

ECE 341: Probability and Random Processes for Engineers, Spring 2012

Homework 8

Name:

Assigned: 02.29.2012

Due: 03.07.2012

Problem 1. Suppose (X, Y) is uniformly distributed (continuous) over the unit circle, i.e. $S_{XY} = \{(x, y) : x^2 + y^2 \leq 1\}$.

1. What is the joint pdf $f_{X,Y}(x, y)$?
2. Find $P[A]$ if A is the event $A = \{(u, v) : u \geq 0, v \geq 0\}$.
3. Find $P[X^2 + Y^2 \leq r^2]$ for $r \geq 0$.
4. Find the marginal pdf of X .
5. Find the conditional pdf of Y given X .

Solution 1:

Problem 2. Suppose one fair die (with six equi-probable sides) is rolled. Let

$$X = \begin{cases} 1 & \text{if "one" shows} \\ 0 & \text{else} \end{cases} \quad Y = \begin{cases} 1 & \text{if "two" shows} \\ 0 & \text{else} \end{cases}$$

1. Find the marginal pmfs of X and Y .
2. Find $E[X], E[Y], \text{Var}(X), \text{Var}(Y)$.
3. Find the joint pmf of (X, Y) , $P_{X,Y}(x, y)$.
4. Find $\text{Cov}(X, Y)$.
5. Find the correlation coefficient between X and Y .

Solution 2:

Problem 3. Let $Z = \frac{Y}{X^2}$ where X, Y have the joint pdf given by

$$f_{X,Y}(x, y) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases} .$$

1. Find $P[Z \leq 0.5]$.
2. Find $P[Z \leq 4]$.

Solution 3:

Problem 4. Suppose $W = \max(X, Y)$, where X and Y are independent, continuous-type random variables. Express the pdf of W in terms of the pdfs of X and Y . (*HINT: First find CDF then differentiate*)

Solution 4: