mile” is in general insufficient. Novel wireless communication technology shows promise in the rural application of telesurgery. Because the delay associated with satellite communication is significant, we continue to explore mobile robotic telesurgery using alternative technologies such as high altitude unmanned airborne vehicle communication systems. For example, Helios (AeroVironment, Inc., Monrovia, CA) is a prototype lightweight solar-electric flying wing that could provide broadband, low latency telecommunication to rural communities that would be ideal for use in telesurgery.
1 Dialogue ≠ or = 2 Monologues?

It depends....

we will use information theory to find out.
Outline

• Information theory - *what, why, when*

• Two-way channel - *channel coding*

• Two-way relay channels
  • *single flow* - canonical example of wireless network coding
  • *multiple flows with a base-station* - pairwise wireless network coding
Information theory’s claims to fame

**Source coding**
- Source = random variable
- Ultimate **data compression limit** is the source’s entropy $H$

**Channel coding**
- Channel = conditional distributions
- Ultimate **transmission rate** is the channel capacity $C$

Reliable communication possible $\leftrightarrow H < C$
Source vs. channel coding

- **Source**
- **Encoder**
- **Channel**
- **Decoder**
- **Destination**

Noise

- **Source**
- **Source coder**
- **Channel coder**
- **Channel**
- **Channel decoder**
- **Source decoder**
- **Destination**

- **Remove redundancy**
- **Controlled adding of redundancy**
- **Decode signals, detect/correct errors**
- **Restore source**
Source vs. channel coding

Limits, not constructions
Communication system model

What is the capacity of this channel?
Channel capacity

- Information channel capacity:
  \[
  C = \max_{p(x)} I(X;Y)
  \]

- Operational channel capacity:
  
  Highest rate (bits/channel use) that can communicate at reliably

- Channel coding theorem says: information capacity = operational capacity
Channel capacity: a cute example
Channel capacity: a cute example

A = A?
A = AAA?
Channel capacity: a cute example

AAA $\rightarrow$ AB.
Channel capacity: a cute example

How to communicate reliably?
Channel capacity: a cute example

Use these 9 symbols!

\[ C = \log_2(9) \]
Capacity in general

- Main idea was to reduce the rate (from a 27-letter input per channel use to a 9-letter input per channel use) so as to produce non-overlapping outputs!
Mathematical description of capacity

- Can achieve reliable communication for all transmission rates $R$: $R < C$

- BUT, probability of decoding error always bounded away from zero if $R > C$
Continuous alphabet channel capacity

Channel: $p(y|x)$

Capacity

$$C = \max_{p(x)} I(X; Y) \text{ bits/channel use}$$

"mutual information" between X and Y

What if

X and Y are not bits, but real numbers?
AWGN channel capacity

Power constrained to $P$

Wireless channel with fading

$hX + N$
**AWGN channel capacity**

\[ C = \max_{p(x): E[XX^T] \leq P} I(X; Y) \]

\[ = \max_{p(x): E[XX^T] \leq P} h(X) - h(X|Y) \]

\[ = \max_{p(x): E[XX^T] \leq P} h(Y) - h(Y|X) \]

\[ = \max_{p(x): E[XX^T] \leq P} h(hX + N) - h(hX + N|X) \]

\[ = \max_{p(x): E[XX^T] \leq P} h(hX + N) - h(N) \]

\[ = \frac{1}{2} \log(2\pi e(|h|^2 P + P_N)) - \frac{1}{2} \log(2\pi e P_N) \]

\[ = \frac{1}{2} \log \left( \frac{|h|^2 P + P_N}{P_N} \right) \]
AWGN channel capacity

\[ C = \frac{1}{2} \log \left( \frac{|h|^2 P + P_N}{P_N} \right) \]

\[ = \frac{1}{2} \log (1 + SNR) \text{ (bits/channel use)} \]

What about bits/second and bandwidth of the channel?

\[ C = W \log_2 \left( 1 + \frac{P}{WN_0} \right) \text{ (bits/second)} \]

[Bandwidth W, h=1, spectral density \(N_0/2\)]
Use?

- Benchmark for performance of practical systems

- Guideline in designing systems - what’s worth shooting for?

- Theoretical insights can lead to practical insights
So now what?

Unsolved

Fundamental
Outline

- Information theory - what, why, when
  
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel

- Two-way channel - channel coding

- Two-way relay channels
  
  - single flow - canonical example of wireless network coding
  
  - multiple flows with a base-station - pairwise wireless network coding
Two-way channel capacity region

One-way **Capacity**

\[ C = \max_{p(x)} I(X;Y) \]

Two-way **Capacity Region**
When is \( \text{Two-way} \) equal to \( \text{Two one-ways} \)?
Models for two-way adaptation

**One-way:**
no adaptation possible

- $x_1^n(w_{12})$
- $W_{12}$
- 1
- 2
- $\widehat{W}_{12}$

**Two-way:**
- no adaptation
- "Restricted two-way channel"

- $x_1^n(w_{12})$
- $W_{12}$
- $\frac{W_{12}}{W_{21}}$
- 1
- 2
- $W_{12}$
- $\widehat{W}_{12}$
- $W_{21}$
- $\widehat{W}_{21}$

**Two-way:**
full adaptation

- $x_1^n(w_{12}, y_1^{n-1})$
- $W_{12}$
- $\frac{W_{12}}{W_{21}}$
- 1
- 2
- $W_{12}$
- $\widehat{W}_{12}$
- $W_{21}$
- $\widehat{W}_{21}$
- $x_2^n(w_{21}, y_2^{n-1})$
Duplex

Two-way: half duplex

Two-way: full duplex

(All on same frequency band)
When is capacity known

- Parallel two-way channel
- Mod-2 adder
- Two-way restricted channel
- Two-way “push-to-talk” channel
- Two-way Gaussian noise channel (full & half duplex, restricted & unrestricted)

When is capacity unknown

- **General** unrestricted discrete memoryless channels
- Binary multiplier channel (BMC)
**General results**

**Inner bound**

\[ R_1 \leq I(X_1; Y_2|X_2) \]
\[ R_2 \leq I(X_2; Y_1|X_1) \]

where \( X_1 \) and \( X_2 \) follow the joint distribution \( p(x_1, x_2) = p(x_1)p(x_2) \).

