A lattice Compress-and-Forward strategy for canceling known interference in Gaussian multi-hop channels

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***Abstract*—We present a nested-lattice encoding and decoding strategy for Compress-and-Forward (CF) relaying. This complements previous work on a nested-lattice encoding and decoding scheme for Decode-and-Forward (DF) relaying, and provides an alternative to random codes for CF relaying which may be a useful coding strategy for larger networks. The proposed nested-lattice CF scheme's utility is demonstrated by using it to cancel interference in a two-hop Gaussian network with a source, a relay and a destination, in which additive interference experienced at the relay and known at the destination (but not the source). The proposed scheme achieves the same rate as a CF scheme without interference, and to within 1/2 bit from the "clean" channel outer bound. We further illustrate this scheme's power by discussing extensions to multi-hop networks in which one or more interference terms are known by receivers "down-the-line."**

I. INTRODUCTION

Lattices are examples of structured, as opposed to random, codes which may achieve the capacity of Additive White Gaussian Noise (AWGN) channels [1]. They may also be used in achieving the capacity of Gaussian channels with interference or state known at the transmitter (but not receiver) [2] using a lattice equivalent [3] of dirty-paper coding (DPC) [4]. In terms of relay channels, lattice codes were recently shown to achieve the decode-and-forward rates of the AWGN relay channel through the application of a recently developed lattice list decoding technique [5][1].

In this work we seek to extend the use of lattices to compress-and-forward relaying, and both motivate and illustrate the power of such a lattice-based CF scheme by looking at a particular two-hop Gaussian channel with a source, relay, and destination without a direct link between the source and destination, with additive interference at the relay which is known to the receiver but not the transmitter (see Fig. 1). This channel model is motivated in channels with cognition, or non-causal side-information of messages at certain transmitters/receivers. As a particular example (though we will abstract this away), consider the multi-hop / line network in which two messages are being transmitted: one from Node 1 to 3 through Node 2 (relay) and another from Node 3 to Node 4 further down the line. If Nodes 1 and 3 transmit simultaneously, the relay Node 2 may see interference from Node 3; but this "interference" is known at Node 3 in the next hop when it attempts to decode Node 1's message. In this scenario, simple dirty-paper coding techniques may not be immediately employed at the relay due to the presence of the potentially large amount of interference seen in decoding Node 1's message (which in turn is needed for a DPC re-encoding) at the relay. Thus, alternative schemes which in some way allow the interference to be forwarded and canceled by the receiver are needed.

[1]The term "lattice list decoding" is used loosely and in this context does not refer to methods for the efficient (computationally) decoding of lattice points.

A. Past work

The nested lattice approach of [3] for the dirty-paper channel is extended to dirty-paper networks in [6], [7], [8], [9], where in some scenarios lattice codes are interestingly shown to outperform random codes. The most similar work to the considered two-hop relay network with interference at the relay, known at the receiver (Fig. 1) is the work of [7] which considers a relay channel with state non-causally available at the *transmitter, relay, or both,* and derives CF-based achievable rates for discrete memoryless channels. In [8] a three terminal relay channel is again considered where the state is known *only at the source,* where upper and lower bounds are derived in Gaussian noise. Our channel model differs in that 1) we consider a two-hop network, and there is no direct link between transmitters and receivers as in the relay channel, and 2) the state is only known at the receiver rather than the transmitter and/or relay.

We seek to develop a Compress-and-Forward (CF) [10] based lattice encoding and decoding strategy, which differs from the DF-based lattice generally used in networks. In Compress-and-Forward schemes, the relay does not decode the exact message, but rather compresses the received signal and sends the compression index. This compression index includes some useful information about the original message, and does not impose decoding rates at the relay(s). In the classic three node relay channel, Compress-and-Forward may be viewed as a Wyner-Ziv coding problem [11] with a noisy link. In [12], [3] the authors describe a lattice version of noiseless Wyner-Ziv coding, where lattice codes quantize/compress the continuous signal; this will form the basis for our proposed lattice-based noisy CF strategy. We note that while in some channel models, structured codes such as lattices may outperform, in terms of achievable rates, classical random codes, that this is not the case for this channel model – i.e. the nested-lattice code-based CF strategy here and the analogous random coding based one achieve identical rates. We are nonetheless motivated to propose a nested-lattice CF scheme as an alternative to the random coding CF scheme as it will provide a building block for nested-lattice coding schemes for larger networks, in which the structure may play a crucial role.

B. Contributions

The main contributions of this paper are:

1) The development of a nested-lattice-based CF scheme which combines source coding lattices (for quantization) and channel coding lattices (for achieving AWGN capacity).

2) The application of the developed nested-lattice-based CF scheme to cancel interference at the relay which is known at the receiver but not the transmitter. We show that the rate achieved is that of a CF scheme without interference, and achieves to within 1/2 bit of a "clean" channel outer bound.

3) The discussion and application of this scheme to multi-hop networks in which multiple interference terms seen at relays are known by other relays or destinations "down-the-line".

C. Outline

In Section II we outline the two-hop channel model considered and introduce our notation. In Section III we present the nested-lattice-based CF scheme and show that it achieves the rate of a CF scheme without interference, and to within 1/2 bit of an outer bound. In Section IV we discuss extensions of the proposed nested-lattice based CF scheme to multiple hops and multiple interference terms, and conclude in Section V.

II. Channel model and notation

A. Channel model

We consider a two-hop AWGN channel where Node 1 wants to communicate with Node 3 through a relay Node 2. The channel inputs X_1, X_2 at Nodes 1 and 2, may be related to the channel outputs Y_2, Y_3 at Nodes 2 and 3 through

$$Y_2 = X_1 + S + Z_2 \tag{1}$$
$$Y_3 = X_2 + Z_3 \tag{2}$$

where S is a random variable modeling interference which is known/detected by Node 3, and Z_2 and Z_3 are i.i.d white Gaussian noises with variance N_2 and N_3. Both Node 1 and 2 are subject to transmit power constraints of P.

We ask: Can we cancel the additive interference S and obtain a clean (interference-free, or $S = 0$) two-hop channel? If not, can we achieve a rate close to capacity using a scheme which is independent of S? We will answer these by deriving a lattice-based Compress-and-Forward scheme which achieves at most 1/2 bit to the clean two-hop channel outer-bound. Interestingly, the achievable rate derived is that of a Compress-and-Forward scheme in a *clean* two-hop relay channel.

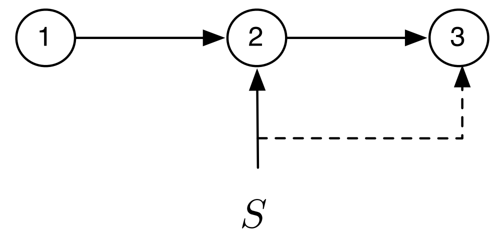

Fig. 1. Canceling known interference in a two-hop network. Solid lines indicate interference, dotted lines indicate its non-causal knowledge.

B. Notation and nested lattice coding preliminaries

Our notation and brief introduction to the needed concepts of nested lattice codes for transmission over AWGN channels follows that of [13], [14], [5]; more thorough treatments are found in [15], [13], [16] and in particular [17].

An n-dimensional lattice Λ is a discrete subgroup of Euclidean space $\mathbb{R}^n$ (of vectors $\mathbf{x}$, though we will denote these without the bold font as x) with Euclidean norm $||\cdot||$ under vector addition. We may define

- The *nearest neighbor lattice quantizer* of Λ as
$$Q_\Lambda(x) = \arg\min_{\lambda\in\Lambda} ||x - \lambda||;$$
- The $\texttt{mod}\ \Lambda$ operation as $x \texttt{ mod } \Lambda := x - Q_\Lambda(x)$, hence
$$x = Q_\Lambda(x) + (x \mod \Lambda);$$
- The *fundamental region of* Λ as the set of all points closer to the origin than to any other lattice point
$$\mathcal{V}(\Lambda) := \{x : Q(x) = \mathbf{0}\}$$
which is of volume $V := \text{Vol}(\mathcal{V}(\Lambda))$.
- The *second moment per dimension of a uniform distribution over* $\mathcal{V}$ as
$$\sigma^2(\Lambda) := \frac{1}{V}\cdot\frac{1}{n}\int_{\mathcal{V}} ||x||^2\, dx$$
- The *crypto lemma* [18]: which states that $(\mathbf{x}+\mathbf{U}) \mod \Lambda$ (where $\mathbf{U}$ is uniformly distributed over $\mathcal{V}$) is an independent random variable uniformly distributed over $\mathcal{V}$.

Standard definitions of *Polytrev good* and *Rogers good* lattices are used [1], [19], and by [19] we are assured the existence of lattices which are both Polytrev and Rogers good, which may intuitively be thought of as being good channel and source codes, respectively.

C. Nested lattice codes

The proposed CF scheme will be based on *nested lattice codes*. To define these, consider two lattices Λ and Λ_c such that $\Lambda \subseteq \Lambda_c$ with fundamental regions $\mathcal{V}, \mathcal{V}_c$ of volumes V, V_c respectively. Here Λ is called the *coarse* lattice which is a sublattice of Λ_c, the *fine* lattice. The set $\mathcal{C}_{\Lambda_c,\mathcal{V}} = \{\Lambda_c \cap \mathcal{V}\}$ may be employed as the codebook for transmission over the AWGN channel, with coding rate R defined as

$$R = \frac{1}{n}\log|\mathcal{C}_{\Lambda_c,\mathcal{V}}| = \frac{1}{n}\log\frac{V}{V_c}.$$

Here $\rho = |\mathcal{C}_{\Lambda_c,\mathcal{V}}|^{\frac{1}{n}} = \left(\frac{V}{V_c}\right)^{\frac{1}{n}}$ is the nesting ratio of this *nested* (Λ, Λ_c) *lattice code*. Nested lattice codes were shown to be capacity achieving (as $n \to \infty$) for the AWGN channel [16].

III. Achievability scheme

We now present a nested-lattice-based CF scheme for the channel model considered in Fig. 1 which consists of a two-hop network in which the interference experienced at the relay is known to the receiver but not the transmitter.

Theorem 1: Using a nested-lattice based CF scheme in the two-hop Gaussian relay channel described by in Section II-A with interference S known at the receiver but not the relay or transmitter, we may achieve rates R which satisfy

$$R < \frac{1}{2}\log\left(1 + \frac{P^2}{PN_2 + PN_3 + N_2N_3}\right).$$

Proof: **Encoding at transmitter (Node 1):** let $\Lambda \subset \Lambda_c$ be a good lattice pair, i.e. Λ is both *Rogers good* and *Poltyrev good* and Λ_c is *Poltyrev good*, where the second moment of Λ is $\sigma^2(\Lambda) = P$. Such lattices are the same as used in [1]. We note that any coding rate R may be achieved. For a given blocklength n, the messages $w \in \{1, 2, \cdots, 2^{nR}\}$ are one-to-one mapped to the fine lattice codewords $t \in \{\Lambda_c \cap \mathcal{V}(\Lambda)\}$. Node 1 transmits

$$X_1 = (t + U) \mod \Lambda,$$

where U is a dither random variable uniformly distributed over $\mathcal{V}(\Lambda)$ and known by the destination (Node 3). Also notice X_1 is uniformly distributed over $\mathcal{V}(\Lambda)$ and independent of t by *crypto lemma*.

Compressing at the relay (Node 2): the relay receives

$$Y_2 = X_1 + S + Z_2$$

and does *not* attempt to decode the message but instead compresses the received signal and transmits its compression index. We note that the proposed scheme is most useful when the power/variance of S is large; if it is small treating it as noise and employing a DF-based scheme and decoding the message may be superior. Compress-and-Forward may be viewed as the Wyner-Ziv coding problem but with a noisy transmission link. Thus we mimic the lattice version of Wyner-Ziv coding [12] , [3].

Let $\Lambda_1 \subset \Lambda_q$ be a good nested-lattice pair: Λ_1 is *Rogers-good* and Λ_q is both *Poltyrev good* and *Rogers-good* (guaranteed to exist with arbitrary nesting ratio [20]) with $\sigma^2(\Lambda_q) = D$ and $\sigma^2(\Lambda_1) = P + N_2 + D$. The relay compresses the received signal as

$$\begin{aligned} t_q &= Q_{\Lambda_q}(Y_2 + U_q) \mod \Lambda_1 \\ &= Q_{\Lambda_q}(X_1 + Z_2 + S + U_q) \mod \Lambda_1, \end{aligned}$$

where U_q is uniformly distributed over $\mathcal{V}(\Lambda_q)$. The source coding rate/compression rate is

$$R_q = \frac{1}{2}\log\left(\frac{\sigma^2(\Lambda_1)}{\sigma^2(\Lambda_q)}\right) = \frac{1}{2}\log\left(1 + \frac{P + N_2}{D}\right).$$

We note that our compression step is not exactly the same as in [12], [3] – we choose the MMSE coefficient α in their work to be 1, i.e. no MMSE estimation is used in the compression step. We do this to ensure that the compression error is independent of the received signal and the transmitted codewords. This allows us to treat the compression error/noise as independent noise in decoding the codewords in the next step (at Node 3). This setting of $\alpha = 1$ (and guaranteeing independence of compression noise and codewords) comes at the expense of a compression rate that is slightly larger than the optimal Wyner-Ziv coding rate in [12], [3].

Encoding at the relay (Node 2) and decoding at the destination (Node 3): The relay treats t_q as its message and sends it with a capacity-achieving lattice code [1]. The destination may decode t_q if the source coding rate of t_q is less than the channel capacity from Node 2 to Node 3, i.e.

$$R_q = \frac{1}{2}\log\left(1 + \frac{P + N_2}{D}\right) < \frac{1}{2}\log\left(1 + \frac{P}{N_3}\right). \quad (3)$$

The destination (Node 3) then de-compresses t_q as:

$$\begin{aligned} Y' &= (t_q - U_q - S) \mod \Lambda_1 \\ &= (Q_{\Lambda_q}(Y_2 + U_q) \mod \Lambda_1 - U_q - S) \mod \Lambda_1 \\ &= (Q_{\Lambda_q}(Y_2 + U_q) - U_q - S) \mod \Lambda_1 \\ &= (X_1 + Z_2 + S + U_q \\ &\quad - (X_1 + Z_2 + S + U_q) \mod \Lambda_q - U_q - S) \mod \Lambda_1 \\ &= (X_1 + Z_2 - E_q) \mod \Lambda_1 \\ &= X_1 + Z_2 - E_q, \end{aligned}$$

where $E_q = (X_1 + Z_2 + S + U_q) \mod \Lambda_q$ is an independent random variable uniformly distributed over $\mathcal{V}(\Lambda_q)$ with variance D. The last step follows as $X_1 + Z_2 - E_q$ is within $\mathcal{V}(\Lambda_1)$ with high probability as in 4.15 in [3] since Λ_1 is *Poltyrev good*. Finally, the destination obtains a signal $Y' = X_1 + Z_2 - E_q$, where Z_2 is Gaussian noise with variance N_2, and E_q approaches an independent Gaussian random variable with variance D when $n \to \infty$. Thus, $Y' = X_1 + Z_2 - E_q$ forms an equivalent AWGN channel from the transmitter to the receiver, and thus proceeds to decode t in the usual nested-lattice fashion employing an MMSE coefficient α, as:

$$\begin{aligned} (\alpha Y' - U) &= (t + U_1 - (1-\alpha)X_1 + \alpha(Z_2 - E_q) - U) \mod \Lambda \\ &= (t - (1-\alpha)X_1 + \alpha(Z_2 - E_q)) \mod \Lambda. \end{aligned}$$

