ECE 534: Elements of Information Theory, Fall 2013
Homework 8
Out: October 23, Due: October 30, 2013

1) Problem 8.8 Consider an additive channel whose input alphabet $\mathcal{X} = \{0, \pm 1, \pm 2\}$ and whose output $Y = X + Z$, where Z is distributed uniformly over the interval $[-1, 1]$. Thus, the input of the channel is a discrete random variable, whereas the output is continuous. Calculate the capacity $C = \max_{p(x)} I(X;Y)$ of this channel.

2) Problem 8.9 Suppose (X, Y, Z) are jointly Gaussian and that $X \to Y \to Z$ forms a Markov chain. Let X and Y have correlation coefficient ρ_1 and let Y and Z have correlation coefficient ρ_2. Find $I(X;Z)$.

3) What is the maximum entropy distribution $p(x, y)$ that has the following marginals? Hint: you may wish to guess and verify a more general result.

		y: 1	2	3	
	1	p_{11}	p_{12}	p_{13}	$\frac{1}{2}$
x	2	p_{21}	p_{22}	p_{23}	$\frac{1}{4}$
	3	p_{31}	p_{32}	p_{33}	$\frac{1}{4}$
		$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	

4) Let $Y = X_1 + X_2$. Find the maximum entropy of Y under the constraint $E[X_1^2] = P_1$ and $E[X_2^2] = P_2$ if

a) X_1 and X_2 are independent

b) X_1 and X_2 are allowed to be dependent.