**Not in general equal!**

**Outer bound**

\[ R_1 \leq I(X_1; Y_2|X_2) \]
\[ R_2 \leq I(X_2; Y_1|X_1) \]

where the joint distribution of random variables \( X_1 \) and \( X_2 \) is \( p(x_1, x_2) \).

[Shannon ’61]
Capacity: two parallel channels

\[ R_{12} \leq C_{K_1} \]

\[ R_{21} \leq C_{K_2} \]

[Shannon ’61]
Capacity: binary mod-2 adder channel

\[ y_1 = y_2 = x_1 + x_2 \text{ (mod 2)} \]

**Figure 3**

<table>
<thead>
<tr>
<th>( Y_1 = Y_2 )</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**MOD. 2 ADDER**

How to achieve capacity region?
Achieving mod 2 adder channel capacity

Receiver 1:

\[ \hat{x}_2 = y_1 \oplus x_1 \]
\[ = x_1 \oplus x_2 \oplus x_1 \]
\[ = x_2 \]

\[ y_1 = y_2 = x_1 + x_2 \pmod{2}. \]

Receiver 2:

\[ \hat{x}_1 = y_2 \oplus x_2 \]
\[ = x_1 \oplus x_2 \oplus x_2 \]
\[ = x_1 \]

EXPLOIT TWO-WAY!
Capacity: restricted channel

\begin{align*}
x_1^n(w_{12}) & \quad \text{1} \\
W_{12} & \\
W_{21} & \\
\end{align*}

\begin{align*}
x_2^n(w_{21}) & \quad \text{2} \\
\widetilde{W}_{12} & \\
\widetilde{W}_{21} & \\
\end{align*}

Capacity region:

\begin{align*}
R_1 & \leq I(X_1; Y_2 | X_2) \\
R_2 & \leq I(X_2; Y_1 | X_1) \\
\end{align*}

where $X_1$ and $X_2$ follow the joint distribution $p(x_1, x_2) = p(x_1)p(x_2)$.

[Shannon ‘61]
Capacity: “push-to-talk” channel

Two-way: half duplex

\[ R_{21} \]

\[ R_{12} \]
Capacity: Gaussian noise channel

\[ Y_1 = aX_1 + bX_2 + N_1 \sim \mathcal{N}(0, \sigma_1^2) \]
\[ Y_2 = cX_1 + dX_2 + N_2 \sim \mathcal{N}(0, \sigma_2^2) \]

**Capacity region:**

\[ R_1 \leq \frac{1}{2} \log(1 + c^2 P_1 / \sigma_2^2) \]
\[ R_2 \leq \frac{1}{2} \log(1 + b^2 P_2 / \sigma_1^2) \]

No dependence on \`a\' or \`d\'

[Han '84]
Capacity: Gaussian noise channel

\[ R_1 \leq \frac{1}{2} \log \left( 1 + \frac{c^2 P_1}{\sigma_2^2} \right) \]
\[ R_2 \leq \frac{1}{2} \log \left( 1 + \frac{b^2 P_2}{\sigma_1^2} \right) \]

- **TWO PARALLEL CHANNELS!!**
- Achieved by Gaussian inputs
- "Feedback" does not help here

\[ Y_1 = aX_1 + bX_2 + N_1 \]
\[ Y_2 = cX_1 + dX_2 + N_2 \]
Why so hard?

Adaptation
Adaptive codewords

Single-letter

\[ C = f(X_1, X_2) \]

Two-letter

\[ C = f(X_1^{(1)}, X_1^{(2)}, X_2^{(1)}, X_2^{(2)}) \]

---


Adaptive codewords

The space over which we can code (x’s) is enormous!

Non-adaptive codewords:

\[ X_1 \in \{0, 1\} \]
\[ Y_1 = X_1 X_2 \]

Code of user 1
\[ u_{11} = 0 \]
\[ 0 \quad 0 \quad 0 \]
\[ u_{11} = 1 \]
\[ 1 \quad 1 \quad 1 \]

Code of user 2
\[ u_{21} = 0 \]
\[ 1 \quad 0 \quad 2 \]
\[ u_{21} = 1 \]
\[ 0 \quad 2 \quad 0 \]

\[ L=N=3 \text{ channel uses} \]

---

**Adaptive codewords:**

\[ X_1 \in \{0, 1\} \]
\[ Y_1 = X_1 X_2 \]

![Diagram](image)

**Adaptive codewords** \( A_1^3 \)

**Adaptive codewords** \( A_2^3 \)

\[ Y_2 = Y_1 \mod 2 \]
\[ X_2 \in \{0, 1, 2\} \]

\( L=N=3 \) channel uses

---

Can we take this adaptation into CAUSAL adaptation - complex and generally deemed unsatisfactory
Take away points - AWGN two-way channel

- If have half-duplex constraint and memoryless channels, time-share
- If have full-duplex - obtain two parallel clean channels

For applications - full duplex gains a lot!
Take away points - Discrete memoryless two-way channel

- If have half-duplex constraint ("push-to-talk"), time-share
- If have parallel two-way channels, mod-2 adder
- If have restricted channel

\[
R_1 \leq I(X_1; Y_2 | X_2) \\
R_2 \leq I(X_2; Y_1 | X_1)
\]

where \(X_1\) and \(X_2\) follow the joint distribution \(p(x_1, x_2) = p(x_1)p(x_2)\).

In general may need adaptive codewords

In general OPEN PROBLEM
## Relationship to feedback channels

- **Feedback channel**

  - **Source**: Encoder \( W \rightarrow X^n \)
  - **Encoder/Decoder**: \( p(y|x) \)
  - **Channel**: \( X^n \rightarrow Y^n \)
  - **Destination**: Decoder \( Y^n \rightarrow \hat{W} \)
  - **Message**: Estimate of message \( Y'_n \)

  - Information is still one-way!!