Choosing α to be the MMSE coefficient: $\alpha = \alpha_{opt} = \frac{P}{P+N_2+D}$ and treating E_q in the same way to the self-noise X_1 (the error probability analysis is similar as [1]) , the destination may decode t at rate

$$R < \frac{1}{2}\log\left(1 + \frac{P}{D + N_2}\right). \quad (4)$$

Combining (3) and (4), we may achieve all rates

$$R < R_a = \frac{1}{2}\log\left(1 + \frac{P^2}{PN_2 + PN_3 + N_2N_3}\right).$$

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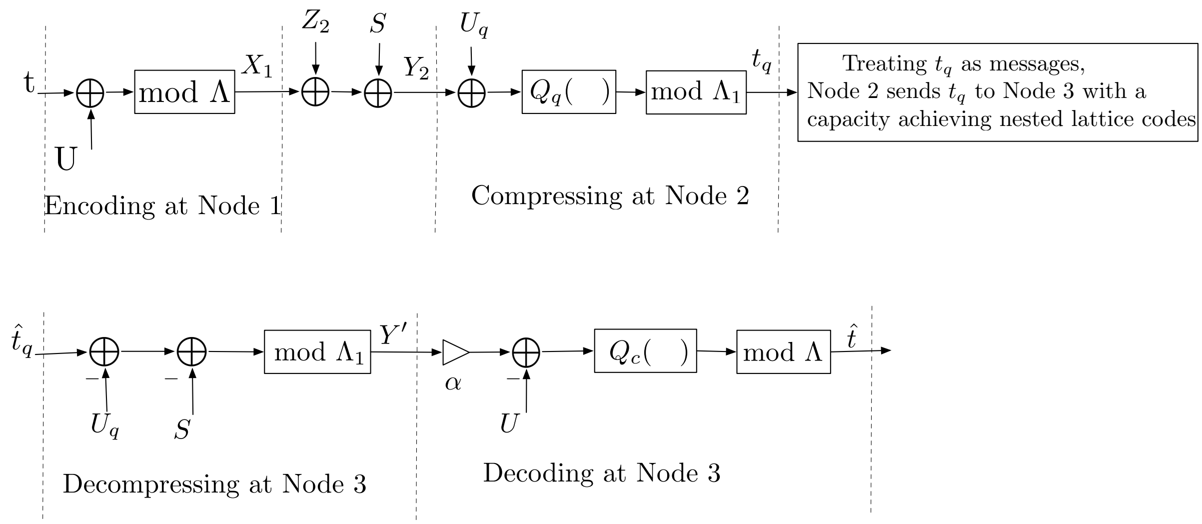

Fig. 2. Nested-lattice encoding and decoding scheme of Theorem 1.

Performance: Clean channel CF capacity. We notice that the rates achieved in Theorem 1 are equal to those achieved in a two-hop interference-free ($S = 0$) Gaussian channel. Thus, the proposed scheme is able to "eliminate" arbitrarily large amounts of interference seen at the relay if it knows this interference at the destination, and is further able to do so using lattice codes. This provides us with another building block for the use of lattices in wireless networks.

Performance: and 1/2 bit gap to outer bound. The rate of Theorem 1 furthermore lies to within 1/2 bit of the "clean" outer bound $\frac{1}{2}\log\left(1+\frac{P}{\max(N_2,N_3)}\right)$ which consists of the concatenation of two interference-free channels (with re-encoding at the relay). This follows from

$$
\begin{aligned}
R_a + \frac{1}{2} &= \frac{1}{2}\log\left(2 + \frac{2P^2}{PN_2 + PN_3 + N_2N_3}\right) \\
&= \frac{1}{2}\log\left(1 + \frac{P}{\max(N_2,N_3)}\right. \\
&\quad \left. * \frac{2P + N_2 + N_3 + \frac{N_2N_3}{P}}{P\frac{N_2}{\max(N_2,N_3)} + P\frac{N_3}{\max(N_2,N_3)} + \frac{N_2N_3}{\max(N_2,N_3)}}\right) \\
&> \frac{1}{2}\log\left(1 + \frac{P}{\max(N_2,N_3)}\right).
\end{aligned}
$$

IV. Generalization to multiple relays and multiple interferences

We now illustrate how the lattice-based CF scheme for channel state known only at the receiver may be extended to multiple relays and even multiple different interferences.

