Survey of MMSE Channel Equalizers

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Abstract
This paper surveys the recent developments in the area of equalization of the wireless propagation channels. It focuses on a subclass of equalization algorithms that minimizes the Mean Square Error (MMSE) between the desired and estimated signal. The following aspects of the equalization problem are considered: system modeling, algorithm classification based on the availability of training sequence and matrix inversion method employed. Various practical application scenarios are also covered.

I. INTRODUCTION

A generic communication system model is given in Fig. 1(a). Where \( s(t) \), is the data sequence to be transmitted, \( w(t) \) is AWGN, and \( y(t) \) is the signal at the output of the receiver filters. The simplified equivalent model is provided in Fig. 1(b), where \( n(t) = w(t) + g_R(t) \) is the noise at the output of the receiver filter. The transmitter filter, propagation channel, and the receiver filters are represented by the composite channel with transfer function

\[
H(f) = G_T(f)G_{ch}(f)G_R(f). \tag{1}
\]

As a result, the problem becomes that of deconvolution.

From here on, we shall refer to the composite channel model as simply channel. According to Fig. 1, the transmitted signal undergoes amplitude and phase distortion given by \( |H(f)| \) and \( \theta(f) \), respectively. The goal of the receiver is to estimate the transmitted signal \( s(t) \) based on the distorted noisy signal \( y(t) \). This can be facilitated by applying the received signal to the filter with the transfer function equal to the inverse of the transfer function of the channel, \( H^{-1}(f) \). The algorithms performing estimation of \( H(f) \) and applying its inverse to the received signal, \( y(t) \), fall under the category of channel equalizers. The scope of this survey is limited to subclass of channel equalization algorithms that minimize the mean square error between the transmitted signal \( s(t) \) and its estimate \( \hat{s}(t) \):

\[
\hat{s}(t) = \arg \min_{\hat{s}(t)} \mathbb{E} \left[ (s(t) - \hat{s}(t))^2 \right] \tag{2}
\]
In its sampled form the model of Fig. 1(b) may be represented as Bayesian linear model [11]:

\[ y = Hs + n \]  \hspace{1cm} (3)

If we assume that \( s(t) \) is a Gaussian process, then the sampled signal \( s[n] \) becomes discrete-time Gaussian process and the LMMSE estimator is given by [11]

\[ \hat{s} = C_s H^T (H C_s H^T + \sigma_n^2 I)^{-1} y \]  \hspace{1cm} (4)

where the matrix \( A = C_s H^T (H C_s H^T + \sigma_n^2 I)^{-1} \) represents the Wiener filter.

II. SURVEY OF THE MAJOR TYPES OF MMSE EQUALIZER ALGORITHMS

In this sections, the MMSE equalizers are classified according to algorithm type.

A. Algorithms Using Training (desired) Sequence

The use of training sequence in channel equalization allows to simplify receiver architecture, while improving the reliability of the data reception. This comes at the expense of reduced maximum data rates, since part of the bandwidth is must be allocated to the training sequence. Most cellular communication standards, incorporate some kind of training signal. In the case of CDMA-based systems this role is played by the code division multiplexed pilot sequence. It has been shown that the pilot PN sequence can be used as a desired signal for chip-level equalization [7], [21]. In the case of GSM, the time-multiplexed midamble is used for the same purpose. In the context of Wiener equation such training sequence is typically used as the desired signal. That is the training sequence is used to estimate the impulse response of the channel. The distortion introduced by the channel is then corrected by convolving the received signal with the inverse of the estimated impulse response. In time-varying channels such as those encountered in wireless communications, the channel estimates must be updated fast enough to track the changes in the channel transfer function.