- **Two-way channel**

  - **Source/Destination**: Encoder/Decoder \( X^n_1 \rightarrow Y^n_2 \)
  - **Encoder/Decoder**: \( p(y_1, y_2|x_1, x_2) \)
  - **Source/Destination**: Decoder \( Y^n_2 \rightarrow X^n_2 \)
  - **Source/Destination**: Encoder/Decoder \( X^n_2 \rightarrow Y^n_1 \)

  - Set rate in \( \leftarrow \) direction to 0

  - Properly choose \( p(y_1, y_2|x_1, x_2) \)
Outline

- Information theory - what, why, when
  
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel

- Two-way channel - channel coding
  
  Adaptive codewords, capacity in Gaussian noise = two parallel channels

- Two-way relay channels
  
  - **single flow** - canonical example of wireless network coding
  
  - **multiple flows with a base-station** - pairwise wireless network coding
So now what?
Motivation

Battlefield Telerobotic Surgery

The future application of telesurgery for patients in extreme environments is currently providing the bulk of the funding for the development of telesurgery (Fig. 7). It is often time and cost prohibitive to evacuate soldiers, mariners, submariners, and astronauts from their extreme environments to undergo urgent or emergent surgery. The mortality rate for injured American armed service members during Operations Enduring Freedom (OEF) and Iraqi Freedom (OIF) (10%) was decreased by 67% compared to the mortality rate for soldiers injured in World War II (30%), and by 58% compared with the mortality rate during Operation Desert Storm (24%). The decrease in mortality rate has occurred despite an increase in the severity and complexity of wounds suffered. While the significant increase in survival is in part because of improved medical care, it is primarily the result of the decrease in time required to receive definitive medical care. The majority of modern war deaths occur within the first hour after injury; the “golden hour.” Limited medical assets and unacceptably high human risk suggest we will not be able to address this unmet medical need by placing multiple surgical teams across the front lines of battle.

As telesurgery gains acceptance within the surgical community, we envision networks of telesurgeons operating on patients located in both remote mobile and fixed telesurgery suites. Widespread application necessitates cooperation of multiple telecommunication providers; network issues increase with the addition of every network provider and interfaces. Unfortunately, the QoS provided by the entire network is only as good as the QoS on the worst leg of the network. Providing telesurgery to underserved rural patients is currently difficult as the final leg or “last mile” is in general insufficient.

Novel wireless communication technology shows promise in the rural application of telesurgery. Because the delay associated with satellite communication is significant, we continue to explore mobile robotic telesurgery using alternative technologies such as high altitude unmanned airborne vehicle communication systems. For example, Helios (AeroVironment, Inc., Monrovia, CA) is a prototype lightweight solar-electric flying wing that could provide broadband, low latency telecommunication to rural communities that would be ideal for use in telesurgery.
(and extensions)


Channel model


Half duplex

Direct link between terminal nodes


Relaying type

Two-way relay channel: half-duplex

Nodes can either transmit

\[ \text{Transmit from } a \text{ to } b \]

or receive

\[ \text{Receive from } b \text{ to } a \]

but not both.
Temporal “phases”: who transmits when

Are 4 phases needed?  **NO!**
Better protocol

\begin{itemize}
\item With a relay, are 4 phases needed?
\end{itemize}

\textbf{NO!}

\textbf{BETTER: 2 phases!}
Message-level network coding

In particular, if the messages of \(a\) and \(b\) are \(w_a\) and \(w_b\) respectively and belong to an algebraic group (such as binary addition), then it is sufficient for the relay node to successfully transmit \(w_a \oplus w_b\) simultaneously to \(a\) and \(b\).
**Key exploits**

- "own message side information" at nodes used to cancel out own message
- "overheard side information" available to nodes when not transmitting
- **broadcast nature** of wireless channels: relay broadcasts one thing, both nodes hear it.

![Diagram of network with nodes and arrows indicating message flows]
Four possible protocols

(i) DT protocol

Time of phase 1 and 2, respectively

(ii) MABC protocol

(iii) TDBC protocol

(iv) HBC protocol
Relaying schemes

What to send?
Relaying schemes

- Amplify and Forward (AF)
- Decode and Forward (DF)
- Compress and Forward (CF)
- Mixed Forward

What to send?
Amplify and forward (AF)

- The relay sends a scaled version of the signal it receives.
- Very little computation is needed.
Decode and forward (DF)

- The relay decodes both $w_a$ and $w_b$.
- Much computation, and transmitter codebooks are needed at the relay.
Compress and forward (CF)

- The relay compresses/quantizes the received signal.
- Less computation than DF and transmitter codebooks are not needed at the relay.

![Diagram](https://via.placeholder.com/150)

- Compress: $y_r^{(1)} \rightarrow \hat{y}_r^{(1)}(w_r)$
- Encode: $w_r \rightarrow x_r^{(2)}$
Mixed Forward (MF)

- The relay decodes $w_a$ and compresses $w_b$, combines them into a new message $w_r$ according to a bijective function, which it encodes and transmits.

\[ w_r = B(\tilde{w}_a, w_{r0}) \]
Comparison of protocols

<table>
<thead>
<tr>
<th>Protocol</th>
<th>Side information</th>
<th>Phase</th>
<th>Interference</th>
</tr>
</thead>
<tbody>
<tr>
<td>MABC</td>
<td>not present</td>
<td>2</td>
<td>present</td>
</tr>
<tr>
<td>TDBC</td>
<td>present</td>
<td>3</td>
<td>not present</td>
</tr>
<tr>
<td>HBC</td>
<td>present</td>
<td>4</td>
<td>present</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relaying</th>
<th>Complexity</th>
<th>Noise</th>
<th>Relay needs</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF</td>
<td>very low</td>
<td>carried</td>
<td>nothing</td>
</tr>
<tr>
<td>DF</td>
<td>high</td>
<td>eliminated</td>
<td>full codebooks</td>
</tr>
<tr>
<td>CF</td>
<td>low</td>
<td>distortion</td>
<td>$p(y_r)$</td>
</tr>
<tr>
<td>Mixed</td>
<td>moderate</td>
<td>partially carried</td>
<td>a codebook, $p(y_r)$</td>
</tr>
</tbody>
</table>
Achievable rate regions: one example

- **Theorem 1**: The capacity region of the half-duplex bi-directional relay channel with the MABC protocol is the union of

\[
R_a < \min \left\{ \Delta_1 I(X_a^{(1)}; Y_r^{(1)}|X_b^{(1)}, Q), \Delta_2 I(X_r^{(2)}; Y_b^{(2)}|Q) \right\}
\]

\[
R_b < \min \left\{ \Delta_1 I(X_b^{(1)}; Y_r^{(1)}|X_a^{(1)}, Q), \Delta_2 I(X_r^{(2)}; Y_a^{(2)}|Q) \right\}
\]

\[
R_a + R_b < \Delta_1 I(X_a^{(1)}, X_b^{(1)}; Y_r^{(1)}|Q)
\]

over all joint distributions \( p(q)p^{(1)}(x_a|q)p^{(1)}(x_b|q)p^{(2)}(x_r|q) \)

with \( |Q| \leq 5. \)
Achievable rate regions: an example

\[ R_a < \min \left\{ \Delta_1 I(X_a^{(1)}; Y_r^{(1)}|X_b^{(1)}, Q), \Delta_2 I(X_r^{(2)}; Y_b^{(2)}|Q) \right\} \]

\[ R_b < \min \left\{ \Delta_1 I(X_b^{(1)}; Y_r^{(1)}|X_a^{(1)}, Q), \Delta_2 I(X_r^{(2)}; Y_a^{(2)}|Q) \right\} \]

\[ R_a + R_b < \Delta_1 I(X_a^{(1)}, X_b^{(1)}; Y_r^{(1)}|Q) \]
Outer bounds: cut-set bound

If the rates \( \{ R^{(ij)} \} \) are achievable with a protocol \( P \) and \( R_{\Sigma}(S \rightarrow S^c) \) denotes the total rate of independent information sent from set \( S \) to set \( S^c \) then for all sets \( S \):

\[
R_{\Sigma}(S \rightarrow S^c) \leq \sum_i \Delta_i I(X^{(i)}_{(S)}; Y^{(i)}_{(S^c)}|X^{(i)}_{(S^c)}, Q).
\]
Simulations for the Gaussian noise channel

The Gaussian Case

- We apply the previous results to the Gaussian channel.
- $h_{ij}$ is the effective channel gain between transmitter $i$ and receiver $j$, which is modeled as a complex number. We assume that the channel is reciprocal.

\[
Y_r = h_{ar}X_a + h_{br}X_b + N_r, \quad N_r \sim \mathcal{N}(0, 1)
\]
\[
Y_a = h_{ba}X_b + h_{ra}X_r + N_a, \quad N_a \sim \mathcal{N}(0, 1)
\]
\[
Y_b = h_{ab}X_a + h_{rb}X_r + N_b, \quad N_b \sim \mathcal{N}(0, 1)
\]

(with appropriate half-duplex constraints)
Gaussian simulations

Through two simple examples we wish to emphasize that the straightforward application of one-way metrics and techniques is not necessarily sufficient or optimal for two-way communications. Motivational examples: dialogues rather than monologues.

Example 1: fundamental differences – why some classical metrics are insufficient for two-way communication.

Consider two nodes which wish to exchange messages with the help of a single relay: one can think of two mobile terminals exchanging messages through a base station or a relay. This example considers two nodes which wish to exchange messages with the help of a single relay: one can think of two mobile terminals exchanging messages through a base station or a relay. This example studies the difference between information rates in the two-terminal nodes exchanging messages through a base station or the relay. This example considers two nodes which wish to exchange messages with the help of a single relay: one can think of two mobile terminals exchanging messages through a base station or a relay.

Example 2: practical differences – designing transmission schemes that exploit the two-way data flow. This example considers two nodes which wish to exchange messages with the help of a single relay: one can think of two mobile terminals exchanging messages through a base station or a relay.

The lack of fully formalized or understood theory for two-way communication and its conceptual difference from one-way communication down to the requirement of different fundamental metrics motivate the study of a new, more general type of network information theory. This example considers two nodes which wish to exchange messages with the help of a single relay: one can think of two mobile terminals exchanging messages through a base station or a relay.

Unfortunately, the mutual information lack of fully formalized or understood theory for two-way communication and its conceptual difference from one-way communication down to the requirement of different fundamental metrics motivate the study of a new, more general type of network information theory. This example considers two nodes which wish to exchange messages with the help of a single relay: one can think of two mobile terminals exchanging messages through a base station or a relay.

Discrete memoryless networks, including the two-way channel, have played a crucial role in the development of capacity theorems for feedback channels and general two-way extensions of one-way protocols. However, due to the complex and directed information-based capacity characterizations have been difficult to evaluate expressions involved. Directed information-based capacity characterizations have been difficult to evaluate expressions involved. Directed information-based capacity characterizations have been difficult to evaluate expressions involved.

To capture the difference between information rates in the two-terminal nodes exchanging messages through a base station or the relay, this example considers two nodes which wish to exchange messages with the help of a single relay: one can think of two mobile terminals exchanging messages through a base station or a relay. This example studies the difference between information rates in the two-terminal nodes exchanging messages through a base station or the relay.

Motivational examples: dialogues rather than monologues.
Gaussian simulations

\[ h_{ar} = h_{br} = 1, \ h_{ab} = 0.2, \ N = 1, \text{ and } P = 50 \text{ dB}. \]
Recent developments: constant gap

- Capacity is known to within a constant # of bits in Gaussian noise regardless of channel parameters!


Recent developments: usefulness of lattice codes

Ideally...

Exploit own-message side information
Recent developments: usefulness of lattice codes

**Reality....**

\[ y_r = x_a(w_a) + x_b(w_b) + n_r \]

**Random coding**
- \( x_a(w_a) + x_b(w_b) \) **NOT** a codeword
- decode both messages
- send \( x_r(w_a \oplus w_b) \)

**Structured (lattice) coding**
- \( x_a(w_a) + x_b(w_b) \) **IS** a codeword
- no multiple access constraints as decode the sum
Recent developments: nested modulo lattice codes

Sum of codewords is a codeword - relay decodes it!!
Relation to network coding?

- bit-level / packet level network coding ➔ **Decode and Forward (DF)**

![Diagram showing network coding](image)

- excellent systems-level demonstration of 2-way relaying gains (all layers, actual testbed)

Relation to network coding?

- physical / analog network coding $\rightarrow$ similar to Amplify and Forward (AF)

- excellent systems-level demonstration of analog network coding (all layers, actual testbed)

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Open questions

- codeword adaptation?

• Potential to increase throughput, but will it be used beyond academic demonstrations?
Outline

- Information theory - what, why, when
  
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel

- Two-way channel - channel coding
  
  Adaptive codewords, capacity in Gaussian noise = two parallel channels

- Two-way relay channels

  - single flow - canonical example of wireless network coding

  - multiple flows with a base-station - pairwise wireless network coding
Motivation
Motivation
Multiple terminals: multiple two-way


Multiple terminals: with base-station


Multiple terminals: with base-station

Arbitrary (m) number of end users

Base-station = node 0


Half-duplex nodes

Decode + forward relay

Compress + forward end user cooperation

Per-flow network coding of messages at relay

"Protocols" = time "phases"

Phase 1 = MAC phase

Phase 2 = BC phase
Protocol 1: FMABC (Full MAC then BC)

Phase 1 = MAC phase

Phase 2 = BC phase

Node number

Time

Multiple-access period

Broadcast period

(a) FMABC protocol
Protocol 2: PMABC (Partial MAC then BC)

Phase 1

Phase 2

Phase 3

Phase 4 = BC phase

Node number

Time

0
1
2
m

Δ1 Δ2

Phase 1 Phase 2

Δm

Phase m

Δm+1

Phase m+1

Multiple-access period

Broadcast period

(b) PMABC protocol
Protocol 3: FTDBC (Full Time Division then BC)

Phase 1

Phase 2

Phase 3

Phase 4

Phase 5

(c) FTDBC protocol
Protocol 4: PTDBC (Partial Time Division then BC)

Phase 1

Phase 2

Phase 3

(d) PTDBC protocol
Which protocol is “better”?

1. Multi-message broadcasting (Marton’s region)

2. Per-flow network coding

3. Exploiting side-information (random binning)

4. CF-based terminal node cooperation
Broadcasting 2 messages
Broadcasting 2 messages

At what rates can we reliably broadcast 2 messages?
Marton’s region

\[ \begin{aligned} R_1 &\leq I(U_1; Y_1) \\ R_2 &\leq I(U_2; Y_2) \\ R_1 + R_2 &\leq I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2) \end{aligned} \]

over all joint distributions \( p(u_1, u_2, x) \)
Extended Marton’s in our notation

\[
Y_1 \quad (U_0, U_1, U_2) \rightarrow X
\]

\[
Y_2 \quad \text{Relay} \quad Y_0
\]

\[
R_{0,1} \leq I(U_1; Y_1)
\]
\[
R_{0,2} \leq I(U_2; Y_2)
\]
\[
R_{1,0} + R_{2,0} \leq I(U_0; Y_0)
\]
\[
R_{1,0} + R_{2,0} + R_{0,1} \leq I(U_0; Y_0) + I(U_1; Y_1) - I(U_0; U_1)
\]
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R_{1,0} + R_{2,0} + R_{0,2} \leq I(U_0; Y_0) + I(U_2; Y_2) - I(U_0; U_2)
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\[
R_{0,1} + R_{0,2} \leq I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2)
\]
\[
R_{1,0} + R_{2,0} + R_{0,1} + R_{0,2} \leq I(U_0; Y_0) + I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_0) - I(U_2; U_1, U_0)
\]

over all joint distributions \( p(u_0, u_1, u_2, x) \)
Use extended Marton’s region at:

Base station

End user

Relay
Which protocol is “better”?

1. Multi-message broadcasting (Marton’s region)
2. Per-flow network coding
3. Exploiting side-information (random binning)
4. CF-based terminal node cooperation
Per-flow Network coding (N)

\[ w_r = w_1 \oplus w_2 \]

\[ w_{r,1} = w_{1,0} \oplus w_{0,1} \]
\[ w_{r,2} = w_{2,0} \oplus w_{0,2} \]

\[ x_r(w_{r,1}, w_{r,2}) \]
Which protocol is “better”?

1. Extended Marton’s region for broadcasting
2. Per-flow network coding
3. Random-binning to exploit side-information
4. CF-based Terminal node cooperation
Random binning (R) for exploiting overheard information

\[ R_{1,0} \leq \Delta_1 I(X_1^{(1)}; Y_r^{(1)}) \]

\[ R_{1,0} \leq \Delta_1 I(X_1^{(1)}; Y_0^{(1)}) + \Delta_3 I(X_r^{(3)}; Y_0^{(3)}) \]

\[ R_{0,1} \leq \Delta_2 I(X_0^{(2)}; Y_r^{(2)}) \]

\[ R_{0,1} \leq \Delta_2 I(X_0^{(2)}; Y_1^{(2)}) + \Delta_3 I(X_r^{(3)}; Y_1^{(3)}) \]
Which protocol is “better”?

1. Extended Marton’s region for broadcasting
2. Per-flow network coding
3. Random-binning to exploit side-information
4. CF-based Terminal node cooperation
Cooperation (C) between terminal nodes

PMABC - NRC

<table>
<thead>
<tr>
<th>Transmit</th>
<th>Process</th>
<th>Slot k</th>
<th>Phase 3</th>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Slot k+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>r: ( x_r^{(3)}(w_r(k)) )</td>
<td>1: ( y_1^{(3)} \rightarrow \hat{y}<em>1^{(3)}(w</em>{1,2}(k)) )</td>
<td>0: ( x_0^{(1)}(w_{0,1}(k+1), w_{0,2}(k+1)) )</td>
<td>r: ( y_r^{(1)} \rightarrow \tilde{w}_{1,0}(k+1) )</td>
<td>1: ( y_1^{(2)} \rightarrow \tilde{w}_{2,1}(k) )</td>
<td>0: ( x_0^{(2)}(w_{0,1}(k+1), w_{0,2}(k+1)) )</td>
<td>2: ( x_0^{(2)}(w_{2,0}(k), w_{2,1}(k)) )</td>
</tr>
<tr>
<td>0: ( x_0^{(1)}(w_{0,1}(k+1), w_{0,2}(k+1)) )</td>
<td>2: ( y_2^{(3)} \rightarrow \hat{y}<em>2^{(3)}(w</em>{2,1}(k)) )</td>
<td>r: ( y_r^{(1)} \rightarrow \tilde{w}_{2,0}(k+1) )</td>
<td>( \ldots )</td>
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<td></td>
</tr>
<tr>
<td>1: ( x_1^{(1)}(w_{1,0}(k+1), w_{1,2}(k)) )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>2: ( x_2^{(2)}(w_{2,0}(k+1), w_{2,1}(k)) )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
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Cooperation (C) between terminal nodes

PMABC - NRC

Transmit

<table>
<thead>
<tr>
<th>slot k</th>
<th>phase 3</th>
<th>phase 1</th>
<th>phase 2</th>
<th>slot k+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 : (x_0^{(1)}(w_{0,1}(k+1),w_{0,2}(k+1)))</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 : (x_1^{(1)}(w_{1,0}(k+1),w_{1,2}(k)))</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>2 : (x_2^{(2)}(w_{2,0}(k+1),w_{2,1}(k)))</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>...</td>
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Process

<table>
<thead>
<tr>
<th>compress</th>
<th>decode</th>
<th>decode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 : (y_1^{(3)} \rightarrow \hat{y}<em>1^{(3)}(w</em>{1,2}(k)))</td>
<td>(r : y_r^{(1)} \rightarrow \tilde{w}_{1,0}(k+1))</td>
<td>(r : y_r^{(2)} \rightarrow \tilde{w}_{2,0}(k+1))</td>
</tr>
<tr>
<td>2 : (y_2^{(3)} \rightarrow \hat{y}<em>2^{(3)}(w</em>{2,1}(k)))</td>
<td>(y_r^{(1)}y_r^{(2)} \rightarrow \tilde{w}<em>{0,1}(k+1), \tilde{w}</em>{0,2}(k+1))</td>
<td>(y_r^{(1)}y_r^{(2)} \rightarrow \tilde{w}<em>{0,1}(k+1), \tilde{w}</em>{0,2}(k+1))</td>
</tr>
<tr>
<td></td>
<td>(y_2^{(3)} \hat{y}<em>1^{(3)}(\tilde{w}</em>{1,2}(k)) \rightarrow \tilde{w}_{0,2}(k))</td>
<td>(y_1^{(2)} \rightarrow \tilde{w}_{2,1}(k))</td>
</tr>
<tr>
<td></td>
<td>(y_1^{(2)} \hat{y}<em>2^{(3)}(\tilde{w}</em>{2,1}(k)) \rightarrow \tilde{w}_{0,1}(k))</td>
<td>(y_1^{(2)} \hat{y}<em>2^{(3)}(\tilde{w}</em>{2,1}(k)) \rightarrow \tilde{w}_{0,1}(k))</td>
</tr>
</tbody>
</table>
Cooperation (C) between terminal nodes

**PMABC - NRC**

- Slot k
- Phase 3
- Slot k+1
- Phase 1
- Phase 2

**Transmit**
- $r : x_r^{(3)}(w_{r(k)})$
- $0 : x_0^{(1)}(w_{0,1(k+1)}, w_{0,2(k+1)})$
- $1 : x_1^{(1)}(w_{1,0(k+1)}, w_{1,2(k)})$
- $2 : x_2^{(2)}(w_{2,0(k+1)}, w_{2,1(k)})$
- $\ldots$

**Compress**
- $1 : y_1^{(3)} \rightarrow \hat{y}_1^{(3)}(w_{1,2(k)})$
- $2 : y_2^{(3)} \rightarrow \hat{y}_2^{(3)}(w_{2,1(k)})$

**Decode**
- $r : y_r^{(1)} \rightarrow \tilde{w}_{1,0(k+1)}$
- $y_r^{(1)} : y_r^{(2)} \rightarrow \tilde{w}_{0,1(k+1)}, \tilde{w}_{0,2(k+1)}$
- $1 : y_1^{(2)} \rightarrow \tilde{w}_{2,1(k)}$
- $y_1^{(3)} : \hat{y}_1^{(3)}(\tilde{w}_{1,2(k)}) \rightarrow \tilde{w}_{0,2(k)}$
- $y_2^{(3)} : \hat{y}_2^{(3)}(\tilde{w}_{1,2(k)}) \rightarrow \tilde{w}_{0,2(k)}$
- $\tilde{w}_{2,1(k)}$
- $\tilde{w}_{0,1(k)}$
Cooperation (C) between terminal nodes

PMABC - NRC

<table>
<thead>
<tr>
<th>Transmit</th>
<th>Process</th>
<th>decode</th>
<th>decode</th>
</tr>
</thead>
<tbody>
<tr>
<td>r : $x_r$ $(w_r(k))$</td>
<td>1 : $y_1^{(3)}$ $\xrightarrow{}$ $\hat{y}<em>1^{(3)}$ $(w</em>{1,2}(k))$</td>
<td>$r : y_r^{(1)}$ $\xrightarrow{}$ $\tilde{w}_{1,0}(k+1)$</td>
<td>$r : y_r^{(2)}$ $\xrightarrow{}$ $\tilde{w}_{2,0}(k+1)$</td>
</tr>
<tr>
<td>0 : $x_0^{(1)}$ $(w_{0,1}(k+1), w_{0,2}(k+1))$</td>
<td>2 : $y_2^{(3)}$ $\xrightarrow{}$ $\hat{y}<em>2^{(3)}$ $(w</em>{2,1}(k))$</td>
<td>$x_1^{(1)}$ $(w_{1,0}(k+1), w_{1,2}(k))$</td>
<td>$y_2^{(1)}$ $\xrightarrow{}$ $\tilde{w}_{1,2}(k)$</td>
</tr>
<tr>
<td>1 : $x_1^{(1)}$ $(w_{1,0}(k+1), w_{1,2}(k))$</td>
<td>2 : $x_2^{(2)}$ $(w_{2,0}(k+1), w_{2,1}(k))$</td>
<td>$y_2^{(3)}$ $\xrightarrow{}$ $\tilde{w}_{1,2}(k)$</td>
<td>$y_1^{(2)}$ $\xrightarrow{}$ $\tilde{w}_{2,1}(k)$</td>
</tr>
<tr>
<td>2 : $x_2^{(2)}$ $(w_{2,0}(k+1), w_{2,1}(k))$</td>
<td></td>
<td>1 : $y_1^{(2)}$ $\xrightarrow{}$ $\tilde{w}_{2,1}(k)$</td>
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Cooperation (C) between terminal nodes

