ECE 534: Elements of Information Theory, Fall 2013
Homework 2

Out: September 4, 2013
Due: September 11, 2013

(Problem 2.23) Consider a sequence of n binary random variables $X_1, X_2, \cdots X_n$. Each sequence with an even number of 1's has probability $2^{(n-1)}$, and each sequence with an odd number of 1's has probability 0. Find the mutual informations

$$I(X_1; X_2), \quad I(X_2; X_3|X_1), \quad \ldots, \quad I(X_{n-1}; X_n|X_1, X_2, \cdots X_{n-2})$$

(Problem 2.29) *Inequalities.* Let X, Y and Z be joint random variables. Prove the following inequalities and find conditions for equality.

(a) $H(X, Y|Z) \geq H(X|Z)$

(b) $I(X, Y; Z) \geq I(X; Z)$

(c) $H(X, Y, Z) - H(X, Y) \leq H(X, Z) - H(X)$

(d) $I(X; Z|Y) \geq I(Z; Y|X) - I(Z; Y) + I(X; Z)$

(Problem 2.37) *Relative entropy.* Let X, Y, Z be three random variables with a joint probability mass function $p(x, y, z)$. The relative entropy between the joint distribution and the product of the marginals is

$$D(p(x, y, z)||p(x)p(y)p(z)) = E\left[\log \frac{p(x, y, z)}{p(x)p(y)p(z)}\right]$$

Expand this in terms of entropies. When is this quantity zero?

(Problem 2.42) *Inequalities.* Which of the following inequalities are generally $\geq$, $=$, $\leq$? Label each with $\geq$, $=$ or $\leq$.

(a) $H(5X)$ vs.$H(X)$

(b) $I(g(X); Y)$ vs.$I(X; Y)$

(c) $H(X_0, X_{-1})$ vs.$H(X_0|X_{-1}, X_1)$

(d) $H(X,Y)/(H(X)+H(Y))$ vs.1

(Problem 2.43) *Mutual information of heads and tails.*

(a) Consider a fair coin flip. What is the mutual information between the top and bottom sides of the coin?

(b) A six-sided fair die is rolled. What is the mutual information between the top side and the front face (the side most facing you)?