B. Blind Adaptive Methods

The blind channel equalization techniques are characterized by the fact that the channel estimation is based solely on the received signal with no access to the transmitted signal. The blind estimation techniques find application in the scenarios where the training sequence is not possible or where significant advantage can be gained by reducing overhead associated with training sequence. Wireless transmission over time varying channel is a typical application. Combination of blind and training sequence-based methods has been explored by [2]. There, it has been assumed that part of the transmitted sequence is known by the receiver, and the rest is unknown. The blind estimation techniques exploit various properties of the channel and the input signal. One such example would be input signal with known statistical properties such as distribution or moments. The general model of the blind estimation scenario is given in Fig. 2.

![General model of blind channel estimation](image)

Fig. 2. General model of blind channel estimation [24].

A classification of blind estimation techniques can be found in [24].
The two main types of channel model used in blind estimation can be classified as multichannel and multirate systems [24]. The multichannel model of Fig. 3 (a) represents single-input P-output channel model. The *channel diversity* is often assumed, which implies that the FIR models of different channels have different zeros. This ensures unique identifiability of each channel as well as ability to construct inverse of the FIR model. The multichannel blind equalization for SIMO and MIMO systems was covered in [9] and [10], respectively, where sequential algorithms were proposed.

In the case of multirate model of Fig. 3 (b), if the input sequence $s_k$ is wide sense stationary, then $y_k$ is *wide sense cyclostationary* with period $P$. This property allows to estimate the channel using the second order statistics.

![Classification of channel models [24].](image)

The multirate model of Fig. 3 (b) basically translates to the requirement for *fractional spacing* of the equalizer taps. This means that the delay between the equalizer taps must be less than the symbol rate of the system. According to the model of Fig. 1, the received complex symbol $y(t)$ is given by

$$y(t) = \sum_{k=-\infty}^{\infty} s_k h(t - kT) + n(t) = x(t) + n(t),$$

where $s_k$ is an information symbol in a signal constellation with symbol interval $T$, $h(t)$ is the composite channel impulse response representing transmitter pulse shaping filter, the wireless multipath channel, and receiver filters. $n(t)$ represents the additive white noise. If the received signal is sampled at $f_s = \frac{1}{T} > \frac{1}{T}$, then equation (5) becomes

$$y(iT_s) = \sum_{k=-\infty}^{\infty} s_k h((i - k)T_s) + n(iT_s) = x(iT_s) + n(iT_s),$$

where

$$x(i) = \sum_{k=-\infty}^{\infty} s_k h((i - k)) = u(i) * h(i),$$

and

$$u(i) = \sum_{k=-\infty}^{\infty} s_k \delta(i - kT_s)$$

It can be shown that the power spectral density of $x(i)$ is given by

$$S_x(\omega, \nu) = \sum_{k} 2\pi H(\omega) H^* (\omega - \nu) \delta(\omega + \nu - \frac{2\pi k}{T_s})$$

If $S_x(\omega, \nu)$ is evaluated at $\nu = -\omega + \frac{2\pi k}{T_s}$, $k = 0, 1, \ldots$ we obtain

$$S_{x,k}(\omega) = 2\pi H(\omega) H^* (\omega - \frac{2\pi k}{T_s}), \quad k = 0, 1, \ldots$$
As can be seen from equation (10), the information regarding the phase of the channel is preserved. However, if $T_s = T$, the noiseless signal $x(i)$ is wide-sense stationary with power spectral density given by equation (10) with $k = 0$:

$$S_{x,k}(\omega) = 2\pi H(\omega)H^*(\omega) = 2\pi |H(\omega)|^2, \quad k = 0$$

Hence, the information about the phase characteristic of the channel is lost and cannot be recovered unless it is known a priori that the channel is either minimum phase or maximum phase [25].

C. Different Matrix Inversion Methods

Since the Wiener solution represents the MMSE solution to the channel estimation problem. The necessary part of deriving the tap weights of the equalizer is the computation of the inverse of the autocorrelation matrix of the received signal. The direct inverse computation is seldom appropriate due to its complexity. Hence, variety of indirect methods have been proposed and successfully employed in channel equalization applications. This section shall describe some of the most popular matrix inversion techniques.