PMABC - NRC

Transmit

<table>
<thead>
<tr>
<th>r : $\mathbf{x}_r$</th>
<th>0 : $\mathbf{x}_0$</th>
<th>0 : $\mathbf{x}_0$</th>
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<tbody>
<tr>
<td>3 : $\mathbf{x}_r$</td>
<td>(1) $\mathbf{x}_0$</td>
<td>(2) $\mathbf{x}_0$</td>
</tr>
<tr>
<td>$(w_r(k))$</td>
<td>$(w_0,1(k+1),w_0,2(k+1))$</td>
<td>$(w_0,1(k+1),w_0,2(k+1))$</td>
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<tr>
<td>1 : $\mathbf{x}_1$</td>
<td>2 : $\mathbf{x}_2$</td>
<td>2 : $\mathbf{x}_2$</td>
</tr>
<tr>
<td>(1) $\mathbf{x}_1$</td>
<td>(2) $\mathbf{x}_2$</td>
<td>$(w_2,0(k+1),w_2,1(k))$</td>
</tr>
<tr>
<td>$(w_1,0(k+1),w_1,2(k))$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 : $\mathbf{x}_0$</td>
<td>0 : $\mathbf{x}_0$</td>
<td>2 : $\mathbf{x}_2$</td>
</tr>
<tr>
<td>1 : $\mathbf{x}_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) $\mathbf{x}_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(w_1,0(k+1),w_1,2(k))$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 : $\mathbf{x}_0$</td>
<td>2 : $\mathbf{x}_2$</td>
<td></td>
</tr>
<tr>
<td>(2) $\mathbf{x}_2$</td>
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<td></td>
</tr>
<tr>
<td>$(w_2,0(k+1),w_2,1(k))$</td>
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</table>

Process

<table>
<thead>
<tr>
<th>1 : $\mathbf{y}_1$</th>
<th>2 : $\mathbf{y}_2$</th>
<th>r : $\mathbf{y}_r$</th>
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</thead>
<tbody>
<tr>
<td>(3) $\mathbf{y}_1$</td>
<td>(3) $\mathbf{y}_2$</td>
<td>(1) $\mathbf{y}_r$</td>
</tr>
<tr>
<td>$(\tilde{w}_1,2(k))$</td>
<td>$(\tilde{w}_2,1(k))$</td>
<td>$(\tilde{w}_1,0(k+1))$</td>
</tr>
<tr>
<td>2 : $\mathbf{y}_2$</td>
<td>2 : $\mathbf{y}_2$</td>
<td>1 : $\mathbf{y}_1$</td>
</tr>
<tr>
<td>(1) $\mathbf{y}_2$</td>
<td>(2) $\mathbf{y}_1$</td>
<td></td>
</tr>
<tr>
<td>$(\tilde{w}_1,2(k))$</td>
<td>$(\tilde{w}_1,0(k+1))$</td>
<td></td>
</tr>
<tr>
<td>1 : $\mathbf{y}_1$</td>
<td>1 : $\mathbf{y}_1$</td>
<td>1 : $\mathbf{y}_1$</td>
</tr>
<tr>
<td>(2) $\mathbf{y}_1$</td>
<td>(3) $\mathbf{y}_2$</td>
<td></td>
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<tr>
<td>$(\tilde{w}_2,1(k))$</td>
<td>$(\tilde{w}_2,1(k))$</td>
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</table>

Decode

<table>
<thead>
<tr>
<th>r : $\mathbf{y}_r$</th>
<th>1 : $\mathbf{y}_1$</th>
<th>1 : $\mathbf{y}_1$</th>
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</thead>
<tbody>
<tr>
<td>(2) $\mathbf{y}_r$</td>
<td>(2) $\mathbf{y}_1$</td>
<td>(3) $\mathbf{y}_2$</td>
</tr>
<tr>
<td>$(\tilde{w}_2,0(k+1))$</td>
<td>$(\tilde{w}_1,0(k+1))$, $(\tilde{w}_0,2(k+1))$</td>
<td>$(\tilde{w}_2,1(k))$</td>
</tr>
<tr>
<td>1 : $\mathbf{y}_1$</td>
<td>1 : $\mathbf{y}_1$</td>
<td></td>
</tr>
<tr>
<td>(2) $\mathbf{y}_1$</td>
<td>(3) $\mathbf{y}_2$</td>
<td></td>
</tr>
<tr>
<td>$(\tilde{w}_1,0(k+1))$, $(\tilde{w}_0,2(k+1))$</td>
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<tr>
<td>(3) $\mathbf{y}_2$</td>
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<tr>
<td>$(\tilde{w}_2,1(k))$</td>
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</table>
### Table I
**Protocols and coding schemes**

<table>
<thead>
<tr>
<th>Protocol</th>
<th>Multiple Access</th>
<th>Marton’s Broadcast</th>
<th>Network coding</th>
<th>Random binning</th>
<th>User cooperation</th>
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<tbody>
<tr>
<td>Simplest</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
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<tr>
<td>FMABC</td>
<td>X</td>
<td>X</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>FMABC-N</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>PMABC</td>
<td>X</td>
<td>X</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>PMABC-NR</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>−</td>
</tr>
<tr>
<td>PMABC-NRC</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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</tr>
<tr>
<td>FTDBC</td>
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<td>X</td>
<td>−</td>
<td>−</td>
<td>−</td>
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<td>−</td>
</tr>
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<td>FTDBC-NRC</td>
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<td>X</td>
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<td>PTDBC</td>
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<td>−</td>
<td>−</td>
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<tr>
<td>PTDBC-NR</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>−</td>
</tr>
</tbody>
</table>

**N** = Network coding  
**R** = Random binning  
**C** = Cooperation between terminals
Outer bounds - half-duplex cut-set