Multiple relays, one interference. We first consider a Gaussian channel model (Fig. 3(a)) in which there may be multiple relays between the one which suffers from interference, and node with knowledge of this interference. We may still use the proposed CF scheme to eliminate interference as long as the links between the node which suffers from interference and the destination are strong enough to transmit the compression indexes. If we assume all these links in the multi-hop channel are the same and characterized by the SNR P for simplicity, then as long as

$$\frac{1}{2}\log\left(1 + \frac{P+1}{D}\right) < \frac{1}{2}\log\left(1+P\right),$$

the achievable rate is given by

$$R < \frac{1}{2}\log\left(1 + \frac{P}{1+D}\right) = \frac{1}{2}\log\left(1 + \frac{P^2}{2P+1}\right),$$

which is again 1/2 bit from the "clean" outer-bound.

Multiple interference terms at a single node. If multiple additive interference terms are experienced at one node and the nodes down the line each only know one of these interferences (or have partial state information, as shown Fig. 3(b)), a more complex CF scheme is needed. Assuming all the links have the same SNR P, the first relay compresses/quantizes the received as in Section III and sends the compression index. The second relay first de-compresses the received index and removes the interference S_1, then it compresses the remaining signal and sends the compression index. Finally, the destination de-compresses the received index, removes the interference S_2 and decodes the message from the remaining signal. Thus, there are two independent compression error E_{q1} and E_{q2} with variances D_1 and D_2, subject to the constraints

$$
\begin{aligned}
\frac{1}{2}\log\left(1 + \frac{P+1}{D_1}\right) &< \frac{1}{2}\log\left(1+P\right) \\
\frac{1}{2}\log\left(1 + \frac{P+1+D_1}{D_2}\right) &< \frac{1}{2}\log\left(1+P\right).
\end{aligned}
$$

Under these constraints, the achievable rate is

$$R < \frac{1}{2}\log\left(1 + \frac{P}{D_1 + D_2 + 1}\right),$$

i.e.

$$R < \frac{1}{2}\log\left(1 + \frac{P^3}{3P^2 + 3P + 1}\right)$$

which is at most $\frac{1}{2}\log(3)$ bit away from the outer bound.

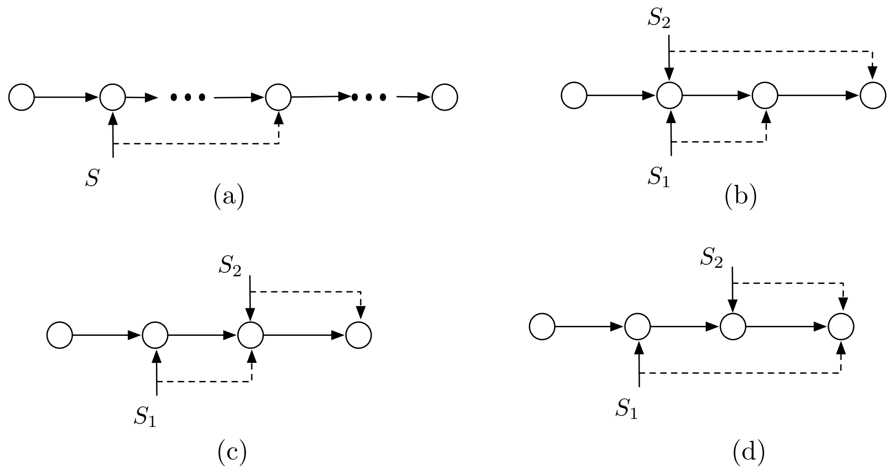

Fig. 3. Extensions to multi-relay and multi-interference channels with channel state known at the receiver.

Different interference terms at different nodes. In Fig. 3(c) and (d), where multiple nodes are corrupted by different interference terms, the compression indices are not transmitted over clean links. In these cases, the simple presented CF scheme may not be immediately applied; however, nested-lattice DF schemes along the lines of [21] may be expected to cancel "most" of the interference terms. Intuitively, in this scheme, the lattice code *structure* may be useful in canceling the interference terms – the DF scheme may decode the sum of the original codeword and the quantization of the known interference (which would furthermore be perfect if this interference were itself structured), while treating the residue/error of quantization as noise. The node(s) with the knowledge of the interference may recover the original codewords by subtracting the quantized interference from the sum.