**Cholesky Factorization** allows to factor the matrix inverse $R^{-1} = (HC, H^T + \sigma_n^2 I)^{-1}$ from equation (4) into the product of an upper triangular and a lower triangular matrices (that are Hermitian transpose of each other):

$$R^{-1} = L^T D^{-1} L = \left( D^{-1/2} L \right)^T \left( D^{-1/2} L \right),$$

where $D = \text{diag}(P_0, P_1, ..., P_M)$ and $P_i = \mathbb{E} [b_i(k)^2]$ = the average power of the $i^{th}$ backward prediction error $b_i(k)$, which is obtained via Gram-Schmidt orthogonalization given by

$$b_i(k) = Ly(k),$$

where $L$ is the lower triangular matrix with 1’s along the main diagonal. Hence, the determinant of $L$ is unity, which guarantees that this matrix is nonsingular. The nonzero elements of each row represent the weights of backward prediction-error filter of order corresponding to the row number. The prediction-error filter coefficients may be found recursively using Levinson-Durbin algorithm as was shown in [14]. Another example of the adaptive method based on Cholesky factorization was proposed in [13], where the Hermitian, Toeplitz properties of the channel correlation matrix were utilized to reduce complexity of the factorization method. These two methods were derived in the context of single-antenna GSM receiver. Yet, the Cholesky-based adaptive algorithms were shown to be applicable for channel equalization in MIMO HSDPA systems as well [18], which will be covered in more detail in Section III.

The matrix inversion techniques used in solving the Wiener equations are not limited to Cholesky factorization. For example, the use of *Conjugate Gradient* (CG) method in the context of MMSE equalizers has been investigated in [4] and [7]. One of the advantages of the CG algorithm is that it is guaranteed to converge in N steps (where N is the number of rows of the autocorrelation matrix, $R_{yy}$). Even faster convergence is attained when eigenvalues of $R_{yy}$ are clustered together. The computational efficiency of the CG method based on the fact that the algorithm does not need to compute the estimate of $R_{yy}^{-1}$ at every iteration but requires computation of only one matrix-vector product per iteration [4]. It uses the estimates of autocorrelation matrix and cross correlation vector as its inputs. The method lands itself well to both sample-by-sample processing (using forgetting factor, $\lambda$) as well as block processing. In the case of sample-by-sample processing, the inputs at the time of reception of $k^{th}$ data sample are

$$\hat{R}_{yy}[k] = \lambda \hat{R}_{yy}[k - 1] + y[k]y^H[k]$$

$$\hat{r}_{dy}[k] = \lambda \hat{r}_{dy}[k - 1] + d^*[k]y[k].$$

If the block processing (of block size $P$ is desired, the input to the algorithm at the m-th block are given by

$$\hat{R}_{yy}[m] = \lambda \hat{R}_{yy}[m - 1] + \sum_{i=(m-1)P}^{mP-1} y[i]y^H[i]$$

$$\hat{r}_{dy}[m] = \lambda \hat{r}_{dy}[m - 1] + \sum_{i=(m-1)P}^{mP-1} d^*[i]y[i].$$

The asymptotic performance of the CG algorithm was investigated in [3]. The results indicate that the this method is numerically stable and in steady-state behaves similarly to method of steepest descent.
III. APPLICATIONS OF MMSE CHANNEL EQUALIZERS

This section covers the classification of the MMSE equalizers according to their intended application.

A. Channel equalization for CDMA and WCDMA systems

List of reference: [15], [21], [12], [7] Terminology: the chip sequence in the context of a CDMA system is the data correlated with the PN spreading sequence. The symbol sequence is the data prior to being spread (at the transmitter) or after being despread (at the receiver) via the PN spreading sequence. The rate of the chip sequence is \( M \) times higher than that of the symbol sequence, where \( M \) is the spreading factor.