FMABC

Phase 1 = MAC phase

Phase 2 = BC phase

\[ R_{0,1} + R_{0,2} \leq \Delta_1 I(X_0^{(1)}; Y_r^{(1)} | X_1^{(1)}, X_2^{(1)}) \]

\[ R_{1,0} + R_{2,0} \leq \Delta_2 I(X_r^{(2)}; Y_0^{(2)}) \]

\[ R_{1,0} \leq \Delta_1 I(X_1^{(1)}; Y_r^{(1)} | X_1^{(1)}, X_0^{(1)}) \]

\[ R_{2,0} \leq \Delta_1 I(X_2^{(1)}; Y_r^{(1)} | X_1^{(1)}, X_0^{(1)}) \]

\[ R_{1,0} + R_{2,0} \leq \Delta_1 I(X_1^{(1)}, X_2^{(1)}; Y_r^{(1)} | X_0^{(1)}) \]

\[ R_{0,1} \leq \Delta_2 I(X_r^{(2)}; Y_1^{(2)}) \]

\[ R_{0,2} \leq \Delta_2 I(X_r^{(2)}; Y_2^{(2)}) \]

\[ R_{0,1} + R_{0,2} \leq \Delta_2 I(X_r^{(2)}; Y_1^{(2)}, Y_2^{(2)}) \]
Simulations in Gaussian noise

\[ Y[k] = HX[k] + Z[k] \]

\[ H_1 = \begin{bmatrix}
0 & 0.3 & 0.05 & 1 \\
0.3 & 0 & 1.5 & 1 \\
0.05 & 1.5 & 0 & 0.2 \\
1 & 1 & 0.2 & 0
\end{bmatrix} \quad \text{and} \quad H_2 = \begin{bmatrix}
0 & 0.9 & 0.4 & 1 \\
0 & 0 & 0.02 & 1 \\
0 & 0.02 & 0 & 0.5 \\
1 & 1 & 0.5 & 0
\end{bmatrix}. \]
Simulations in Gaussian noise

\[ Y[k] = HX[k] + Z[k] \]

\[
H_1 = \begin{bmatrix}
0 & 0.3 & 0.05 & 1 \\
0.3 & 0 & 1.5 & 1 \\
0.05 & 1.5 & 0 & 0.2 \\
1 & 1 & 0.2 & 0
\end{bmatrix} \quad \quad H_2 = \begin{bmatrix}
0 & 0.9 & 0.4 & 1 \\
0 & 0 & 0.02 & 1 \\
0 & 0.02 & 0 & 0.5 \\
1 & 1 & 0.5 & 0
\end{bmatrix}.
\]

Evaluate expressions assuming Gaussian input distributions and optimize over:

- phase durations
- correlation matrices of Marton binning RVs subject to power constraints
- compression parameters
Simple - MB - MB+NR
Network coding + random binning

**Fig. 7.** Comparison with $P_0 = P_1 = P_2 = P_r = 0$ dB, $H = H_1$.

**Fig. 8.** Comparison with $P_0 = P_1 = P_2 = P_r = 0$ dB, $H = H_2$.

**Fig. 9.** Comparison with $P_0 = P_r = 20$, $P_1 = P_2 = 0$ dB, $H = H_1$.

**Fig. 10.** Comparison with $P_0 = P_1 = P_2 = P_r = 20$ dB, $H = H_1$. 

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**C. Cooperation coding gain**

To show the cooperation coding gain, we plot the achievable rate region of the different protocols with and without cooperation. In Fig. 12, we fixed the data rates $(R_{0}, R_{1}, R_{2}, R_{0})$ to the rate pair $(0.19, 0.01)$ and plot rate regions in the $(R_{1}, R_{0}, R_{0}, R_{2})$ domain. We do this to highlight the cooperation gain, which comes from re-allocating node 1's transmission resources (i.e. relative power) to the two information flows; $1 \rightarrow r(R_{1}, 0)$ and $1 \rightarrow 2(R_{0}, 2)$. As expected, -NRC protocols achieve much better performance than -NR protocols. Notably, the cooperation protocols improve $R_{0}, R_{2}$ without any degradation of $R_{1}, R_{0}$.
Lemma 21: For a given subset $S \subseteq B$, $|S| > 1$, we define $w = \{w_i, i \in S\}$, $w_0 = \{w_i^0 | i \in S\}$, $u(w_0) = \{u_i(w_i^0) | i \in S\}$, $U = \{U_i | i \in S\}$ and the set $D_w := \{u(w_0) \in A(U) | w_0 \in \bigotimes_{i \in S} B_i w_i\}$.

Then for any choice of $w, \epsilon > 0$ and sufficiently large $n$:

$$P[\|D_w\| = 0] \leq \epsilon$$

with

$$\sum_{i \in S} R_i < \sum_{i \in S} (I(U_i; Y_i) - I(U_i; U_S(i)) - |S| \epsilon - \delta(\epsilon))$$

where $\delta(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$.

Proof: We use the similar proofs to Lemma in [9]. From Chebychev's inequality, we have

$$P[\|D_w\| - E[\|D_w\|] > \epsilon E[\|D_w\|]] \leq \sigma[\|D_w\|]^2 \epsilon^2$$

and

$$P[u(w_0) \in D_w] \geq 2^{-n}(H(U) - \sum_{i \in S} H(U_i) - \delta(\epsilon))$$

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Multi-flow take-away points

- Most schemes use *per-flow network coding*

\[ w_r = w_1 \oplus w_2 \]

\[ w_{r,1} = w_{1,0} \oplus w_{0,1} \]
\[ w_{r,2} = w_{2,0} \oplus w_{0,2} \]
\[ x_r(w_{r,1};w_{r,2}) \]

**One flow**

**Multiple flows**
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- Significantly more complex: protocols and opportunities abound. Only starting to understand when to do what.

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- Due to practical relevance - crucial to develop insight and thereafter demonstrations (there are none)
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- Two-way nature must be explicitly accounted for (side-information, ability to network code and broadcast) in order to see gains.
Future areas of two-way channels

- one-way information theory “fairly” well understood
- advances in processing power
- never ending desire for bandwidth and limited wireless spectrum

Two-way wireless networks

When is two-way processing needed?
Questions?

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