V. Conclusion

We presented a nested-lattice-based scheme for CF relaying to cancel arbitrarily large interference experienced at the relay and known only to the destination. The scheme presented achieves at most 1/2 bit to the capacity of this channel and achieves the same rate as the interference-free CF scheme. We pointed out generalizations of this scheme to the cases with multiple relays and multiple interference terms. An alternative to the Compress-and-Forward lattice scheme presented here is the nested-lattice code based Decode-and-Forward (DF) [21], which, in contrast to the CF scheme explicitly relies of the structure of lattice codes.

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References

[1] U. Erez and R. Zamir, "Achieving $\frac{1}{2}\log(1+SNR)$ on the AWGN channel with lattice encoding and decoding," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2293–2314, Oct. 2004.

[2] S. Gel'fand and M. Pinsker, "Coding for channels with random parameters," *Probl. Contr. and Inf. Theory*, vol. 9, no. 1, pp. 19–31, 1980.

[3] R. Zamir, S. Shamai, and U. Erez, "Nested Linear/Lattice codes for structured multiterminal binning," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1250 – 1276, Jun. 2002.

[4] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. IT-29, pp. 439–441, May 1983.

[5] Y. Song and N. Devroye, "List decoding for nested lattices and applications to relay channels," in *Proc. Allerton Conf. Commun., Control and Comp.*, Sep. 2010.

[6] Y. H. Kim, A. Sutivong, and S. Sigurjosson, "Multiple user writing on dirty paper," in *Proc. IEEE Int. Symp. Inf. Theory*, 2004.

[7] B. Akhbari, M. Mirmohseni, and M. R. Aref, "Compress-and-Forward strategy for the relay channel with non-causal state information," in *Proc. IEEE Int. Symp. Inf. Theory*, 2009.

[8] A. Zaidi, S. Shamai, P. Piantanida, and L. Vandendorpe, "Bounds on the capacity of the relay channel with noncausal state information at the source," in *Proc. IEEE Int. Symp. Inf. Theory*, 2010.

[9] T. Philosof, R. Zamir, and A. Khisti, "Lattice strategies for the dirty multiple access channel," Submitted to *IEEE Trans. Inf. Theory*, 2009.

[10] T. M. Cover and A. El Gamal, "Capacity theorems for relay channels," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572 – 584, Sep. 1979.

[11] A. Wyner and J. Ziv, "The rate-distortion function for source coding with side information at the decoder," *IEEE Trans. Inf. Theory*, vol. 22, no. 1, pp. 1–10, 1976.

[12] R. Zamir and S. Shamai, "Nested linear/lattice codes for Wyner-Ziv encoding," in *IEEE Inf. Theory Workshop*, 1998.

[13] R. Zamir, S. Shamai, and U. Erez, "Nested linear/lattice codes for structured multiterminal binning," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1250–1276, 2002.

[14] W. Nam, S.-Y. Chung, and Y. Lee, "Nested lattice codes for gaussian relay networks with interference," 2009. [Online]. Available: http://arxiv.org/PS cache/arxiv/pdf/0902/0902.2436v1.pdf

[15] H. Loeliger, "Averaging bounds for lattices and linear codes," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1767–1773, 1997.

[16] U. Erez and R. Zamir, "Achieving $\frac{1}{2}\log(1+SNR)$ on the AWGN channel with lattice encoding and decoding," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2293–2314, Oct. 2004.

[17] R. Zamir, "Lattices are everywhere," in *4th Annual Workshop on Information Theory and its Applications, UCSD*, 2009.

[18] G. D. Forney Jr., "On the role of MMSE estimation in approaching the information theoretic limits of linear Gaussian channels: Shannon meets Wiener," in *Proc. Allerton Conf. Commun., Control and Comp.*, 2003.

[19] U. Erez, S. Litsyn, and R. Zamir, "Lattices which are good for (almost) everything," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3401–3416, 2005.

[20] D. Krithivasan and S. S. Pradhan, "A proof of the existence of good nested lattices," in *www.eecs.umich.edu/techreports/systems/cspl/cspl-384.pdf*, 2007.

[21] Y. Song and N. Devroye, "Structured interference-mitigation in two-hop networks," in *Information Theory and Applications Workshop*, La Jolla, Feb. 2011.