ECE 534: Elements of Information Theory, Fall 2013
Homework 10
Out: November 13, Due: November 20, 2013

Problem 1 (9.14)

Additive noise channel. Consider the channel $Y = X + Z$, where X is the transmitted signal with power constraint P, Z is independent additive noise, and Y is the received signal. Let

$$Z = \begin{cases} 0 & \text{with probability } \frac{1}{10} \\ Z^* & \text{with probability } \frac{9}{10} \end{cases}$$

where $Z^* \sim \mathcal{N}(0, N)$. Thus, Z has a mixture distribution that is the mixture of a Gaussian distribution and a degenerate distribution with mass 1 at 0.

(a) What is the capacity of this channel? This should be a pleasant surprise.

(b) How would you signal to achieve capacity?

Problem 2 (9.16)

Gaussian mutual information. Suppose that (X, Y, Z, W) are jointly Gaussian and that $X \to Y \to Z \to W$ forms a Markov chain. Let X and Y have correlation coefficient ρ_1 and le t Y and Z have correlation coefficient ρ_2. Find $I(X; Z)$.

Problem 3 (10.1)

One-bit quantization of a single Gaussian random variable. Let $X \sim \mathcal{N}(0, \sigma^2)$ and let the distortion measure be squared error. Here we do not allow block descriptions. Show that the optimum reproduction points for 1-bit quantization are $\pm\sqrt{\frac{2}{\pi}}\sigma$ and that the expected distortion for 1-bit quantization is $\frac{\pi-2}{\pi}\sigma^2$. Compare this with the distortion rate bound $D = \sigma^2 2^{-2R}$ for $R = 1$.

Problem 4 (10.5)

Rate distortion for uniform source with Hamming distortion. Consider a source X uniformly distributed on the set $\{1, 2, \ldots, m\}$. Find the rate distortion function for this source with Hamming distortion; that is,

$$d(x, \hat{x}) = \begin{cases} 0 & \text{if } x = \hat{x} \\ 1 & \text{if } x \neq \hat{x} \end{cases}$$