With increase in available computing power in the mobile receivers, the chip-level equalization has been gaining popularity. Its main advantage is that it allows to resolve multipath rays with relative arrival delays less than the chip interval. This is accomplished by oversampling the received signal at the rates equal to multiple of the chip rate of the system and using the fractionally-spaced equalizer taps [7], [19]. From the statistical perspective, the received data sequences at symbol level (after despreading), are non-cyclostationary due to application of the long scrambling code [6]. Finally, another advantage of chip-level equalization, is that it allows to model the received sampled signal as i.i.d. process [4]. Under this assumption, the LMMSE equalizer expression (4) reduces to

\[
\hat{s} = H^T \left( HH^T + \frac{\sigma_n^2}{\sigma_s^2} I \right)^{-1} y.
\] (18)

With increase in the number of user applications requiring the wireless data transfers, the optimization of the mobile receiver performance becomes more important. It has been shown that in most multipath fading channel conditions the MMSE equalizer-based receiver outperforms the rake-based receiver [7], [15], [21]. The gain due to channel equalization is especially significant at high SNR levels.

B. Channel equalization for OFDMA systems

The OFDM-based systems are widely used for high-speed wireless communications. Some their attractive features include its spectral efficiency and its robustness against multi-path fading due to elimination of ISI. The application of channel equalization for OFDM systems was investigated in [8], [22], and [23]. Denoting the received OFDM symbol \( y_i = [y_0,i, \ldots, y_{N-1},i]^T \) and corresponding transmitted symbol as \( s_i = [s_0,i, \ldots, s_{N-1},i]^T \), we can obtain the following channel model:

\[
y_i = Fx_i + n_i = FHu_i + n_i
\] (19)

The corresponding MMSE equalization matrix is then given by [23]

\[
\hat{s} = F^H H^H \left( HH^H + \frac{\sigma_n^2}{\sigma_s^2} I \right)^{-1}.
\] (20)

where \( F \) represents the inverse FFT. The simplified form of OFDM channel model is also depicted in Fig. 4.

![Simplified OFDM system model](image)

Fig. 4. Simplified OFDM system model [22].

Some of the typical tradeoffs involve embedding different types of redundancies in the data in order to improve the receiver performance. The detailed study of this topic was described in [22].
C. Channel equalization for MIMO systems

Recently, some of the existing commercial wireless data standards have included MIMO enhancements. This spurred more research devoted to finding computationally efficient equalization methods for MIMO channels. In the context of CDMA-based MIMO systems, the equalization is needed in order to restore the orthogonality of the spreading codes, which is degraded due to multipath propagation and code reuse by multiple transmit antennas. In the case of MIMO system with $N_T$ transmit antennas and $N_R$ receiver antennas, the convolution matrix of the channel is given by

$$
H = \begin{bmatrix}
H^{(1,1)} & \cdots & H^{(1,N_T)} \\
\vdots & \ddots & \vdots \\
H^{(N_R,1)} & \cdots & H^{(N_R,N_T)}
\end{bmatrix}
$$

(21)

where, assuming the impulse response of the length of individual propagation channel equal to $L_h$ and equalizer length equal to $L_f$, the channel matrix between $k^{th}$ transmit and the $m^{th}$ receive antenna is given by

$$
H^{(m,k)} = \begin{bmatrix}
h_0^{(m,k)} & \cdots & h_{L_h-1}^{(m,k)} & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & h_0^{(m,k)} & \cdots & h_{L_h-1}^{(m,k)}
\end{bmatrix}.
$$

(22)

Defining the stacked received signal vector at time instant $i$ as $\mathbf{r}_i = [\mathbf{r}_i^{(1)} T, \ldots, \mathbf{r}_i^{(N_R)} T]^T$, it is possible represent the overall model of the MIMO transmission channel in the form of Bayesian linear model similar to that of equation (3):

$$
\mathbf{r}_i = \mathbf{H} \mathbf{s}_i + \mathbf{n}_i.
$$

(23)

Hence, the LMMSE estimator of same form as that of equation (4) is required, where the matrix inversion can be facilitated by an iterative method based on the `Singular Value Decomposition` that was proposed in [18].

**References**


